 Name: ____________________

5.3.D1 – The Behavior of Rational Functions

For each rational function, compare the degrees of the numerator and denominator and then find the end behavior and the horizontal asymptote.

1. $f(x) = \dfrac{-x^3}{3x+5}$

$\lim_{x\to-\infty} f(x) =$ $\lim_{x\to\infty} f(x) =$ Horizontal asymptote: $y =$

2. $f(x) = \dfrac{x^7 + 2x^5 + x^2 + 9}{3x^7 + 5x^4 + 24}$

$\lim_{x\to-\infty} f(x) =$ $\lim_{x\to\infty} f(x) =$ Horizontal asymptote: $y =$

3. $f(x) = \dfrac{8x^2 + 6}{3x^4 + 1}$

$\lim_{x\to-\infty} f(x) =$ $\lim_{x\to\infty} f(x) =$ Horizontal asymptote: $y =$

4. $f(x) = \dfrac{-2x^5}{3x^2 + 4}$

$\lim_{x\to-\infty} f(x) =$ $\lim_{x\to\infty} f(x) =$ Horizontal asymptote: $y =$

5. $f(x) = \dfrac{-2x^2}{x^2 - 9}$

$\lim_{x\to-\infty} f(x) =$ $\lim_{x\to\infty} f(x) =$ Horizontal asymptote: $y =$

6. $f(x) = \dfrac{x-1}{x^2 + 2x + 2}$

$\lim_{x\to-\infty} f(x) =$ $\lim_{x\to\infty} f(x) =$ Horizontal asymptote: $y =$

7. $f(x) = \dfrac{2x^3}{3x^2 - 1}$

$\lim_{x\to-\infty} f(x) =$ $\lim_{x\to\infty} f(x) =$ Horizontal asymptote: $y =$

8. $f(x) = \dfrac{-12}{x^2 + 4}$

$\lim_{x\to-\infty} f(x) =$ $\lim_{x\to\infty} f(x) =$ Horizontal asymptote: $y =$

Analyze each rational function for its long-run behavior (end behavior and horizontal asymptote) and its short-run behavior (intercepts, vertical asymptote, and holes). *Write DNE if the function doesn't have a particular property.*

9.
$f(x) = \dfrac{x-4}{-4x-16} = \dfrac{x-4}{-4(x+4)}$

$\lim_{x\to-\infty} f(x) =$	$\lim_{x\to\infty} f(x) =$	Horizontal asymptote: $y =$	y-intercept:
Vertical asymptote: $x =$	x-intercept:	Hole:	Domain: $x \neq$

10.
$f(x) = \dfrac{2x+4}{x-1} = \dfrac{2(x+2)}{x-1}$

$\lim_{x\to-\infty} f(x) =$	$\lim_{x\to\infty} f(x) =$	Horizontal asymptote: $y =$	y-intercept:
Vertical asymptote: $x =$	x-intercept:	Hole:	Domain: $x \neq$

11.
$f(x) = \dfrac{x+2}{x^2-4} = \dfrac{x+2}{(x+2)(x-2)}$

$\lim_{x\to-\infty} f(x) =$	$\lim_{x\to\infty} f(x) =$	Horizontal asymptote: $y =$	y-intercept:
Vertical asymptote: $x =$	x-intercept:	Hole:	Domain: $x \neq$

12.
$$f(x) = \frac{3x^2 - 12x}{x^2 - 2x - 3} = \frac{3x(x-4)}{(x-3)(x+1)}$$

$\lim_{x\to-\infty} f(x) =$	$\lim_{x\to\infty} f(x) =$	Horizontal asymptote: $y =$	y-intercept:
Vertical asymptote: $x =$	x-intercept:	Hole:	Domain: $x \neq$

13.
$$f(x) = \frac{x^2 + 2x - 3}{-3x - 6} = \frac{(x+3)(x-1)}{-3(x+2)}$$

$\lim_{x\to-\infty} f(x) =$	$\lim_{x\to\infty} f(x) =$	Horizontal asymptote: $y =$	y-intercept:
Vertical asymptote: $x =$	x-intercept:	Hole:	Domain: $x \neq$

14.
$$f(x) = \frac{2x^2 - 12x + 16}{x^2 - x - 12} = \frac{2(x-4)(x-2)}{(x-4)(x+3)}$$

$\lim_{x\to-\infty} f(x) =$	$\lim_{x\to\infty} f(x) =$	Horizontal asymptote: $y =$	y-intercept:
Vertical asymptote: $x =$	x-intercept:	Hole:	Domain: $x \neq$

15.
$$f(x) = \frac{x^2 + x - 6}{4x^2 + 16x + 12} = \frac{(x+3)(x-2)}{4(x+1)(x+3)}$$

$\lim_{x\to-\infty} f(x) =$	$\lim_{x\to\infty} f(x) =$	Horizontal asymptote: $y =$	y-intercept:
Vertical asymptote: $x =$	x-intercept:	Hole:	Domain: $x \neq$

16.
$$f(x) = \frac{x^2 + 5x}{x^2 + 7x + 10} = \frac{x(x+5)}{(x+5)(x+2)}$$

$\lim_{x\to-\infty} f(x) =$	$\lim_{x\to\infty} f(x) =$	Horizontal asymptote: $y =$	y-intercept:
Vertical asymptote: $x =$	x-intercept:	Hole:	Domain: $x \neq$

17.
$$f(x) = \frac{x^2 - x - 6}{x^3 - 3x - 2} = \frac{(x+2)(x-3)}{(x+1)^2(x-2)}$$

$\lim_{x\to-\infty} f(x) =$	$\lim_{x\to\infty} f(x) =$	Horizontal asymptote: $y =$	y-intercept:
Vertical asymptote: $x =$	x-intercept:	Hole:	Domain: $x \neq$

18.
$$f(x) = \frac{x^3 + 2x^2 - 8x}{-4x^2 + 12x} = \frac{x(x+4)(x-2)}{-4x(x-3)}$$

$\lim_{x\to-\infty} f(x) =$	$\lim_{x\to\infty} f(x) =$	Horizontal asymptote: $y =$	y-intercept:
Vertical asymptote: $x =$	x-intercept:	Hole:	Domain: $x \neq$