

### 5.1.D1 ~ Higher-Order Polynomial Functions

Numerical representations of either a linear or quadratic function are shown in a table. Find successive rates of change to determine if the function is linear, quadratic, or cubic. Identify intervals where the function is increasing and/or decreasing and concave up, concave down, or neither.

1.  $f(x)$

| $x$ | $f(x)$ | FIRST DIFFERENCES | SECOND DIFFERENCES | THIRD DIFFERENCES |
|-----|--------|-------------------|--------------------|-------------------|
| -4  | -48    |                   |                    |                   |
| -3  | -15    | _____             |                    |                   |
| -2  | 0      | _____             |                    |                   |
| -1  | 3      | _____             |                    |                   |
| 0   | 0      |                   |                    |                   |

Linear / Quadratic / Cubic

Increasing:  $\_\_\_ \leq x \leq \_\_\_$

Decreasing:  $\_\_\_ \leq x \leq \_\_\_$

Concave Up

Concave Down

Neither

2.  $g(x)$

| $x$ | $g(x)$ | FIRST DIFFERENCES | SECOND DIFFERENCES | THIRD DIFFERENCES |
|-----|--------|-------------------|--------------------|-------------------|
| 1   | 9      |                   |                    |                   |
| 2   | 16     | _____             |                    |                   |
| 3   | 21     | _____             |                    |                   |
| 4   | 24     | _____             |                    |                   |
| 5   | 25     |                   |                    |                   |

Linear / Quadratic / Cubic

Increasing:  $\_\_\_ \leq x \leq \_\_\_$

Decreasing:  $\_\_\_ \leq x \leq \_\_\_$

Concave Up

Concave Down

Neither

3.  $g(x)$

| $x$ | $g(x)$ | FIRST DIFFERENCES | SECOND DIFFERENCES | THIRD DIFFERENCES |
|-----|--------|-------------------|--------------------|-------------------|
| 1   | 5      |                   |                    |                   |
| 2   | 15     | _____             |                    |                   |
| 3   | 25     | _____             |                    |                   |
| 4   | 35     | _____             |                    |                   |
| 5   | 45     |                   |                    |                   |

Linear / Quadratic / Cubic

Increasing:  $\_\_\_ \leq x \leq \_\_\_$

Decreasing:  $\_\_\_ \leq x \leq \_\_\_$

Concave Up

Concave Down

Neither

4.  $h(x)$

| $x$ | $g(x)$ | FIRST DIFFERENCES | SECOND DIFFERENCES | THIRD DIFFERENCES |
|-----|--------|-------------------|--------------------|-------------------|
| 1   | 0      |                   |                    |                   |
| 2   | -6     | _____             |                    |                   |
| 3   | -8     | _____             |                    |                   |
| 4   | 0      | _____             |                    |                   |
| 5   | 24     |                   |                    |                   |

Linear / Quadratic / Cubic

Increasing:  $\_\_\_ \leq x \leq \_\_\_$

Decreasing:  $\_\_\_ \leq x \leq \_\_\_$

Concave Up

Concave Down

Neither

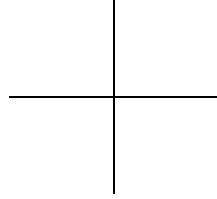
Sketch the graph of a polynomial function that has the given characteristics.

5. Third degree

As  $x \rightarrow \infty, f(x) \rightarrow -\infty$

1 maximum

1 minimum



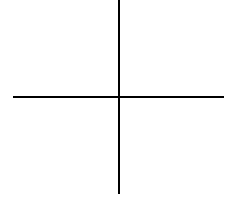
6. Fourth degree

As  $x \rightarrow \infty, f(x) \rightarrow -\infty$

2 maximums

1 minimum

$f(x) = 0$  exactly twice

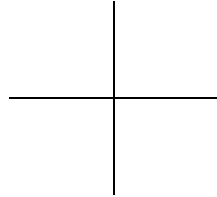


7. Fourth degree

A zero @ 3

Maximum @  $x = 2$

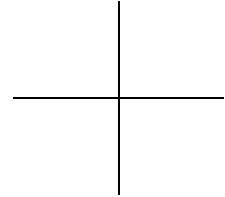
Minimum @  $x = -1$



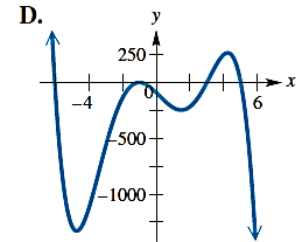
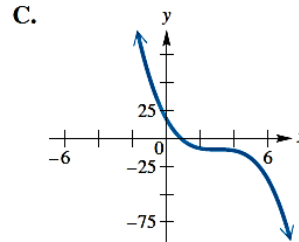
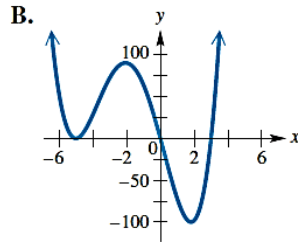
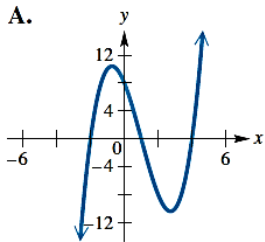
8. Fifth degree

Zeros @ -4, -1, & 3

Maximum @  $x = -2$



Problems 9 – 17, use graphs A – D:



9. Which graph(s) are that of an odd-degree polynomial function?
10. Which one of the graphs shows a polynomial function with no maximum or minimum values?
11. Which one of the graphs is that of a function whose range is *not*  $(-\infty, \infty)$ ?
12. Which one of the graphs has the most turning points/local extrema?
13. Which graph(s) have an end behavior of  $\lim_{x \rightarrow -\infty} f(x) = -\infty$ ?
14. Which graphs have an equal amount of  $x$ -intercepts?
15. Which one of the graphs shows that  $f(x)$  is a polynomial function with  $f(x) = 0$  at exactly three different values of  $x$ , and  $f(x) \rightarrow \infty$  as  $x \rightarrow \pm\infty$ ?
16. Which graphs have only one inflection point?
17. Recall that the graph of a polynomial function of degree  $n$  will have at most  $n - 1$  turning points (local extrema). What is the degree of graph D?