Unit 2: Understanding Functions

2.1 Pre-Algebra Review – VARIABLE EXPRESSIONS

- A numerical expression consists of numbers and operations.
- A variable is a letter used to represent one or more numbers.
- A variable expression consists of numbers, variables, and operations.
- To evaluate a variable expression, substitute a number for each variable and simplify the resulting numerical expression.
    - Follow the order of operations!

Order of Operations

1. Evaluate expressions inside grouping symbols.
2. Evaluate powers.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.

Examples

Evaluate the variable expression when $x = 4$ & $y = -2$. Remember to use the order of operations!

1. $3x - 2y$

2. $x^2 - y$

3. $2(x - y)^2$

4. $x \div y + y^2 - 6$

5. $(x + y - 5) \times 3$

6. $1 - \big(x + (6y \div 3)\big)$

Summary:

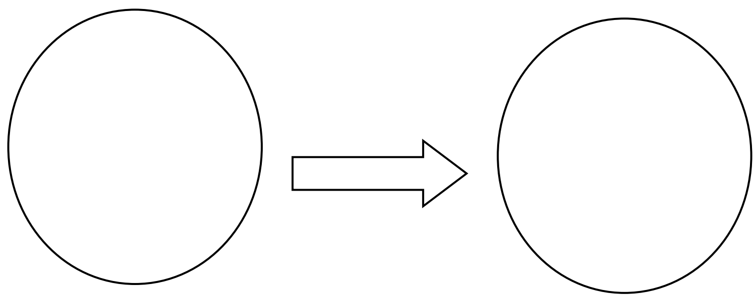
2.1 What is a Function? (Day 1)

Main Idea: Interpreting Functions
Objectives: Recognize Functions

- Understand that a function is a relation in which every input (domain) has exactly one output (range)
- Determine if a graph, table, or set of ordered pairs represents a function

What class do you have second hour?

Domain Range

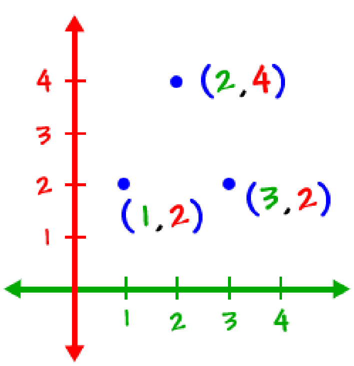


THINK ABOUT THIS. . .

- Is it possible for each student to go to a different 2nd hour?
  YES / NO
- Is it possible for two or more students to go to the same 2nd hour?
  YES / NO
- Is it possible for the one student to go to two different 2nd hours?
  YES / NO

~~~~~~

- Relations
  - A relation is a ________________________________
    - Domain – The set of the _________ coordinates of the ordered pairs
      - Input, independent variable, *x*-coordinates, *x*
    - Range – The set of _____________ coordinates
      - Output, dependent variable, *y*-coordinates, *f* (*x*)



This is a graph of the relation.

$\{(1,2),(2,4),(3,2)\}$

WHAT IS THE DOMAIN OF THIS RELATION?

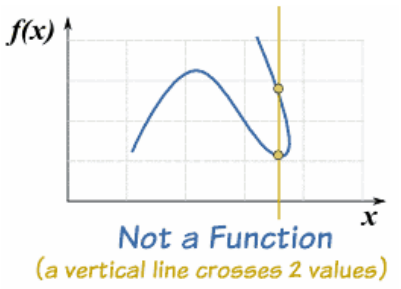
WHAT IS THE RANGE?
~~~~~~

- Functions
  - A function is _____
    _____
    _____
  - How can a function be represented?
    - Verbally (by a description in words)
    - Numerically (by a set of ordered pair or a table of values)
    - Visually (by a graph or a mapping diagram)
    - Algebraically (by an explicit formula – a function rule)

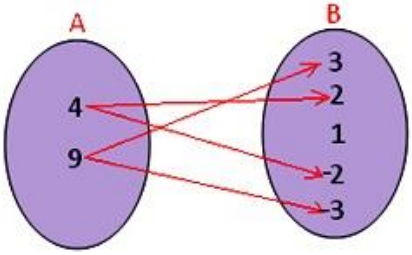
VirtualNerd

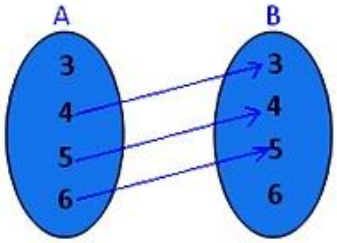
http://www.virtualnerd.com/algebra-1/relations-functions/functions/function-notation/function-definition

- Is it a Function?
  - Given a graph…
    - Does it pass the vertical line test?
      - If any vertical line passes through more than one point of the graph, the relation is not a function.
  - Given a relation or a table of values…
    - Do any *x*–values repeat?
      - Yes – Not a function
      - No – Function!
  - Given a mapping diagram…
    - Functions have no more than one arrow starting from each value in the domain.
      - List the domain values and the range values in order.
      - Draw arrows from the domain values to their range values.



WHICH ONE OF THESE MAPPING DIAGRAMS SHOWS A FUNCTION?





Summary:

Examples

*State whether the following are **functions**. Explain your reasoning.*
*If yes, then define the values in the **domain** and **range.***

1. {(3, 5), (4, 0), (6, 7), (-3, 5)}
   a. Function? YES NO
   b. Explanation:
   c. Domain:
   d. Range:

2. {(6, 4), (1, 8), (6, 4), (2, 0)}
   a. Function? YES NO
   b. Explanation:
   c. Domain:
   d. Range:

3. {(-5, 5), (-2, 0), (-2, 7), (11, 6)}
   a. Function? YES NO
   b. Explanation:
   c. Domain:
   d. Range:

4. {(-10, 4), (-5, 4), (1, 4), (6, 0)}
   a. Function? YES NO
   b. Explanation:
   c. Domain:
   d. Range:

5.

x	***y***
4	0
6	-2
-3	5

   a. Function? YES NO
   b. Explanation:
   c. Domain:
   d. Range:

6.

x	9	6	5	9
y	2	-1	3	0

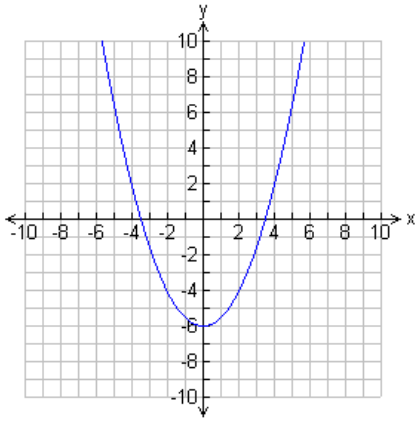
   a. Function? YES NO
   b. Explanation:
   c. Domain:
   d. Range:

7.

x	2	2	3	3
y	0	0	1	1

   a. Function? YES NO
   b. Explanation:
   c. Domain:
   d. Range:

8.



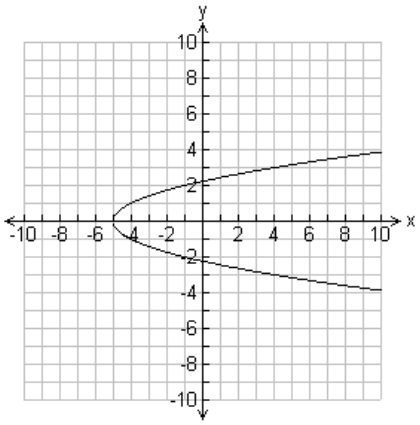
a. Function? YES NO

b. Explanation:

c. Domain:

d. Range:

9.



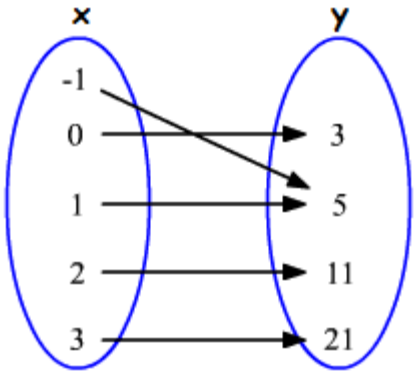
a. Function? YES NO

b. Explanation:

c. Domain:

d. Range:

10.



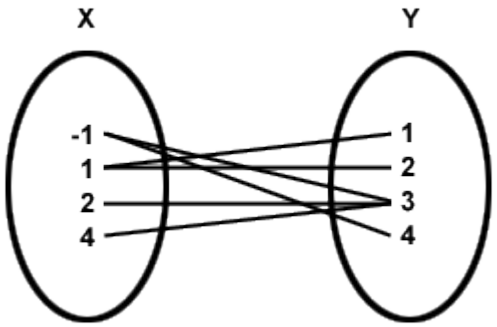
a. Function? YES NO

b. Explanation:

c. Domain:

d. Range:

11.



a. Function? YES NO

b. Explanation:

c. Domain:

d. Range:

2.1 Pre-Algebra Review – VERBAL EXPRESSIONS

- A verbal model describes a problem using words as labels and using math symbols to relate the words.

Common Words and Phrases that Indicate Operations			
Addition	**Subtraction**	**Multiplication**	**Division**
plus the sum of increased by total more than added to	minus the difference of decreased by fewer than less than subtracted from	times the product of multiplied by of	divided by divided into the quotient of

Examples
Write a variable expression to represent the phrase.

1. The product of 72 and a number
2. The difference of a number and 1
3. 13 more than a number
4. The sum of a number and 9.4
5. The quotient of a number and 3
6. 4 less than a number

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2.1 What is a Function? (Day 2)

- Describe what a function is in your own words. How do we tell whether a relationship is a function or not?

- What is a relation? How can a relation be represented?
~~~~~~

- Are all relations functions? Are all functions relations? Explain. (A non-example works if the answer is no.)

- What other names can we use to refer to the domain? The range?

  Domain:

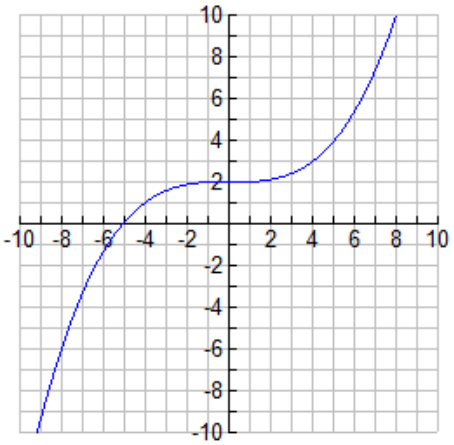
  Range:

Examples

State whether the following are functions. Provide an explanation.

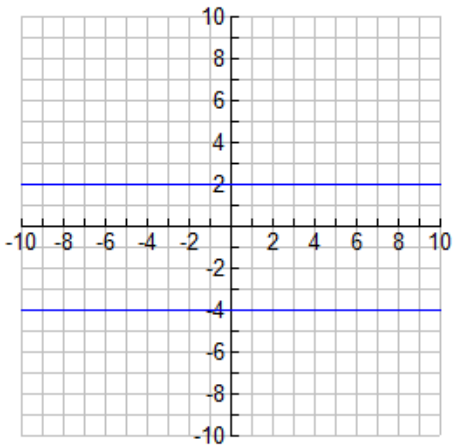
1. YES NO

   Explanation:



2. YES NO

   Explanation:



3. Two students were discussing whether or not a graph is a function. One of them remembered he heard someone mention that the graph has to pass The Vertical Line Test in order to be a function. How could something called The Vertical Line Test tell whether a relationship is a function or not?

Given a scenario: (a) identify the domain and range, (b) write a relation for the domain and range, and (c) determine whether it is a function or not and provide an explanation.

4.

Name	**Phone Number**
Bob	home: 480-555-2222 cell: 480-321-4444 work: 602-666-1234
Jerry	cell: 480-456-9876
Sarah	home: 480-555-3333 cell: 480-555-0018

Domain:

Range:

Relation:

Function: YES NO

Explanation:

5. When Sam woke up at 6:15 AM. Every morning last week, he noticed the following temperatures: Monday – 85˚, Tuesday – 83˚, Wednesday – 86˚, Thursday – 88˚, and Friday – 90˚. (Time is the independent variable.)

Domain:

Range:

Relation:

Function: YES NO

Explanation:

6. In a snack machine, A-1 corresponds to Snickers, B-1 corresponds to Reese's Pieces, C-2 corresponds to Doritos, and D-4 corresponds to Fritos.

Domain:

Range:

Relation:

Function: YES NO

Explanation:

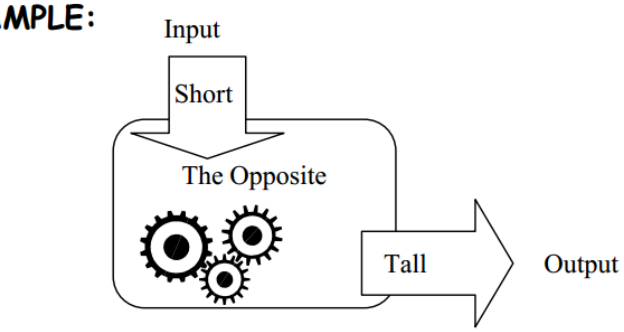
Summary:

Introduction to Functions

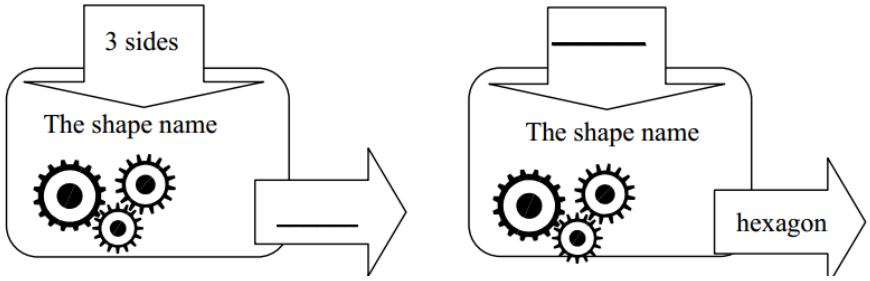
MACHINES AT WORK

Use the example below to determine the input or output of the following machines.

EXAMPLE:



1.



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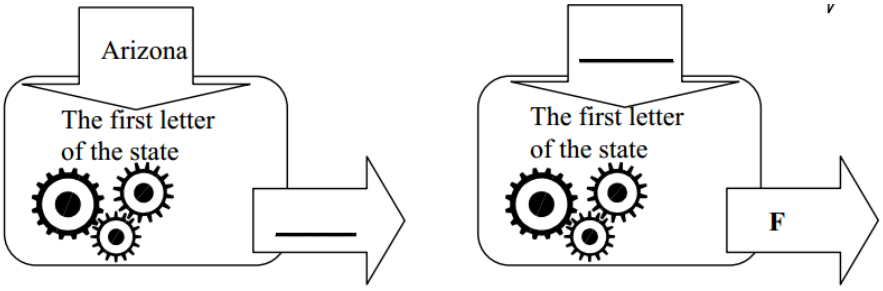

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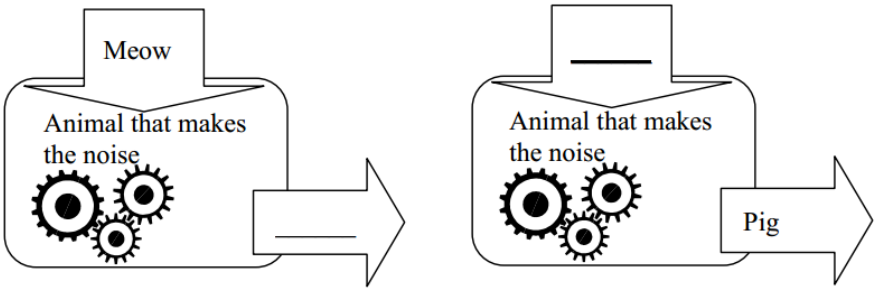

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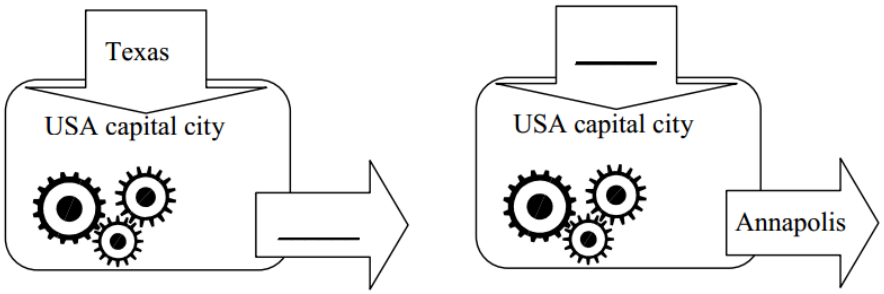

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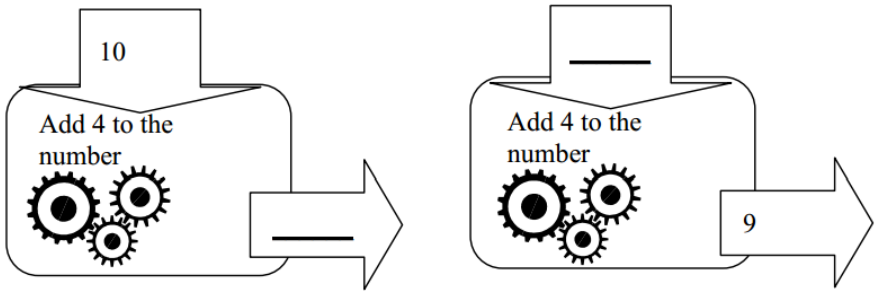

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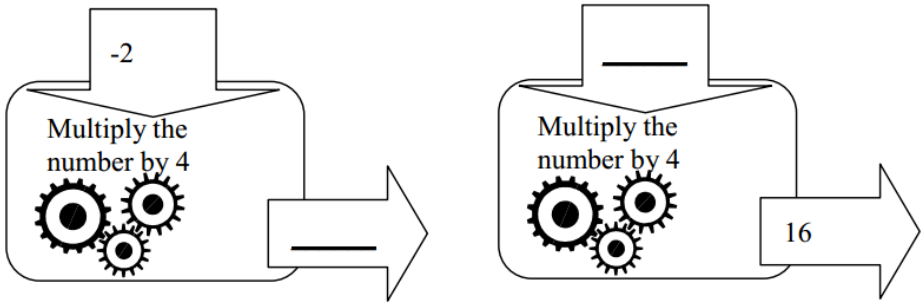


Fill in the missing values and state the rule that relates each number in the top row to the entry below it.

1	2	3	4	5	6		8	9	10
2	4	6	8			14		18	

7. Rule: _____

1	2	3	4	5	6	7		9	10
1	4	9	16	25			64		

8. Rule: _____

5	10	15	20	25	30	35		45	50
0	5	10	15	20			35		

9. Rule: _____

INS & OUTS OF FUNCTIONS

Fill in each table, complete the rule, and provide a description of the domain & range.

#10

In	Out
Apple	A
Banana	
Kiwi	
Orange	
Grapes	

Rule:

Domain:

Range:

#11

In	Out
Soccer	
	
Baseball	
	
Basketball	

Rule:

Domain:

Range:

#12

In	Out
Night	Day
Heavy	
Short	
	Little
Skinny	

Rule:

Domain:

Range:

#13

In	Out
10	4
12	
8	
4	
-2	

Rule:

Domain:

Range:

#14

In	Out
Valentines	February
Christmas	
Independence Day	
Labor Day	
Halloween	

Rule:

Domain:

Range:

#15

In	Out
11	
8	
-3	
2	
0	

Rule: *Multiply each number by 4, then add 2*

Domain:

Range:

FUNCTION TABLES

Complete the following tables. If the x value is given, evaluate the function to find y; if y is given, work backwards to find x. If there is no value for x or y, use your own value for x to find y.

16. $y = x + 1$

x	y
–1	
0	
2	
	5

17. $y = 2x - 3$

x	y
–2	
–1	
0	
	5

18. $y = 1 - 2x$

x	y
4	
3	
0	
	5

2.2 PRE-ALGEBRA REVIEW – THE DISTRIBUTIVE PROPERTY

- Two variable expressions that have the same value for all vales of the variable(s) are called equivalent variable expressions.
- You can use the distributive property to write equivalent variable expressions.

❖ The Distributive Property

➢ When you use the distributive property, you multiply each term inside the parentheses with the term on the outside of the parentheses.

- $a(b + c) = ab + ac$
  - Works w/subtraction too: $a(b - c) = ab - ac$

Schultz says:

The number of terms in the parentheses represents the number of terms in your answer.

- Try setting up your answer first:
  - $-3(x + 5) \rightarrow$ ___ + ___
- And then multiply & simplify:
  - $\underline{-3x}$ + $\underline{(-15)}$ = $-3x - 15$

Examples

Use the distributive property to write an equivalent variable expression.

1. $8(x + 2)$

2. $(7 - y)(-4)$

3. $9(3m + 5)$

4. $-2(n - 6)$

5. $-(x + 3)$

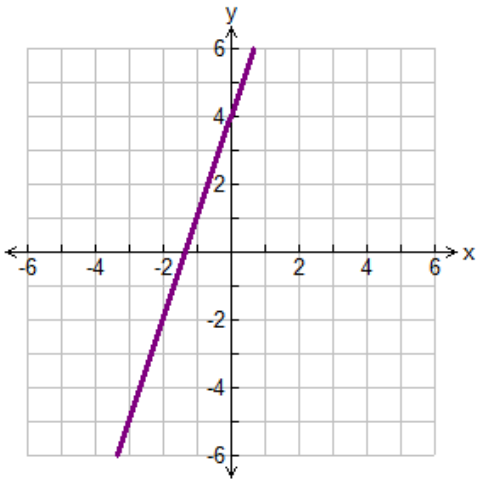
6. $-(-2 - 7h)$

2.2 Function Notation (Day 1)

Main Idea: Interpreting Functions
Objectives: Use Function Notation

- Explain that when x is an element of the input of a function, $f(x)$ represents the corresponding output of the function
- Decode function notation and explain how the output of a function is matched to its input
- Use the order of operations to evaluate a function for a given domain (input) value

Suppose we have the function shown on the graph below.



It can be described by the function rule:

$$y = 3x + 4$$

Given the input or the output values of the function we can figure out the corresponding output or input values.

Input (x)	1		6
Output (y)		4	

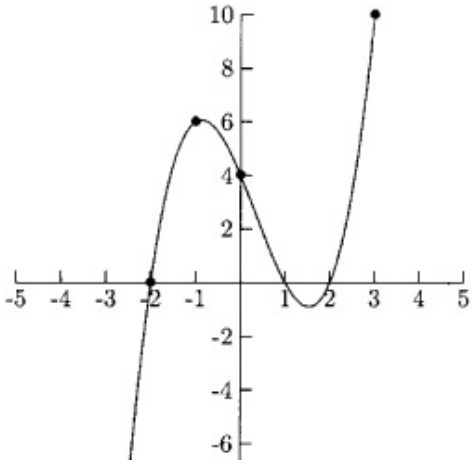
We can also use function notation to describe the function rule:

$y = 3x + 4 \quad \rightarrow \qquad\qquad f(1) \quad \rightarrow$

- You read $f(x)$ as "f of x" or "f is a function of x"
- The notations $g(x)$, $h(x)$, and $j(x)$ also indicate functions of x.

❖ What is the relationship between function notation and ordered pairs?

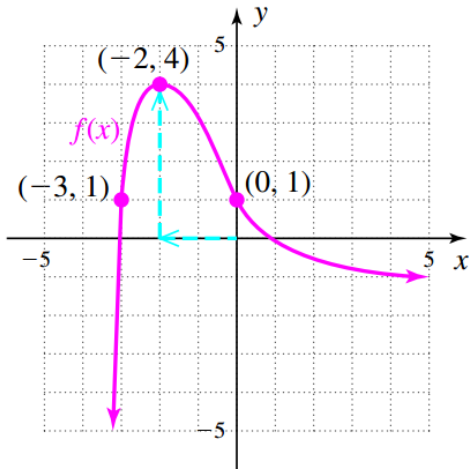
➢ $(x, y) = \big(x, f(x)\big)$



The point $(-1, 6)$ on the graph means that $f(-1) = 6$.

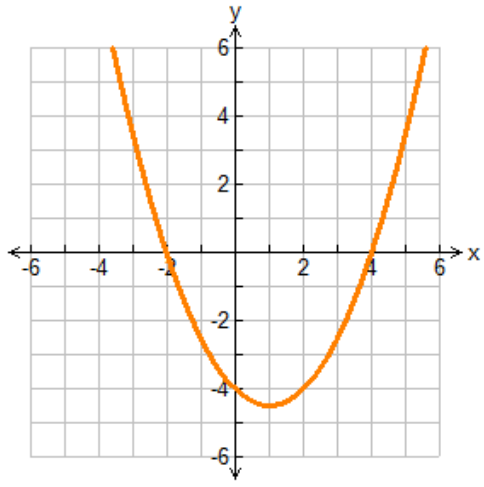
The point $(0, 4)$ on the graph means that $f(0) = 4$.

1. What is the value of $f(-2)$?
2. What value(s) of *x* satisfy $f(x) = 10$?



Use the graph of $f(x)$ given to answer the following questions:

3. What is the value of $f(-2)$?
4. What value(s) of *x* satisfy $f(x) = 1$?



A function is represented by the following graph.

5. What does the point $(0, -4)$ represent on this graph?

6. What is the value of output when the input is -2?

Evaluate the following expressions given the functions below.

$f(x) = 2x + 3$ $\quad g(x) = -\frac{1}{2}x^2$ $\quad h(x) = \frac{5 - x}{x + 1}$

7. $f(4)$

8. $g(-2)$

9. $h(-4)$

10. $h(1)$

11. Find x if $f(x) = -7$

12. Find x if $g(x) = -8$

Example

The domain of the function $y = 3x - 5$ is $\{2, 3, 4, 5\}$. Make a table for the function, then identify the range of the function.

x	2	3	4	5
$y = 3x - 5$	$3(2) - 5 = 1$	$3(3) - 5 = 4$	$3(4) - 5 = 7$	$3(5) - 5 = 10$

The range of the function is $\{1, 4, 7, 10\}$.

Find the range of each function for the given domain.

13. $f(x) = x - 6$; D: $\{-3, 2, 12\}$

x	-3	2	12
$f(x) = x - 6$			

14. $g(x) = -4x$; D: $\left\{-1, \frac{1}{2}, 4\right\}$

x	-1	$\frac{1}{2}$	12
$g(x) = -4x$			

Given the table of values find the following functions for $f(x)$.

15. $f(0)$

16. $f(2)$

17. $f(x) = 0$

18. $f(x) = -2$

x	$f(x)$
0	-3
1	-2
2	-1
3	0
4	1

19. Fill in the table with the missing values.

x		-2		4		8	
$f(x) = -x + 7$	14		7		1		-3

Summary:

What is function notation?

How do you do problems involving function notation?

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2.2 Function Notation (Day 2)

Main Idea: Interpreting Functions
Objectives: Use Function Notation

Error Analysis ~ Describe and correct the error in the student's work.

①

The relation given by the ordered pairs $(-4, 2)$, $(-1, 5)$, $(3, 6)$, and $(7, 2)$ is not a function because the inputs -4 and 7 are both mapped to the output 2.

②

x	0	1	2	1	0
y	5	6	7	8	9

The relation given by the table is a function because there is only one value of x for each value of y.
~~~~~~

Evaluate the following expressions given the functions below:

$f(x) = x + 2$ $g(x) = 3x - 1$ $h(x) = x^2$

3. $f(2) + g(5) =$

4. $h(3) - g(3) =$

5. $g(0) * h(1) =$

6. $\frac{f(2)}{g(-1)} =$

7. $g(x) + f(x) =$

8. $f(x) - g(x) =$

9. Given $f(x) = 2x - 1$, write a sequence when $x = 1, 2, 3, 4, ...$

10. Given $m(x) = 2x^2$, write a sequence when $x = -3, -2, -1, 0, ...$

11. Martin earns $7.50 per hour proofreading ads at a local newspaper. His weekly wage *w* can be described by the equation $w = 7.5h$, where *h* is the number of hours worked.
    a. Write the equation in functional notation.
    b. Find $w(15)$ and explain the meaning.
    c. Find *h* when $w(h) = 45$. Explain the meaning of your answer.

12. A hot air balloon descends its maximum height at a rate of $h = 250 - 10t$, where *t* is the number of minutes and *h* is in feet.
    a. Write the equation in function notation.
    b. Find $h(5)$ and explain the meaning.
    c. Find t when $h(t) = 100$. Explain the meaning of your answer.

13. Swine flu is attacking Porkopolis. The function below determines how many people have the swine flu where t = time in days and *S* = the number of people in thousands.

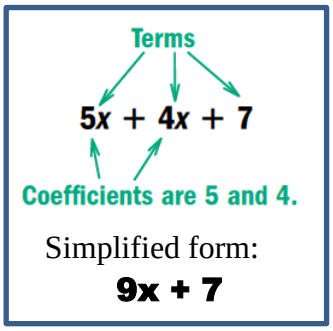
$$S(t) = 9t - 4$$

    a. Find *S*(4) and explain what it means.
    b. Find t when $S(t) = 23$. Explain its meaning.

2.3 PRE-ALGEBRA REVIEW – SIMPLIFYING EXPRESSIONS

Recall ~ A variable expression consists of numbers, variables, and operations.

- The parts of an expression that are added (or subtracted) together are called terms.
- The coefficient of a term with a variable is the number part of the term.
- A constant term has a number but no variable.
- Like terms are terms that have identical variable parts.
- A variable expression is simplified if it contains no grouping symbols and all like terms are combined.



Examples
Simplify each expression.

1. $6x + 1 - 8x$
2. $7 - 7x + 3 - x$
3. $-7 - 6m + 10$
4. $8n + 7 + n + 4$
5. $6n - 7(n - 8)$
6. $6m + 5(1 - 4m)$

2.3 Domain & Range

Main Idea: Applications of Functions
Objectives: Identify appropriate values for the domain and range

Domain	Range
x-values	y-values
X	$f(x)$
Input	Output
Independent variable	Dependent variable

❖ How do we know whether a relationship is a function or not?

Look back in your notes at Lesson 2.1.

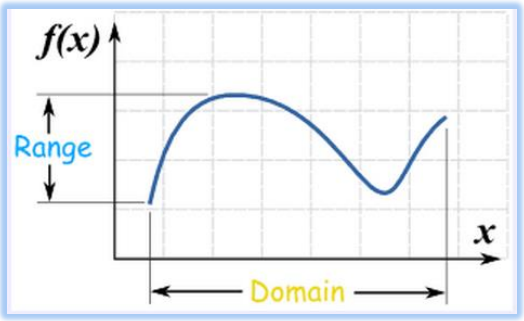
➢ Given a table of values:

➢ Given a mapping diagram:

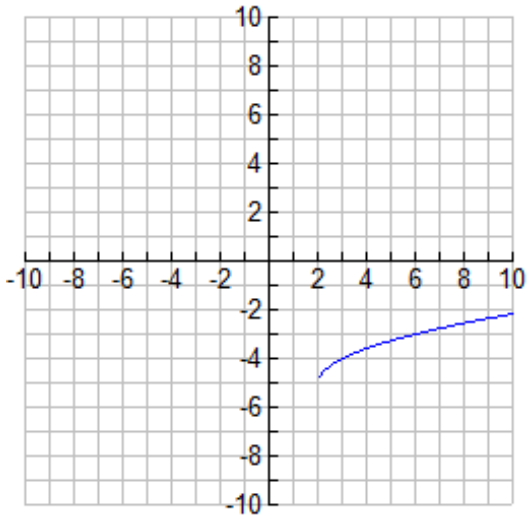
➢ Given a set of ordered pairs:

➢ Given a graph:

- Finding the domain and range given a graph:



Examples:



This is called a radical function

a. In the graph, what are all the *x*-values shown?

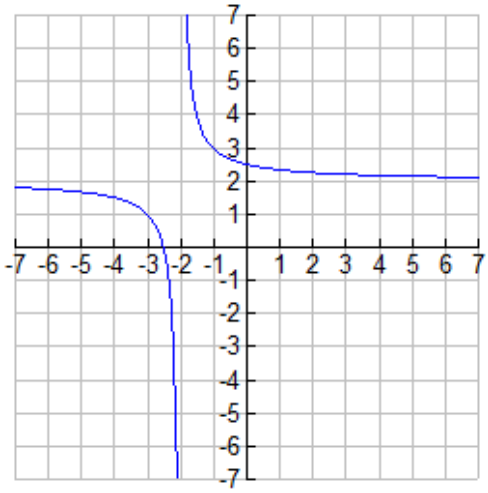
b. How can we express the domain as an inequality?

c. What are all values that are included in the range?

d. How can we express the range as an inequality?

e. Is the relationship a function or not? Explain how you know.

Sometimes, when looking at the domain and range of a function, it helps to think about what values are *not* included.



This is called a rational function

6. What *x*-value(s) does it look like this graph does not include?

7. What is the domain?

8. Are there any *y*-values that are not included?

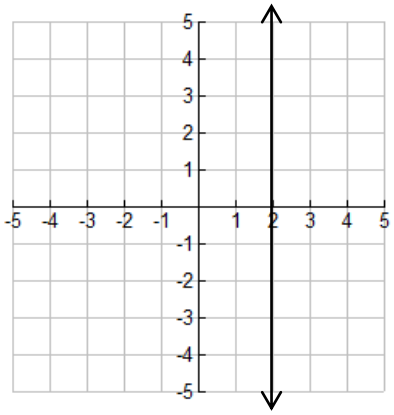
9. What is the range?

10. Is the relationship a function or not? Explain how you know.

Examples

Use each graph given to find the domain and range; determine whether the graph represents a function and explain your reasoning.

11.



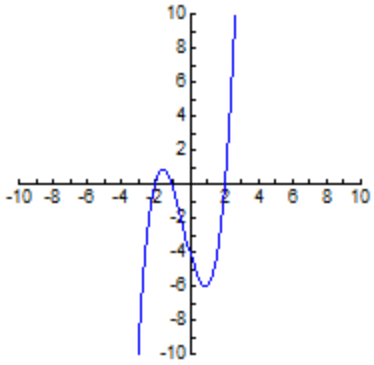
Domain:

Range:

Function: YES NO

Explanation:

12.



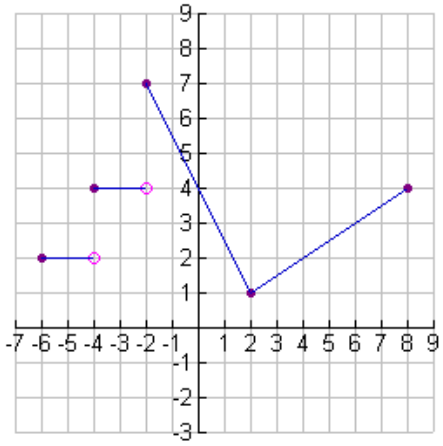
Domain:

Range:

Function: YES NO

Explanation:

13.

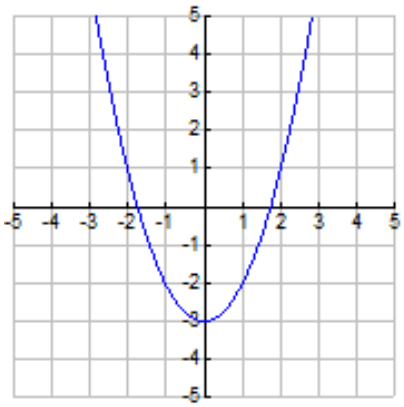


Domain:

Range:

Function: YES NO

Explanation:

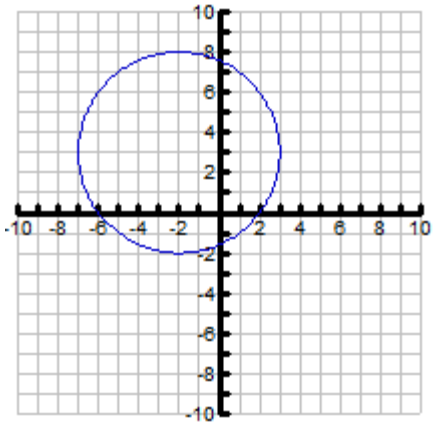
14. 

Domain:

Range:

Function: YES NO

Explanation:

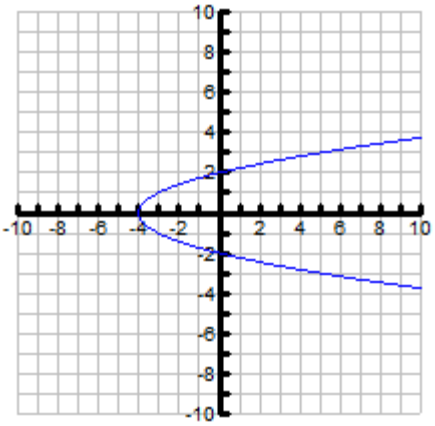
15. 

Domain:

Range:

Function: YES NO

Explanation:

16. 

Domain:

Range:

Function: YES NO

Explanation:

Summary:

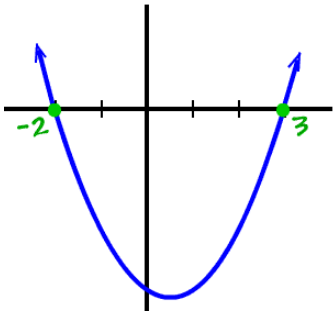
How can you find the domain and range of a function, given its graph?

2.4 Key Features of Graphs

Main Idea: Applications of Functions
Objectives: Analyzing Functions

- Identify appropriate values for the domain and range
- Identify and interpret the x-intercept(s) and the y-intercept
- Identify other key features of the function: increasing, decreasing; positive, negative; relative maximums and minimums; symmetries; end behaviors; and periodicity

Function Analysis 101

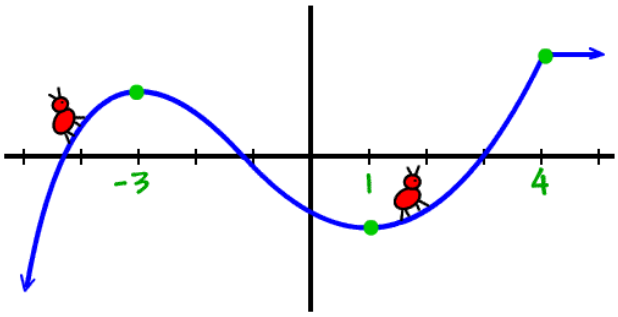


- Domain – x-values
- Range – y-values
- x-intercepts: where the graph crosses or touches the x-axis
  - The x-intercepts are $(-2, 0)$ & $(3, 0)$
- y-intercept: value of the function at $y = 0$; where the graph crosses the y-axis

- Increasing Interval(s) – where the graph is going up from left to right
  - Refer to the x-values when describing
- Decreasing Interval(s) – where the graph is going down from left to right
  - Refer to the x-values when describing

- Relative maximum(s) – where the graph changes from increasing to decreasing
  - Identify the y-value of the maximum(s) & its location "at $x =$ ___"
- Relative minimum(s) – where the graph changes from decreasing to increasing
  - Identify the y-value of the minimum(s) & its location "at $x =$ ___"

If Pierre is climbing uphill, then the graph is increasing:



This graph is...

Increasing from $-\infty < x \leq -3$

Decreasing from $-3 \leq x \leq 1$

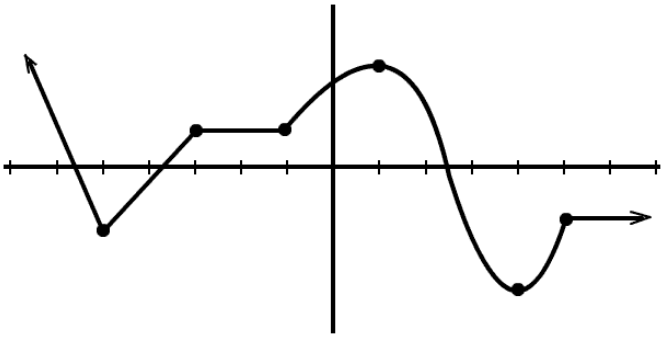
Increasing from $1 \leq x \leq 4$

There is a...

Relative maximum at $x = -3$

Relative minimum at $x = 1$

Your Turn: Identify the intervals where is the graph increasing and where it is decreasing.

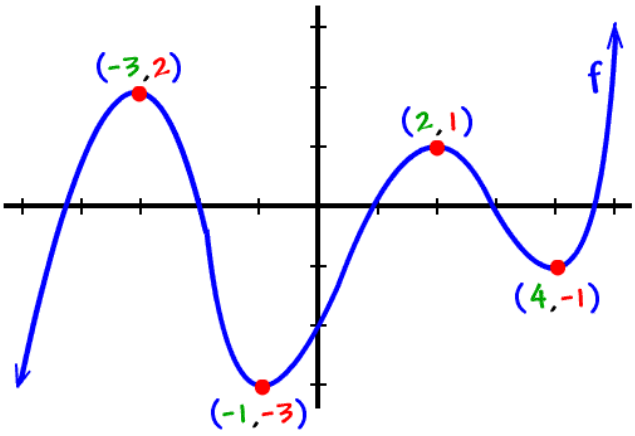


a. Increasing:

b. Decreasing:

c. What is happening in the interval $-3 \leq x \leq -1$?

Your Turn: In the graph below, identify the y-intercept, any x-intercept(s), relative maximum(s) and relative minimum(s): "*Relative max/min of __ at x = __.*"



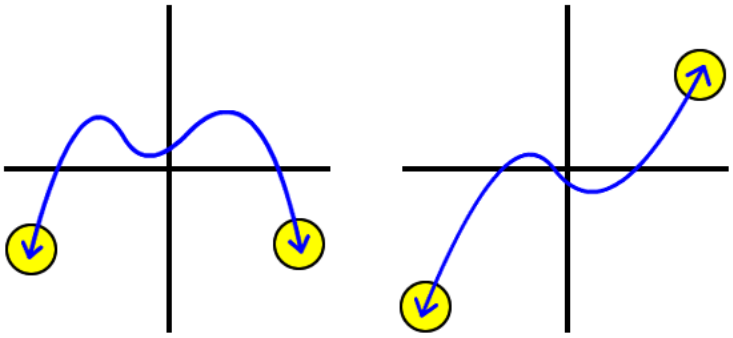
a. y-intercept:

b. x-intercept(s):

c. Relative maximum(s):

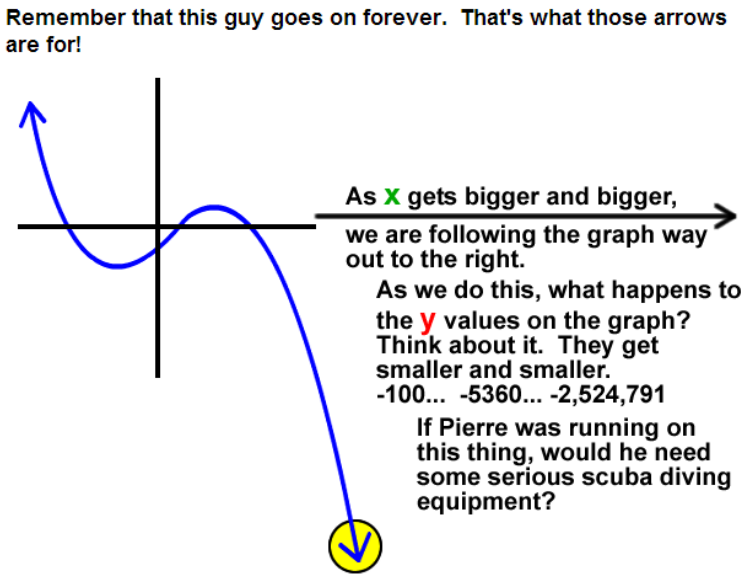
d. Relative minimum(s):

- Symmetry – y-axis, origin, or vertical line: $x = \#$
- End Behavior:



What happens to the height of the graph as the x-values gets really, really big?

What happens to the height of the graph as the x-values gets really, really small?



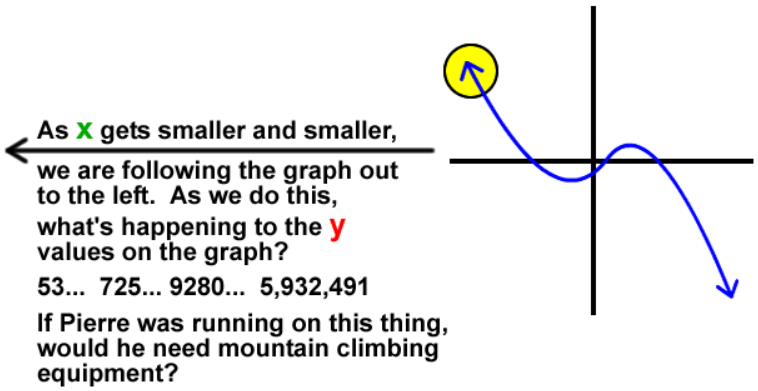
End behavior notation:

$$\textbf{As } x \to \infty, y \to -\infty$$

"As x approaches positive infinity, y approaches negative infinity."

Meaning:

As the x-values get bigger, the y-values are getting smaller.



End behavior notation:

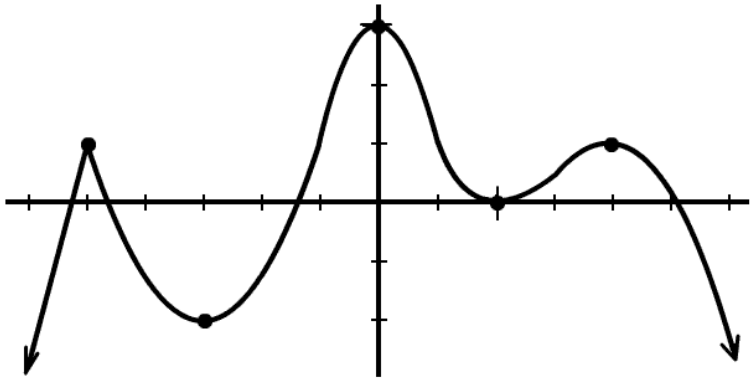
$$\textbf{As } x \to -\infty, y \to \infty$$

"As x approaches negative infinity, y approaches positive infinity."

Meaning:

As the x-values get smaller, the y-values are getting bigger.

Your Turn: Identify the functions below for the characteristics that follow. Approximate, if necessary.



a. Domain

b. Range

c. x-intercept(s)

d. y-intercept

e. Increasing intervals

f. Decreasing intervals

g. Relative maximum(s)

h. Relative minimum(s)

i. End behavior:
As $x \to -\infty, y \to$ _____

j. End behavior:
As $x \to \infty, y \to$ _____

Summary:

Given the graph of a function, how do you identify x- and y-intercepts; increasing/decreasing intervals; relative maximums and minimums; and end behavior?

2.5 Graph Stories

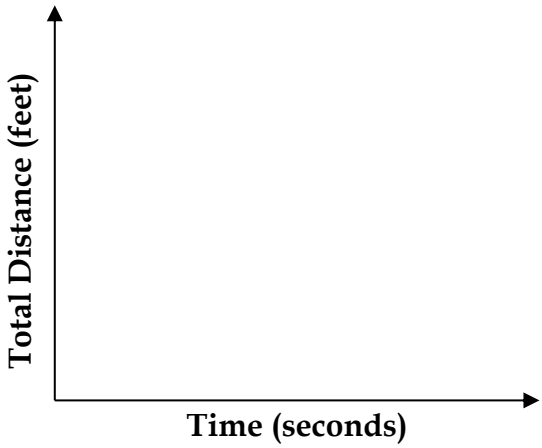
Main Idea: Applications of Functions
Objectives: Given a written situation, locate information that explains what each quantity represents

APK – ACCESSING PRIOR KNOWLEDGE

1) In a walking experiment, Josephine walked a total distance of 40 feet. At the half way point, she had walked for 25 seconds. She stopped for 5 seconds to tie her shoe and then continued walking for 25 more seconds.

Sketch a graph that shows Josephine's distance from the starting point over time.

How can we describe the function representing Josephine's walk?

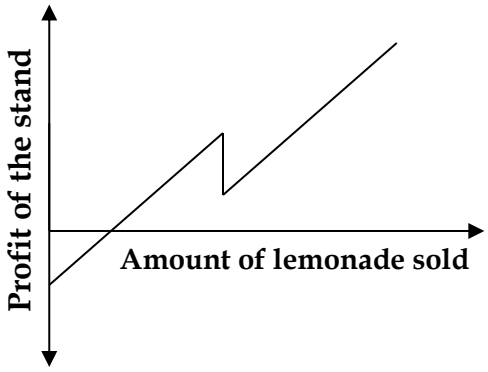


2) The graph represents the relationship between the profit and the amount of lemonade sold at a lemonade stand.

a) What is being represented when the graph is below the x-axis?

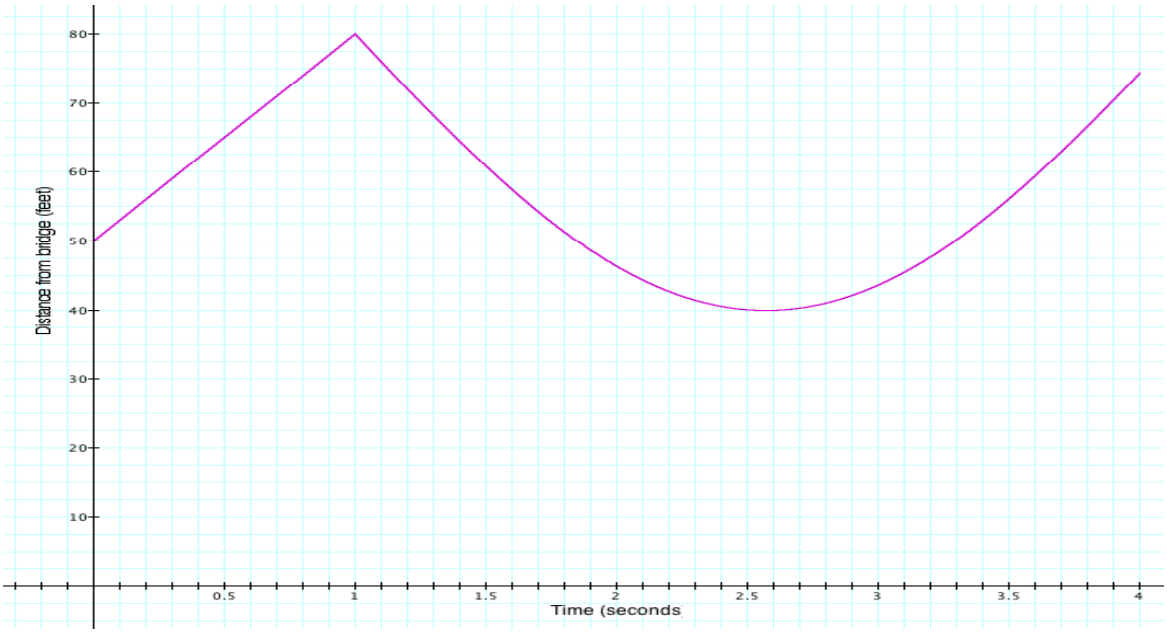
b) What does the point $f(0)$ represent?

c) What event might happen to cause the vertical section of the graph?



INTERPRETING GRAPHS:

This is a graph of Aaron's distance from the bridge in relation to time mid-flight during one of his bungee rides.



a) During the period shown on the graph, when was Aaron the furthest from the bridge? How far from the bridge was he at that time?

b) At the beginning of the time period is Aaron on the bridge? How do you know?

c) What does the curve on the graph represent in this situation?

d) Select a point on the graph; tell what your point represents in this situation.

e) Complete the following table:

Time Period	Change in Aaron's distance from the bridge during time interval	Average Speed in ft/sec (Average Rate of Change)
$0 - 1$		
$1 - 2$		
$2 - 2.6$		
$2.6 - 4$		