Pre-Calculus – 1st Semester Summary Sheet

Exponents

If all bases are nonzero:

$u^m u^n = u^{m+n}$ $\quad u^0 = 1$

$(uv)^m = u^m v^m$ $\quad (u^m)^n = u^{mn}$

$u^{-n} = \frac{1}{u^n}$ $\quad \frac{u^m}{u^n} = u^{m-n}$

$\left(\frac{u}{v}\right)^m = \frac{u^m}{v^m}$

Radicals & Rational Exponents

If all roots are real numbers…

$\sqrt[n]{uv} = \sqrt[n]{u} \cdot \sqrt[n]{v}$ $\quad \left(\sqrt[n]{u}\right)^n = u$

$\sqrt[m]{\sqrt[n]{u}} = \sqrt[mn]{u}$ $\quad \sqrt[n]{u^m} = \left(\sqrt[n]{u}\right)^m$

$\sqrt[n]{\frac{u}{v}} = \frac{\sqrt[n]{u}}{\sqrt[n]{v}}\ (v \neq 0)$ $\quad \sqrt[n]{u^n} = \begin{cases} |u| & n \text{ even} \\ u & n \text{ odd} \end{cases}$

$u^{1/n} = \sqrt[n]{u}$ $\quad u^{m/n} = \left(u^{1/n}\right)^m = \left(\sqrt[n]{u}\right)^m$

$u^{m/n} = (u^m)^{1/n} = \sqrt[n]{u^m}$

Special Products

$(u+v)(u-v) = u^2 - v^2$

$(u+v)^2 = u^2 + 2uv + v^2$

$(u-v)^2 = u^2 - 2uv + v^2$

$(u+v)^3 = u^3 + 3u^2v + 3uv^2 + v^3$

$(u-v)^3 = u^3 - 3u^2v + 3uv^2 - v^3$

Factoring Polynomials

$u^2 - v^2 = (u+v)(u-v)$

$u^2 + 2uv + v^2 = (u+v)^2$

$u^2 - 2uv + v^2 = (u-v)^2$

$u^3 + v^3 = (u+v)(u^2 - uv + v^2)$

$u^3 - v^3 = (u-v)(u^2 + uv + v^2)$

Quadratic Formula

If $a \neq 0$, the solutions of the equation $ax^2 + bx + c = 0$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

"De-Foiling"

1. *Multiply first & last terms*
2. *Find factors that give you #1 & combine to be the middle term*
3. *Replace the middle term w/these factors*
4. *Factor by grouping*

Coordinate Geometry

Given two points: $P(x_1, y_1)$ & $Q(x_2, y_2)$

Distance $\quad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint $\quad (x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Slope $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$

Equations of Lines

The point-slope form, slope *m* and through (x_1, y_1):

$$y - y_1 = m(x - x_1)$$

The slope-intercept form, slope *m* & *y*-int. *b*:

$$y = mx + b$$

Condition for parallel lines: $m_1 = m_2$

Condition for perpendicular lines: $m_2 = \frac{-1}{m_1}$

Logarithms

If $0 < b \neq 1$, $0 < a \neq 1$, x, R, S, > 0, then…

$y = \log_b x$ if & only if $b^y = x$

$\log_b 1 = 0$ $\quad \log_b b = 1$

$\log_b b^y = y$ $\quad b^{\log_b x} = x$

$\log_b RS = \log_b R + \log_b S$

$\log_b \frac{R}{S} = \log_b R - \log_b S$

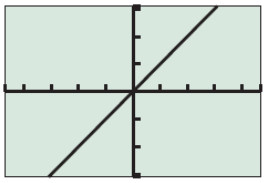
$\log_b R^c = c \log_b R$ $\quad \log_b x = \frac{\log_a x}{\log_a b}$

First Semester Functions

Let *a*, *b*, *c*, *k* & *m* be constants…

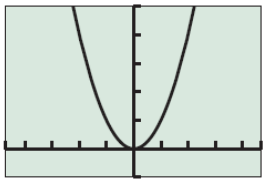
Linear	$f(x) = mx + b, m \neq 0$
Quadratic	$f(x) = ax^2 + bx + c, a \neq 0$ $f(x) = a(x-h)^2 + k$
Power	$f(x) = k \cdot x^a$
Rational	$r(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$
Exponential	$f(x) = a \cdot b^x, a \neq 0, b > 0, b \neq 1$
Logistic	$f(x) = \frac{c}{1 + a \cdot b^x}$

Library of Functions



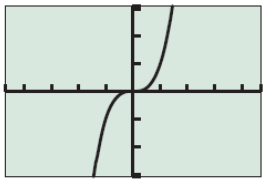
[–4.7, 4.7] by [–3.1, 3.1]

Identity Function
$f(x) = x$
Domain $= (-\infty, \infty)$
Range $= (-\infty, \infty)$



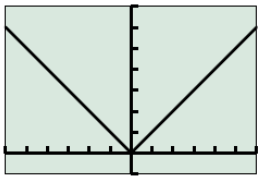
[–4.7, 4.7] by [–1, 5]

Squaring Function
$f(x) = x^2$
Domain $= (-\infty, \infty)$
Range $= [0, \infty)$



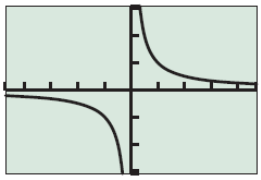
[–4.7, 4.7] by [–3.1, 3.1]

Cubing Function
$f(x) = x^3$
Domain $= (-\infty, \infty)$
Range $= (-\infty, \infty)$



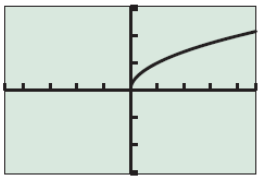
[–6, 6] by [–1, 7]

Absolute Value Function
$f(x) = |x| = \text{abs}\,(x)$
Domain $= (-\infty, \infty)$
Range $= [0, \infty)$



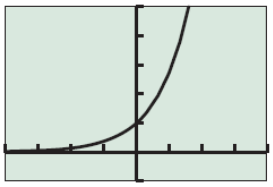
[–4.7, 4.7] by [–3.1, 3.1]

Reciprocal Function
$f(x) = \dfrac{1}{x}$
Domain $= (-\infty, 0) \cup (0, \infty)$
Range $= (-\infty, 0) \cup (0, \infty)$



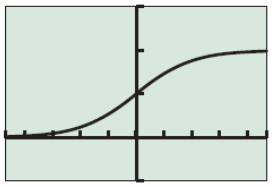
[–4.7, 4.7] by [–3.1, 3.1]

Square Root Function
$f(x) = \sqrt{x}$
Domain $= [0, \infty)$
Range $= [0, \infty)$



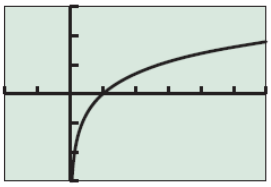
[–4, 4] by [–1, 5]

Exponential Function
$f(x) = e^x$
Domain $= (-\infty, \infty)$
Range $= (0, \infty)$



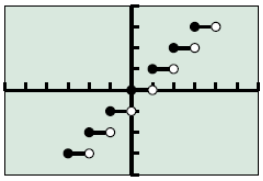
[–4.7, 4.7] by [–0.5, 1.5]

Logistic Function
$f(x) = \dfrac{1}{1 + e^{-x}}$
Domain $= (-\infty, \infty)$
Range $= (0, 1)$



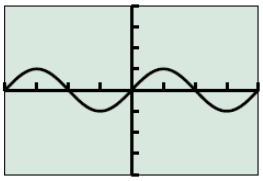
[–2, 6] by [–3, 3]

Natural Logarithmic Function
$f(x) = \ln x$
Domain $= (0, \infty)$
Range $= (-\infty, \infty)$



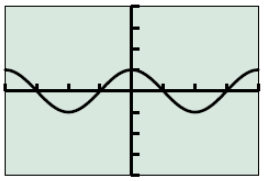
[–6, 6] by [–4, 4]

Greatest Integer Function
$f(x) = \text{int}\,(x)$
Domain $= (-\infty, \infty)$
Range = all integers



[–2π, 2π] by [–4, 4]

Sine Function
$f(x) = \sin(x)$
Domain $= (-\infty, \infty)$
Range $= [-1, 1]$



[–2π, 2π] by [–4, 4]

Cosine Function
$f(x) = \cos(x)$
Domain $= (-\infty, \infty)$
Range $= [-1, 1]$