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## 5.REV. 4 - Rational Functions

$\qquad$ Period: $\qquad$
For each rational function, compare the degrees of the numerator and denominator and then find the end behavior and the horizontal asymptote.

1. $f(x)=\frac{4 x+3 x^{3}}{4 x^{2}+3 x}$

$$
\lim _{x \rightarrow-\infty} f(x)=\quad \lim _{x \rightarrow \infty} f(x)=
$$

Horizontal asymptote:
$y=$
2. $f(x)=\frac{3 x^{2}+x}{2 x^{2}+5 x^{3}}$

$$
\lim _{x \rightarrow-\infty} f(x)=\quad \lim _{x \rightarrow \infty} f(x)=
$$

Horizontal asymptote:
4. $f(x)=\frac{-x^{2}}{x+5}$

Horizontal asymptote:
$\lim _{x \rightarrow-\infty} f(x)=\quad \lim _{x \rightarrow \infty} f(x)=$
$y=$

$$
\lim _{x \rightarrow-\infty} f(x)=\quad \lim _{x \rightarrow \infty} f(x)=
$$

Horizontal asymptote:
6. $f(x)=\frac{2 x-4 x^{2}}{x^{2}-4 x+8}$

$$
\lim _{x \rightarrow-\infty} f(x)=\quad \lim _{x \rightarrow \infty} f(x)=\quad \begin{aligned}
& \text { Horizo } \\
& y=
\end{aligned}
$$

Write the rational function in its factored form. Analyze each rational function for its long-run behavior (end behavior and horizontal asymptote) and its short-run behavior (intercepts, vertical asymptote, and holes). Write DNE if the function doesn't have a particular property.
7.
$f(x)=\frac{-4 x+12}{x^{2}-x-6}$

| $\lim _{x \rightarrow-\infty} f(x)=$ | $\lim _{x \rightarrow \infty} f(x)=$ | Horizontal <br> asymptote: <br> $y=$ | $y$-intercept: |
| :--- | :--- | :--- | :--- |
| Vertical asymptote: | $x$-intercept: | Hole: | Domain: <br> $x=$ |
|  |  |  |  |

8. 

$f(x)=\frac{3 x^{2}-3 x-6}{x^{2}-2 x-3}$

| $\lim _{x \rightarrow-\infty} f(x)=$ | $\lim _{x \rightarrow \infty} f(x)=$ | Horizontal <br> asymptote: <br> $y=$ | $y$-intercept: |
| :--- | :--- | :--- | :--- |
| Vertical asymptote: <br> $x=$ | $x$-intercept: | Hole: | Domain: <br> $x \neq$ |
|  |  |  |  |

Find a possible formula for the rational function.
9.

10.

11. The graph crosses the $x$-axis at 2 , touches the $x$ axis at -1 ; vertical asymptotes at $x=-5 \& x=6$; a horizontal asymptote at $y=3$; the $y$-intercept is ( $0,0.04$ ).
12. There is a hole at $x=2$, a zero at $x=3$, a vertical asymptote at $x=-1$, and a horizontal asymptote at $y=1$.

Write the rational function in its factored form. Then analyze the rational function for its long-run behavior (horizontal asymptote) and its short-run behavior (intercepts, vertical asymptote, and holes). Write DNE if the function doesn't have a particular property. Lastly, sketch its graph.
13. $f(x)=\frac{-2 x-2}{x-3}$

| Horizontal asymptote $y=$ | 4-intercept: | Domain: $x \neq$ |
| :---: | :---: | :---: |
| Hole: | $x$-intercept(s): | vertical asymptote: $x=$ |


15. $f(x)=\frac{3 x^{2}+6 x-24}{3 x^{2}-9 x+6}$

| Horizontal asymptote $y=$ | 4-intercept: | Domain: $x \neq$ |
| :---: | :---: | :---: |
| Hole: | $x$-intercept(s): | Vertical asymptote: $x=$ |


14. $f(x)=\frac{x^{2}-4 x}{x^{2}-3 x}$

| Horizontal asymptote $y=$ | y-intercept: | Domain: $x \neq$ |
| :---: | :---: | :---: |
| Hole: | $x$-intercept(s): | Vertical asymptote: $x=$ |


16. $f(x)=\frac{2 x^{3}-18 x}{x^{3}+x^{2}-6 x}$

| Horizontal asymptote | y-intercept: | Domain: |
| :--- | :--- | :--- |
| Hole: | $x \neq$ |  |
|  | $x$-intercept(s): |  |
|  |  | $x=$ |



