Unit 3 – Linear Functions

Name: ______________________________

Main Idea 1: Rates of Change

You may be asked to view a Virtual Nerd video in order to prepare for the next day's lesson. You can access the videos in a number of ways:

(1) Go to my website – www.schultzjen.weebly.com From the Math Topics ½ ~ Algebra menu, select "Lesson Guides & More." Scroll down to Unit 3 ~ Linear Functions and click on the appropriate link.

(2) Type in the links provided.

(3) Follow the directions below:

Using Virtual Nerd:

- Go to www.virtualnerd.com
  - It's best to use Google Chrome.
- Select 'Algebra 1'
- From the Algebra 1 Topics List, select 'Analyzing Linear Equations'
  - Lesson 3.1 ~ Rate of Change
    - Click on 'Rate of Change & Slope'
    - Click on 'Defining Slope'

      Watch the lessons:
      - How do you find rate of change between two points in a table?
      - How do you find rate of change between two points on a graph?
      - What does the slope of a line mean?
      - What the formula for slope?
    - Click on 'Finding Slopes'
    - *Watch the lessons:*
      - How do you find the slope of a line from two points?
  - Lesson 3.2 ~ Constant Rate of Change
    - Click on 'Relations & Functions'
    - Click on 'Function'
    - Click on 'Linear & Non-Linear Functions'

      Watch the lesson:
      - How can you tell if a function is linear or non-linear from a table?

3.1 Rates of Change

Objectives:

- Calculate and interpret the average rate of change of a function — presented symbolically or as a table — over a specified interval.
- Estimate the rate of change from a graph.
- Explain the connection between rate of change and the slope formula

LESSON PREP

Watch the following lesson(s) on www.virtualnerd.com

- *Identify any key ideas or vocabulary presented in the lesson.*
- *Summarize the lesson.*
- *Do the problem(s) that follow.*

LESSON: **How do you find the rate of change between two points in a table?**

http://www.virtualnerd.com/algebra-1/linear-equation-analysis/slope-rate-of-change/understanding-slope/rate-of-change-two-points-table

KEY IDEAS & TERMS	SUMMARY

PROBLEM #1:

The table shows the cost of mailing a 1-ounce letter in different years. Find the rate of change in cost between the years 1988 and 2008.

Year	1988	1990	1991	2004	2008
Cost (¢)	25	25	29	37	42

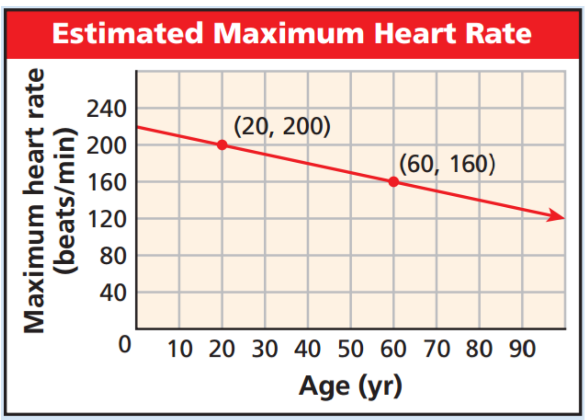
LESSON: How do you find the rate of change between two points on a graph?

http://www.virtualnerd.com/algebra-1/linear-equation-analysis/slope-rate-of-change/understanding-slope/rate-of-change-two-points-graph

KEY IDEAS & TERMS	SUMMARY

PROBLEM #2:

The graph shows a relationship between a person's age and his or her estimated maximum heart rate in beats per minute. Find the rate of change.



LESSON: What does the slope of a line mean?

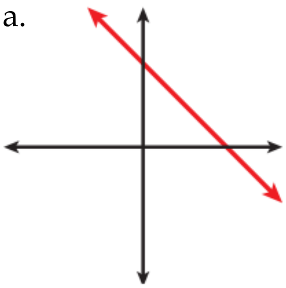
http://www.virtualnerd.com/algebra-1/linear-equation-analysis/slope-rate-of-change/understanding-slope/slope-definition

KEY IDEAS & TERMS	SUMMARY

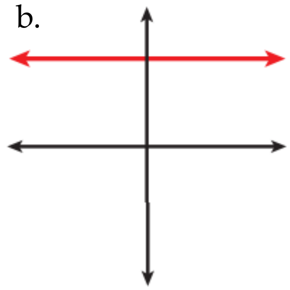
PROBLEMS:

Tell whether the slope of each line is positive, negative, zero, or undefined.

a.



b.



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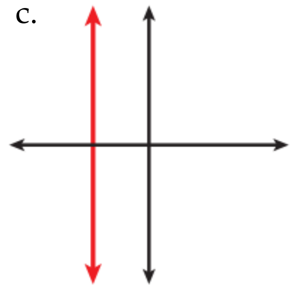
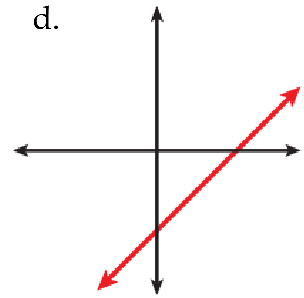


d.



LESSON: **What's the formula for slope?**

http://www.virtualnerd.com/algebra-1/linear-equation-analysis/slope-rate-of-change/understanding-slope/slope-formula-definition

KEY IDEAS & TERMS	SUMMARY

LESSON: **How do you find the slope of a line from two points?**

http://www.virtualnerd.com/algebra-1/linear-equation-analysis/slope-rate-of-change/slope-examples/slope-from-two-points

KEY IDEAS & TERMS	SUMMARY

PROBLEM #1:

Find the slope of a line that passes through the points $(6, -3)$ & $(-3, 4)$.

- Rate of Change
  - A <u>rate of change</u> is a ratio that compares the amount of change in a dependent variable to the amount of change in an independent variable.

$$\text{rate of change} = \frac{\text{change in dependent variable}}{\text{change in independent variable}}$$

- Constant Rate of Change
  - If all the connect segments have the same rate of change.
  - The <u>constant rate of change</u> of a non-vertical line is called the <u>slope</u> of the line.

Slope of a Line

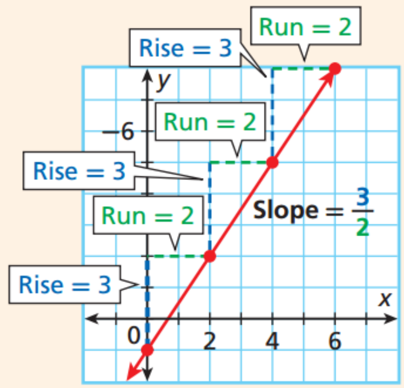
The **rise** is the difference in the ***y*-values** of two points on a line.

The **run** is the difference in the ***x*-values** of two points on a line.

The **slope** of a line is the ratio of rise to run for any two points on the line.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$

(Remember that y is the **dependent variable** and x is the **independent variable**.)



- Slope of a Line
  - A line's slope is a measure of its steepness.

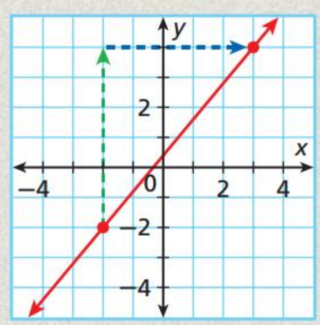
MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. What is the rise shown in the graph? What is the run? What is the slope?

2. The rate of change of the profits of a company over one year is negative. How have the profits of the company changed over that year?

3. Would you rather climb a hill with a slope of 4 or a hill with a slope of $\frac{5}{2}$? Explain your answer.



3.2 Constant Rate of Change

Objectives:

- Demonstrate that a linear function has a constant rate of change (slope)
- Identify situations that display equal rates of change over equal intervals and can be modeled with linear functions

LESSON PREP

Watch the following lesson(s) on www.virtualnerd.com

- *Identify any key ideas or vocabulary presented in the lesson.*
- *Summarize the lesson.*
- *Do the problem(s) that follow.*

LESSON: **How can you tell whether a function is linear or non-linear from a table?**

http://www.virtualnerd.com/algebra-1/relations-functions/functions/linear-nonlinear/table-linear-vs-nonlinear

KEY IDEAS & TERMS	SUMMARY

PROBLEM #1:

Do any of the tables below represent a linear function? Explain.

a.

x	*y*
2	4
5	3
8	2
11	1

b.

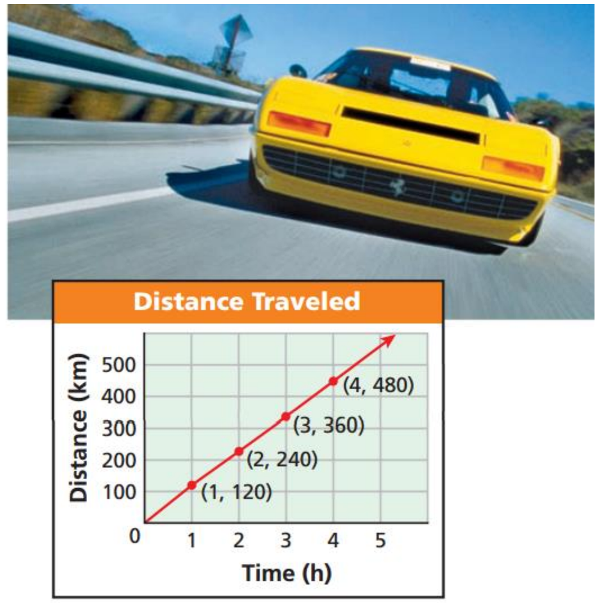
x	*y*
−10	10
−5	4
0	2
5	0

INTRODUCTION

Why learn this?
Linear functions can describe many real-world situations, such as distances traveled at a constant speed.

Most people believe that there is no speed limit on the German autobahn. However, many stretches have a speed limit of 120 km/h. If a car travels continuously at this speed, $y = 120x$ gives the number of kilometers y that the car would travel in x hours. Solutions are shown in the graph.

The graph represents a function because each domain value (x-value) is paired with exactly one range value (y-value). Notice that the graph is a straight line. A function whose graph forms a straight line is called a **linear function**.



Examples:

1. Juan is running on a treadmill. The table shows the number of calories Juan burns as a function of time.

   a. Explain how you can tell that this relationship is linear by using the table.

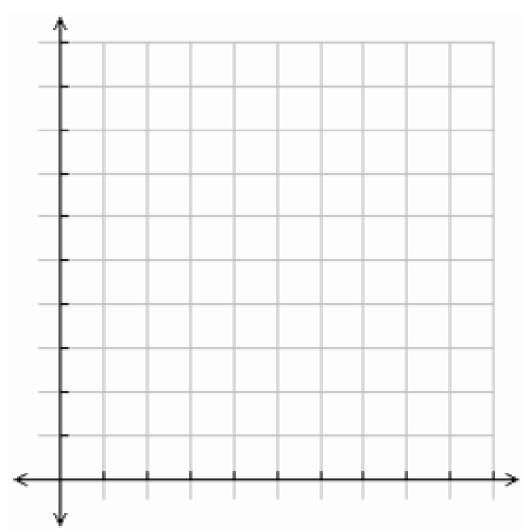
Time (min)	Calories
3	27
6	54
9	81
12	108
15	135
18	162
21	189

   b. Which quantity is the independent variable?

   c. Which quantity is the dependent variable?

   d. Draw a graph of this data; do not forget to correctly label the axes.

   e. What is Juan's rate of change in calories per minute?

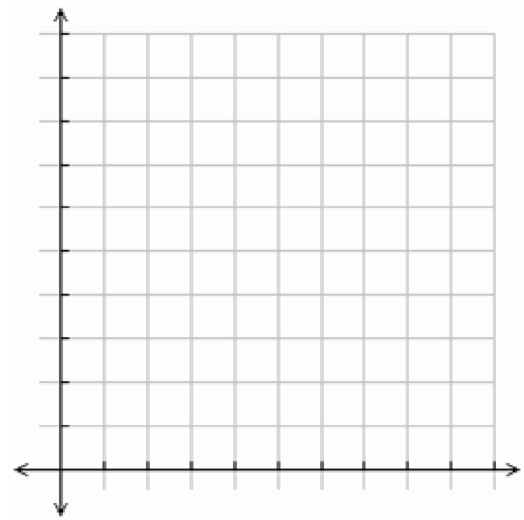


2. Maria drove from Phoenix to Page in five hours. She drove her car at a constant rate of change covering 114 miles in 2 hours.
    a. What was Maria's constant rate of change in miles per hour?

    b. When you describe Maria's constant rate of change, what are you actually describing?

    c. Use what you know about Maria's constant rate of change to complete the table (below).
    d. Draw a graph of this data; do not forget to correctly label the axes.

Time (hours)	*Show work in this column*	*Distance (miles)*
0		
1		
2		
3		
4		
5		
m		

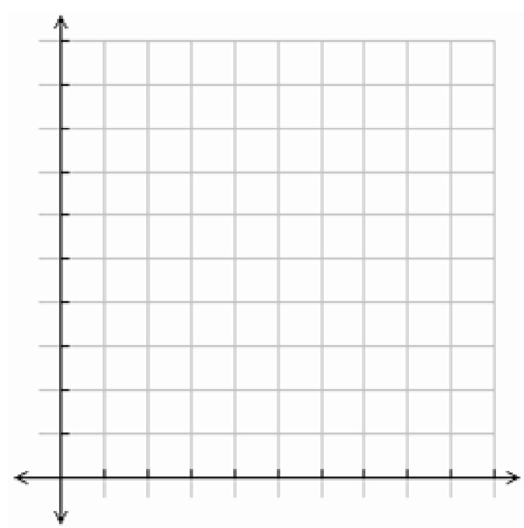


3. At Cali's Coffee Café, it costs $1.25 for a medium cup of coffee.
    a. What is the constant rate of change in this situation?

    b. You bought yourself and three friends a medium cup of coffee. How much did it cost?

    c. How much would it cost to by x medium cups of coffee? Write a function to represent this relationship.

    d. Draw a graph of this data; do not forget to correctly label the axes.



4. A swimming pool is being drained. The table shows the function relating the volume of water in a pool and the time in hours that the pool has been draining.

Draining a Pool	
Time (h)	Volume (gal)
x	f(x)
0	10,080
2	8640
6	5760
10	2880
12	1440
14	0

a. What is the domain in this situation?

b. What is the range?

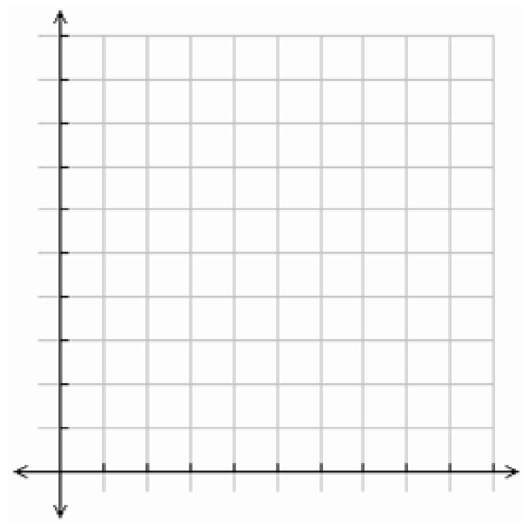
c. What is the constant rate of change in this situation?

d. What is the volume of water remaining in the pool after 5 hours?

e. What is the volume of water remaining in the pool after 5 ½ hours?

f. Write a function to represent this relationship.

g. Draw a graph of this data; do not forget to correctly label the axes.



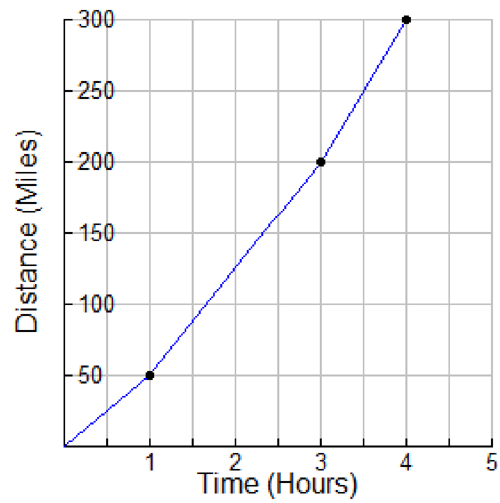
3.3 Average Rate of Change

Objectives:

- Explain the connection between average rate of change and the slope formula
- Calculate and compare the average rate of change of a function, represented either by function notation, a graph, or a table, over a specific input interval
- Interpret the meaning of the average rate of change — using appropriate units — as it relates to a real-world problem

Can an object, starting from rest (such as a car, a swimmer, etc.) ever cover some given distance, from start to finish, at a constant speed? Why or why not?
Need help?…Have you ever gone anywhere on vacation and traveled by car? Think about that car trip.

1) The following graph shows a four-hour time period of the distance a trucker drove on his way from Boston to Los Angeles.



a) What is the distance the trucker drives in the first hour?

Between the 1st and 3rd hours?

In the last hour?

b) Using your answers from *a*, what was the trucker's average speed during each time segment?

In the first hour?

Between the 1st and 3rd hours?

In the last hour?

In the first three hours: 0 – 3?

In all four hours?

We can find average speed by looking at a table or graph. Instead of finding $\frac{\text{total distance}}{\text{total time}}$, we can find:

$$\text{average speed} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

2) *Use the table to the right to answer the following questions.*

a) Does this table have a constant rate of change? Show work that explains your answer.

x	y
-2	-6
0	-8
4	-12
10	-18

b) What is the average rate of change between $x = 0$ and $x = 10$?

c) What is the average rate of change between $x = -2$ and $x = 10$?

d) Is the relationship in the table a function? Is it a linear function? Explain.

3) *Use the table to the right to answer the following questions.*

a) Does this table have a constant rate of change? Show work that explains your answer.

x	y
-4	4
0	-1
2	-2
6	-6

b) What is the average rate of change between $x = -4$ and $x = 6$?

c) What is the average rate of change between $x = -4$ and $x = 2$?

d) Is the relationship in the table a function? Is it a linear function? Explain.

4) A climber is hiking South Mountain. It takes her 45 minutes to climb the first 900 feet. She stops and takes a 15-minute break to get water and take pictures of the incredible scenery. She then proceeds to climb another 600 feet in 40 minutes.

a) What was her average speed over the first 900 feet?

b) What was her average speed in the last 40 minutes?

c) What was her average rate of speed over the entire climb?

5) Mr. Jeanson's friend, Tom, weighed 350 lbs. in college when he decided that he wanted to start losing weight. Once he started running and lifting weights, he lost 25 lbs. in the first month (30 days). Over the course of 2 years, he lost half his total weight.

a) What was Tom's average amount of weight lost per day in the first month?

b) What was his average rate of change, in weight loss per month, over the course of the 2 years?

3.4 Linear Functions

<u>Objectives</u>:

- Demonstrate that a linear function has a constant rate of change.
- Identify situations that display equal rates of change over equal interval and can be modeled with linear functions.

When a word problem involves a constant rate of change or speed and an initial value (beginning amount), it can be written as a linear function in the form: $f(x) = mx + b$, where m represents the rate of change and b represents the initial value.

1) The function below shows the relationship between the cost ($) of placing an online advertisement in terms of the number of days the ad runs for.

$$y = 1.5x + 2.25$$

Match the parts of the function to the correct description of what each part represents based on the situation described.

x $\quad$ y $\quad$ 1.5 $\quad$ $1.5x$ $\quad$ 2.25

a) ________ The flat service fee charged for placing the advertisement.

b) ________ The total cost of the ad relative to the number of days the ad is online.

c) ________ The cost being charged for each day the ad is run online.

d) ________ The number of days that the ad is run.

e) ________ The change in the cost of placing the ad relative to the number of days the ad is run.

2) The current temperature in Alaska is 6° C and has been increasing 3° C each hour.

a) What is the <u>independent</u> and <u>dependent</u> variables in this situation?

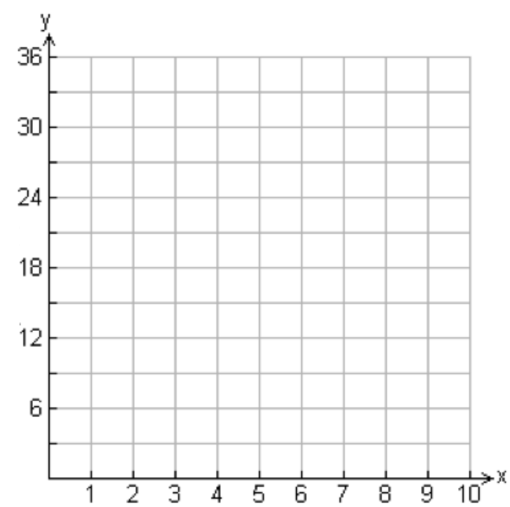
Independent (x):

Dependent (y):

b) What is the rate of change in this situation?

c) What is the initial value in this situation?

d) What is the function that describes the change in temperature in relation to the number of hours?



e) Graph this situation. (Remember to label the x- and y-axes.)

f) How long until the temperature is 24°C?

3) A crate weighs 3 kilograms when empty. It is filled with oranges weighing 0.2 kilogram each. The crate will hold 45 oranges.

a) What is the <u>independent</u> and <u>dependent</u> variables in this situation?

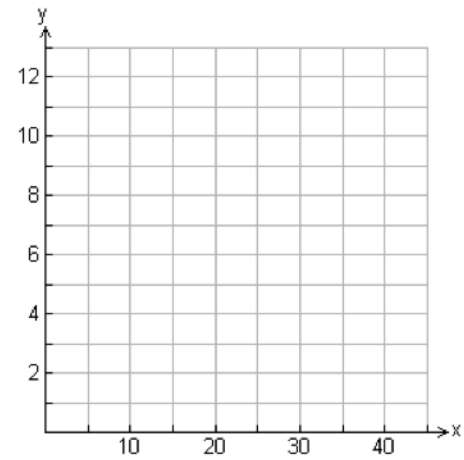
Independent (x):

Dependent (y):

b) What is the rate of change in this situation?

c) What is the initial value in this situation?

d) What is the function that describes the change in weight in relation to the number of oranges?



e) Graph this situation. (Remember to label the x- and y-axes.)

f) How much will the crate weigh when it contains 31 oranges?

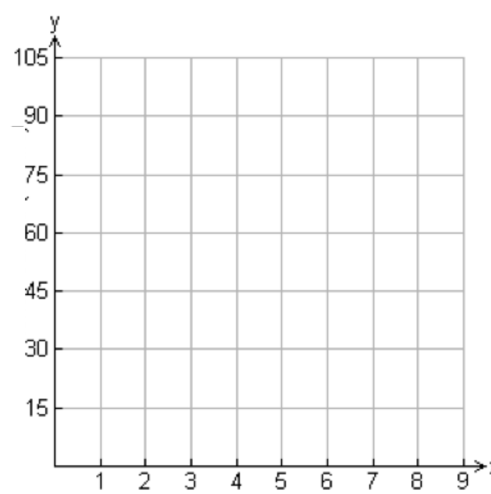
4) Rick receives an allowance of $15 per week for doing chores around the house.

a) What is the function that describes the change in Rick's allowance in relation to the number of weeks?

b) Graph this situation – imagining that Rick earns money consistently throughout the week. (Remember to label the x- and y-axes.)

c) How many weeks until Rick will have earned $50?

d) What is the rate of change in terms of allowance earned per day?



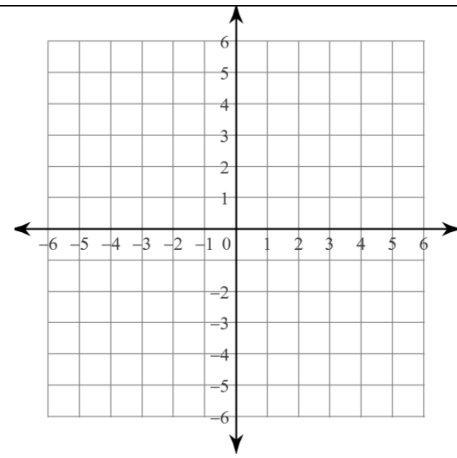
3.5 Slope-Intercept Form of a Linear Equation

Objectives:

- Identify the slope-intercept form of a linear function
- Graph a line in slope-intercept form and use the graph to show where the y-intercept (b) and the slope (m) are represented on the graph

LESSON: What is slope-intercept form of a linear equation?

http://www.virtualnerd.com/algebra-1/linear-equation-analysis/slope-intercept-form/slope-intercept-form-examples/slope-intercept-form-definition

KEY IDEAS & TERMS	GRAPH
$$y = mx + b$$	

EXAMPLES:

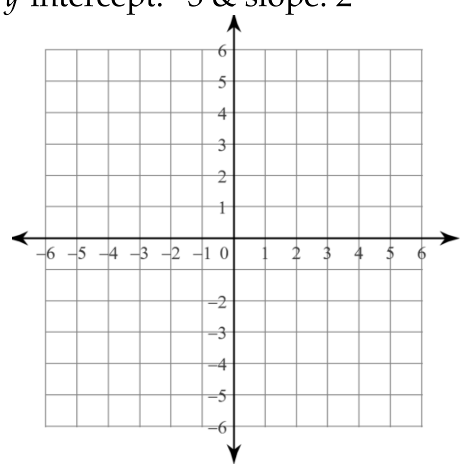
Given the slope and y-intercept write an equation then graph the linear function.

Graphing a Linear Equation: $y = mx + b$

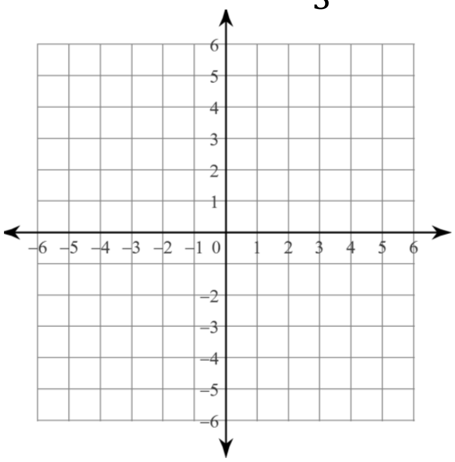
- Identify the y-intercept.
- Plot the y-intercept on the y-axis
- Identify the slope: $\pm \frac{N}{D}$
- Go up if the slope is positive
- Go down if the slope is negative
- Always go right

$$\frac{N}{D}$$

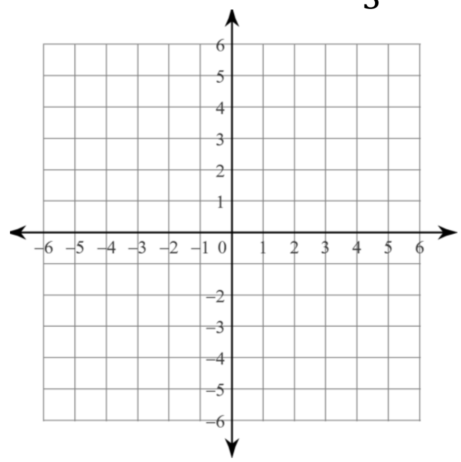
1. y-intercept: –3 & slope: 2



2. y-intercept: 4 & slope: $\frac{2}{3}$



3. y-intercept: 2 & slope: $-\frac{4}{3}$

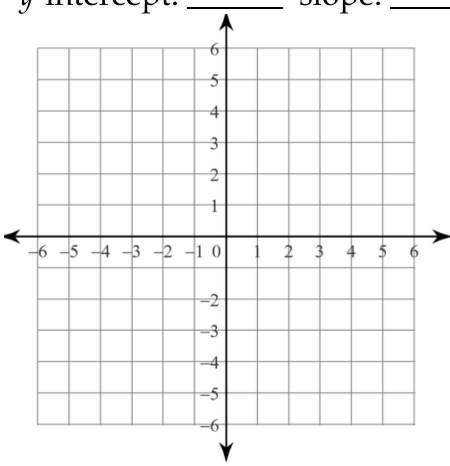


EXAMPLES:

Identify the y-intercept and the slope of the functions given the following linear equations, and then graph the function.

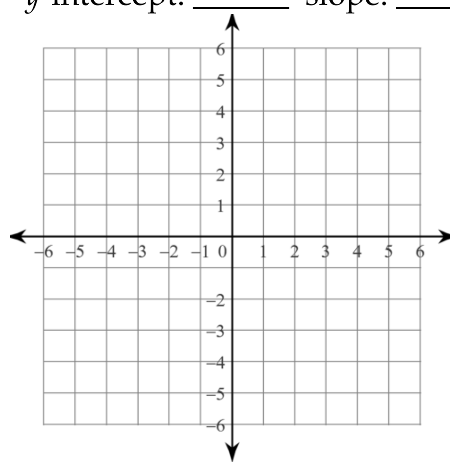
4. $y = \frac{1}{4}x + 1$

y-intercept: ______ slope: ______



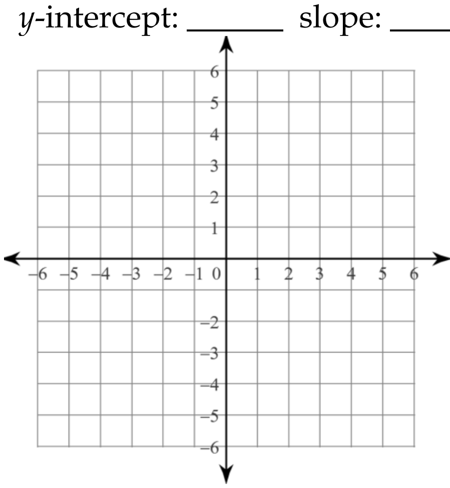
5. $y = -\frac{2}{3}x + 3$

y-intercept: ______ slope: ______



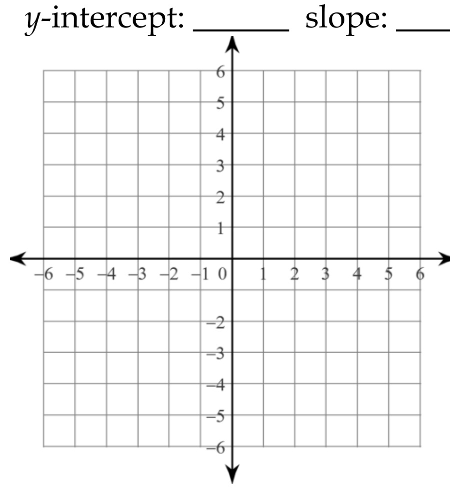
8. $y = -x - 5$

y-intercept: ______ slope: ______



9. $y = 3x$

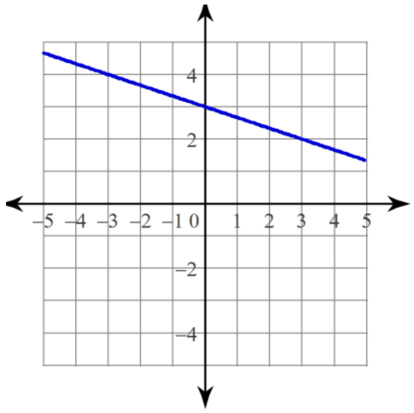
y-intercept: ______ slope: ______



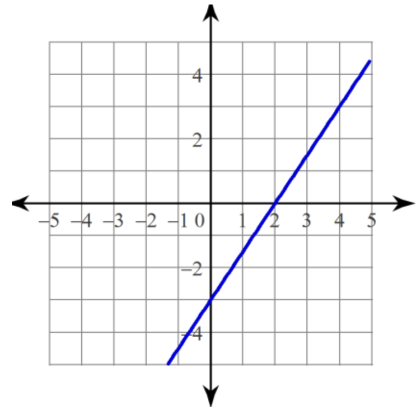
EXAMPLES:

Identify the y-intercept and the slope of the linear function. Then use this information to write an equation to describe this function in slope-intercept form.

10. y-intercept: ______ & slope: ______



11. y-intercept: ______ & slope: ______



EXAMPLES: **Problem Solving**

12. A group of mountain climbers begin an expedition with 265 pounds of food. They plan to eat a total of 15 pounds of food per day.
    a. Write an equation in slope-intercept form relating the remaining food supply r to the number of days d.
    b. The group plans to eat the last of their food the day their expedition ends. How many days does the group expect the expedition to last?

3.6 Point-Slope Form of a Linear Equation

Objectives:

- Identify the point-slope form of a linear function
- Graph a line in point-slope form and use the graph to show where the y-intercept (b) and the slope (m) are represented on the graph

ACCESSING PRIOR KNOWLEDGE ~ MULTIPLYING BY FRACTIONS

- Multiplying an integer by a fraction is simple.
- You multiply the numerator of the fraction together with the integer and place the product over the denominator.
- If possible, simplify the resulting fraction.

$$\frac{2}{3} \times (-2) = \frac{2 \times (-2)}{3} = -\frac{4}{3}$$

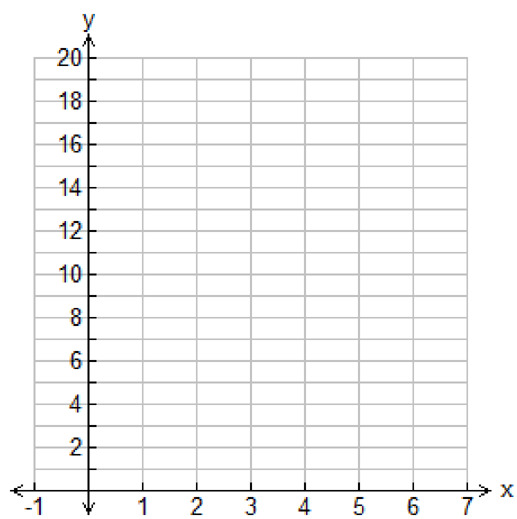
$$\frac{1}{2} \times 6 = \frac{1 \times 6}{2} = \frac{6}{2} = 3$$

$$-\frac{3}{4} \times (-2) = \frac{-3 \times (-2)}{4} = \frac{6}{4} = \frac{3}{2}$$

APPLICATIONS & MODELING

Chuck was cruising in the nerd herder (a car) at a constant speed away from the Buy More. Five minutes after Chuck started his stopwatch he was a total of 19 miles from the Buy More and he was traveling at 3.5 miles per minute.

Elapsed Time (minutes)	Distance from the Buy More (miles)
0	
1	
2	
3	
4	
5	19
6	
7	



a) Write a rule (linear function) in slope-intercept form to describe the relationship between Chuck's distance from the Buy More and the time elapsed since starting the stopwatch.

b) How did you find the *y*-intercept for your function?

LESSON: What is point-slope form of a linear equation?

http://www.virtualnerd.com/algebra-1/linear-equation-analysis/point-slope-standard-form/point-slope-examples/point-slope-form-definition

KEY IDEAS & TERMS	SUMMARY

HOW DO YOU RID AN EQUATION OF FRACTIONS?

$$y - 1 = \frac{1}{5}(x - 3)$$

EXAMPLES:

The point-slope form of a linear equation is given. Rewrite it so that it is in slope-intercept form.

1. $y - 3 = -\frac{3}{8}(x + 5)$

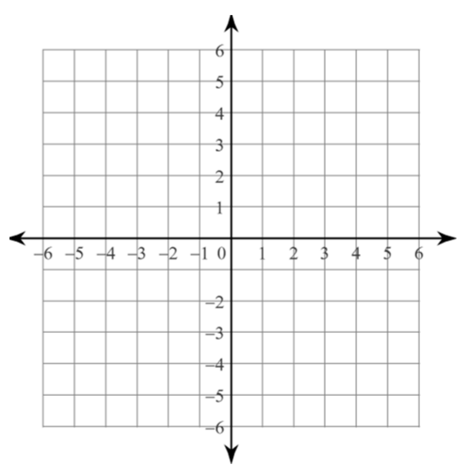
2. $y + 2 = \frac{5}{3}(x + 2)$

EXAMPLES:

For the following problems, use the given information to write an equation in both point-slope form and slope-intercept form of the linear function that passes through the given point with the given slope. Then graph the function.

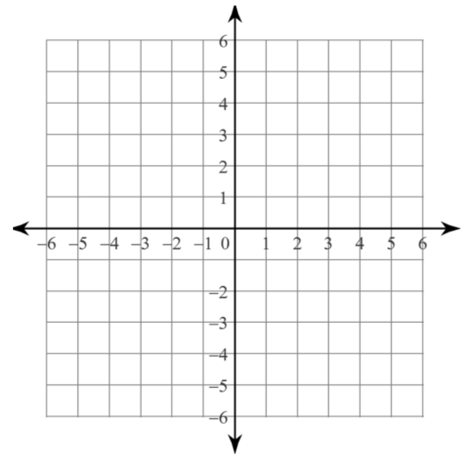
3. Point $(2, -1)$, slope: -3
   Point-slope form:

   Slope-intercept form: ________________



4. Point $(4, 3)$, slope: $\frac{3}{2}$

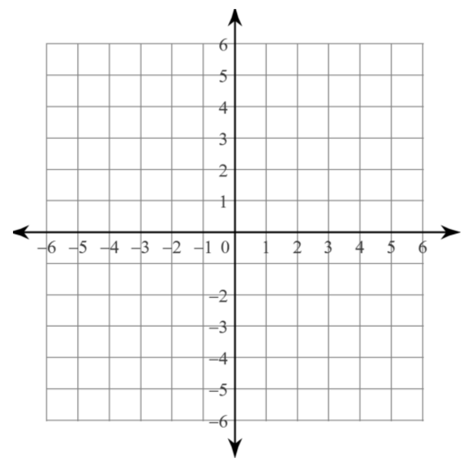
   Point-slope form:



   Slope-intercept form: ________________

5. Point $(-2, -3)$, slope: $\frac{1}{2}$

   Point-slope form:



   Slope-intercept form: ________________

3.6 More Point-Slope Form

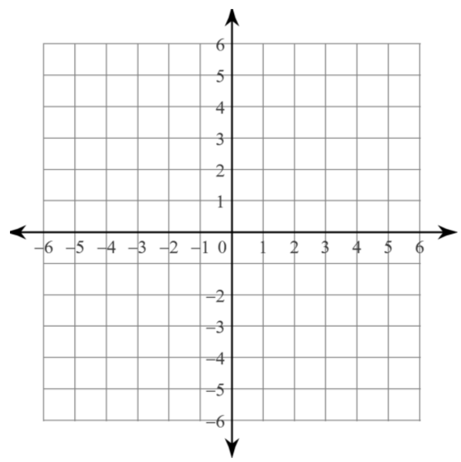
LESSON: How do write an equation of a line in point-slope form if you have two points?
http://www.virtualnerd.com/algebra-1/linear-equation-analysis/point-slope-standard-form/point-slope-examples/point-slope-line-two-points-example

KEY IDEAS & TERMS	SUMMARY

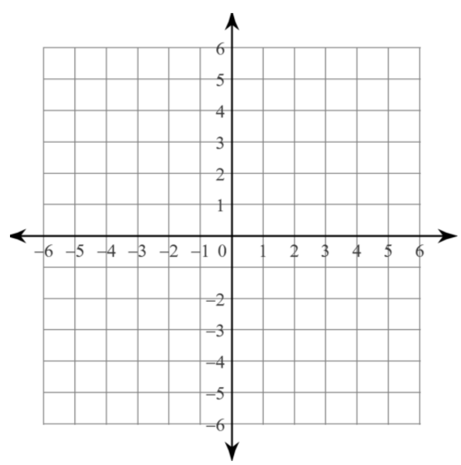
EXAMPLES:

Graph the line that passes through the given points. Write an equation for the line in point-slope form. Then rewrite the equation in slope-intercept form.

1. $(5, -3)$ & $(-5, -5)$



2. $(-3, 4)$ & $(3, -6)$



EXAMPLES: Problem Solving

3. The cost to place an ad in a newspaper for one week is a linear function of the number of lines in the ad. Write an equation in slope-intercept form that represents the function. Then find the cost of an ad that is 18 lines long.

City Gazette

Newspaper Ad Costs

Lines	3	5	10
Cost ($)	13.50	18.50	31

   a. First find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

   b. Then use point-slope form to write the equation.

   c. Finally, write the equation in slope-intercept form.

   d. Find the cost of an ad that is 18 lines long.

4. Tim decided to go on a road trip. Three hours after burning gas, he had traveled 220 miles, and five hours after buying gas, he had traveled 350 miles.
   a. Independent variable (x):
   b. Dependent variable (y):
   c. Points on the function:
   d. Write is the constant rate of change (slope) for the described function?
   e. Write an equation in both point-slope form and slope-intercept form that represents the function described.

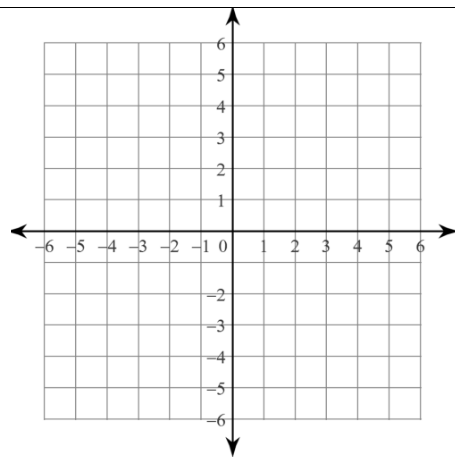
3.7 Standard Form of a Linear Equation

Objectives:

- Identify the standard form of a linear function
- Use the definitions of the x-intercept and y-intercept to find the intercepts of a standard form line and graph the line.
- Relate the constants A, B, and C to the values of the x-intercept, y-intercept, and slope.

LESSON: How do use x- and y-intercepts to graph a line in standard form.

http://www.virtualnerd.com/algebra-1/linear-equation-analysis/intercept/intercept-examples/x-y-intercepts-graph-standard-line

KEY IDEAS & TERMS	GRAPH
$$\boldsymbol{Ax + By = C}$$ Where A, B, and C are real numbers, and A and B are not both zero.	

❖ Intercepts

- ➢ x-intercept: the x-coordinate of the point where a line crosses the x-axis: $(x \cdot \text{int.}, 0)$
  - ▪ find the x-intercept by substituting 0 in for y and solving for x
- ➢ y-intercept: the y-coordinate of the point where a line crosses the y-axis: $(0, y \cdot \text{int.})$
  - ▪ find the y-intercept by substituting 0 in for x and solving for y

EXAMPLES:

Find the x- and y-intercepts of each linear equation. Use the intercepts to find the slope.

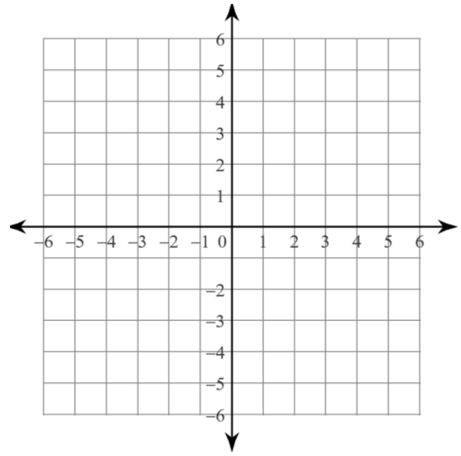
1. $-6x + 3y = -9$ Slope:

2. $9x - 6y = -72$ Slope:

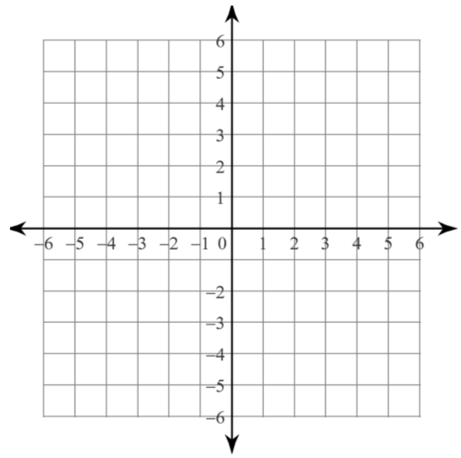
EXAMPLES:

Graph each linear equation using the x- and y-intercepts.

3. $2x + 3y = 12$

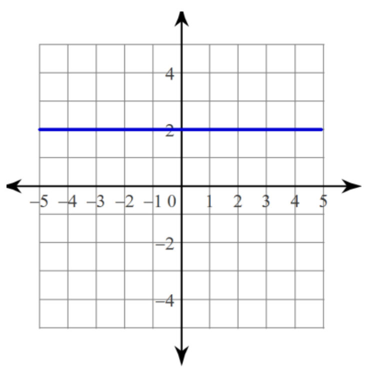


4. $5x - 3y = 15$

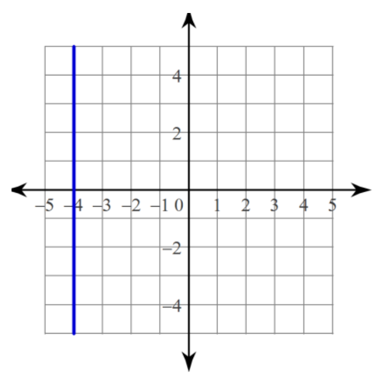


In the standard form of a linear equation $Ax + By = C$, either A or B – but not both – may be zero. If A or B is zero, then the line is either horizontal or vertical.

- Horizontal Lines
  - Slope:
  - y-intercept:
  - x-intercept:
  - Equation:



- Vertical Lines
  - Slope:
  - y-intercept:
  - x-intercept:
  - Equation:



EXAMPLES:

For each linear equation, tell whether its graph is a horizontal or a vertical line.

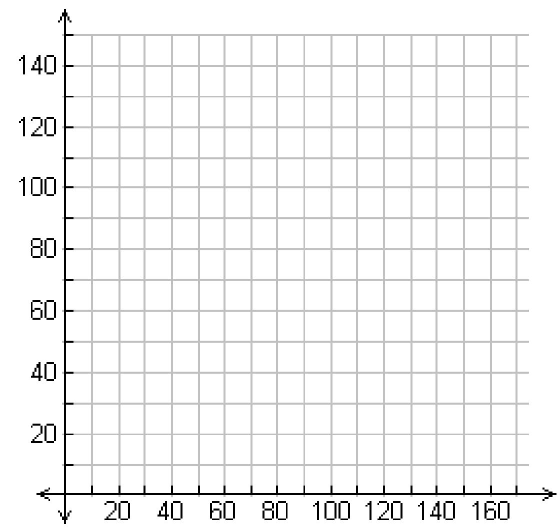
5. $y = 3$ 6. $x = 4$ 7. $x = 4.5$ 8. $y = 0$

EXAMPLES: Problem Solving

9. The freshman class holds a car wash to raise money. A local merchant donates all of the supplies. A wash costs $5 per car and $6.50 per van or truck.

   a. Let _______ represent the number of cars and _______ represent the number of vans or trucks.

   b. Write an equation, in standard form, to relate the number of cars and vans or trucks the students must wash to raise $800.



   c. How many cars will the students need to wash if no trucks or vans come to the car wash?

   d. How many trucks or vans will the students need to wash if no cars come to the car wash?

   e. Sketch a graph of the relationship between the number of cars and the number of vans or trucks that need to be washed in order to raise $800.

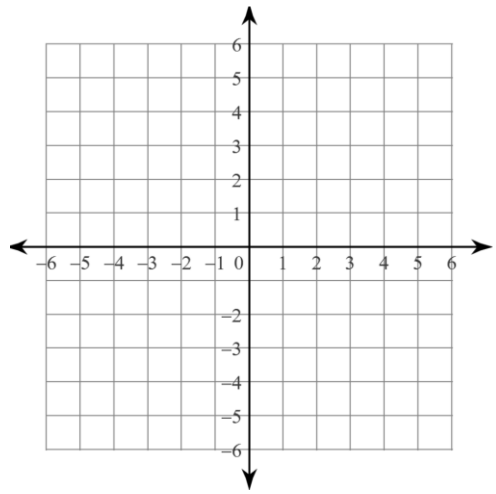
3.8 Graphing Transformations & the Absolute Value Function

Objectives:

- Develop the parent function for an absolute value function
- Identify the effect on the graph of the absolute value function replacing $f(x)$ by $f(x)+k$, $kf(x)$, $f(kx)$, and $f(x+k)$ for specific values of k (both positive or negative)
- Graph absolute value functions using graphing transformations

❖ The Absolute-Value Function: $f(x) = |x|$

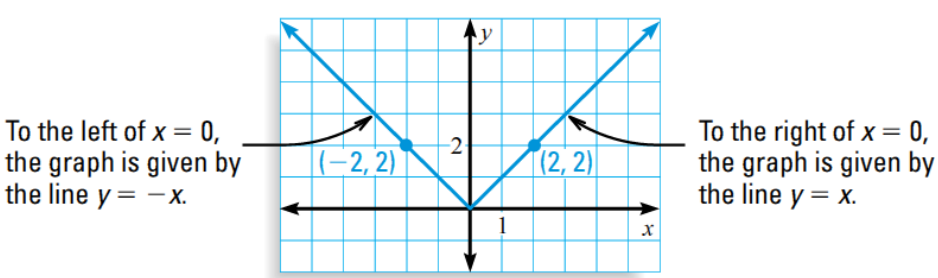
x	$f(x) = \lvert x \rvert$
-3	
-2	
-1	
0	
1	
2	
3	



❖ Characteristics of the Absolute-Value Function

- Axis of symmetry: the y-axis $(x = 0)$
- Vertex: $(0, 0)$
- Domain: ____________________
- Range: ____________________
- x-intercept: __________
- y-intercept: __________
- Increasing: ____________________
- Decreasing: ____________________
- Maximum/minimum of _____ at x = _____

The graph of this piecewise function consists of two rays, is V-shaped, and opens up. The corner point of the graph, called the **vertex,** occurs at the origin.



Notice that the graph of $y = |x|$ is symmetric in the y-axis because for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph.

- Transformations
  - Transformations – functions that map real numbers to real numbers
- Graphing Transformations
  - ALL functions can be written in such a way:

$$\boldsymbol{y = \pm a\, f\big(b(x \pm h)\big) \pm k}$$

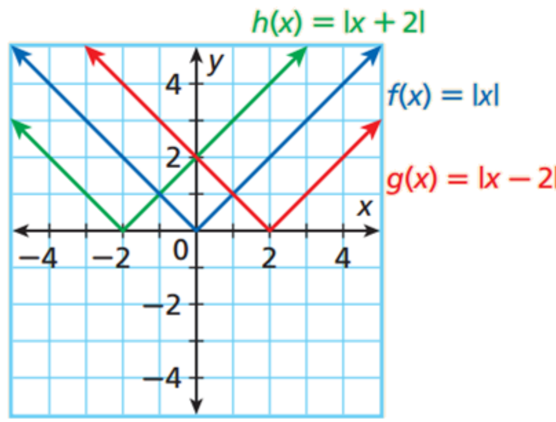
- Reflections Across Axes (flips)
  - The following transformations result in reflections of the graph $y = f(x)$
    - $y = -f(x)$: a reflection across the x-axis
    - $y = f(-x)$: a reflection across the y-axis
- Size Changes: Stretches & Compressions
  - Let a be a positive real number. Then the following transformations result in VERTICAL size changes of the graph of $y = f(x)$.
    - $y = af(x)$
      - A stretch by a factor of a if $a > 1$
      - A compression by a factor of a if $0 < a < 1$
  - Let b be a positive real number. Then the following transformations result in HORIZONTAL size changes of the graph of $y = f(x)$.
    - $y = f\left(\frac{x}{b}\right)$
      - A stretch by a factor of b if $b > 1$
      - A compression by a factor of b if $0 < b < 1$
- Translations (shifts)
  - Let h be a positive real number. Then the following transformations result in HORIZONTAL translations of the graph $y = f(x)$
    - $y = f(x - h)$ a shift right h units
    - $y = f(x + h)$ a shift left h units
  - Let k be a positive real number. Then the following transformations result in VERTICAL translations of the graph $y = f(x)$
    - $y = f(x) + k$ a shift up k units
    - $y = f(x) - k$ a shift down k units

TODAY'S TRANSFORMATION ~ Translations

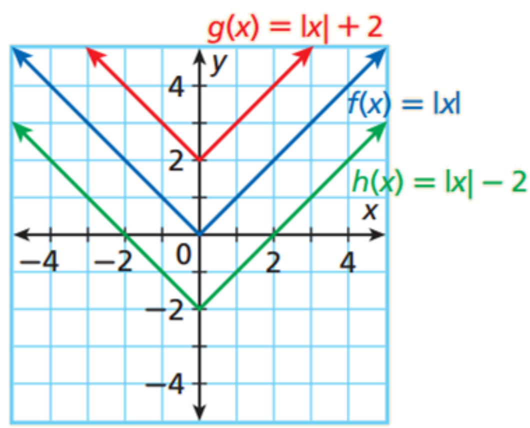
$$f(x) = |x - h| + k$$

- Translations (shifts)
  - Let h be a positive real number. Then the following transformations result in HORIZONTAL translations of the graph $y = |x|$
    - $y = |x - h|$ a shift right h units
    - $y = |x + h|$ a shift left h units
  - Let k be a positive real number. Then the following transformations result in VERTICAL translations of the graph $y = |x|$
    - $y = |x| + k$ a shift up k units
    - $y = |x| - k$ a shift down k units

What do you observe when $|x + h|$ or $|x - h|$?



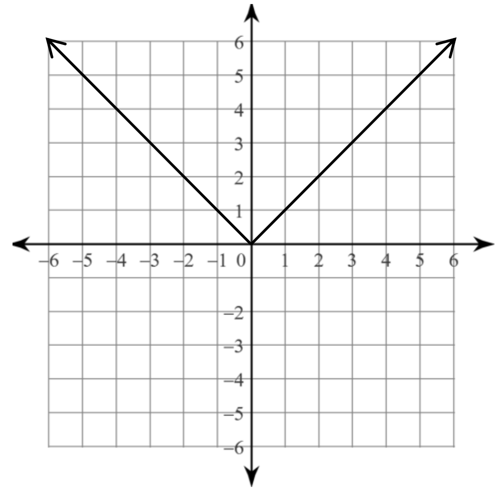
What do you observe when $|x| + k$ or $|x| - k$?



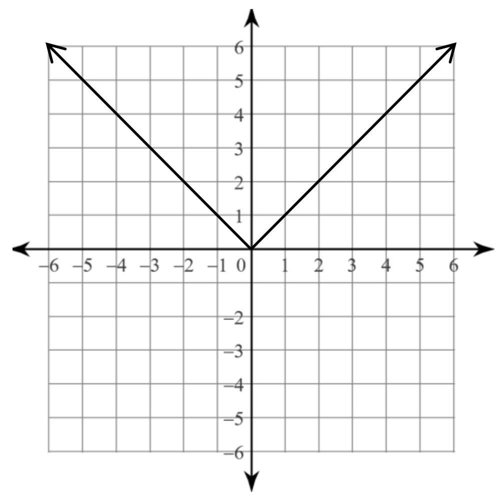
EXAMPLES

Describe the graphing transformation made to the graph of $y = |x|$ and then graph.

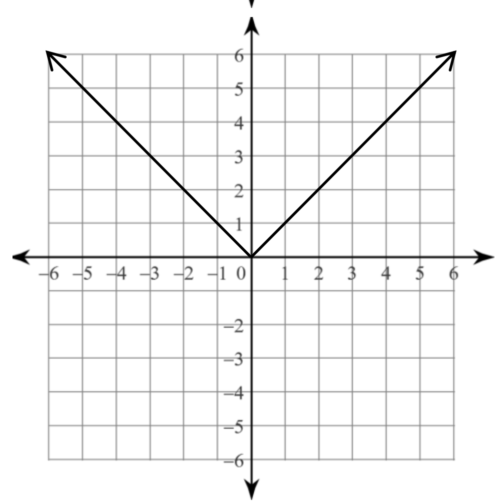
1. $y = |x| - 3$



2. $y = |x - 4|$



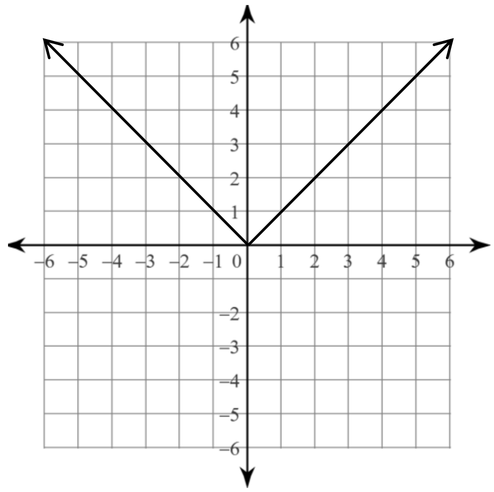
3. $y = |x - 4| - 3$



4. How do you think the graph of $y = |x| + 3$ would be related to the graph of the parent function $y = |x|$?

5. How do you think the graph of $y = |x+4|$ would be related to the graph of the parent function $y = |x|$?

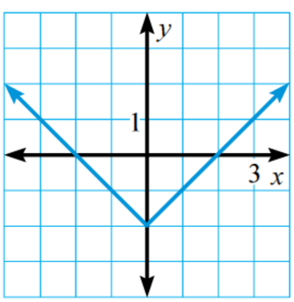
6. Predict how the graph of $y = |x+3| + 1$ is related to the graph of the the parent function $y = |x|$? Check your prediction by graphing the function.

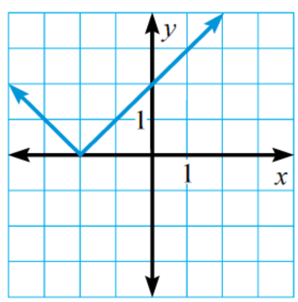


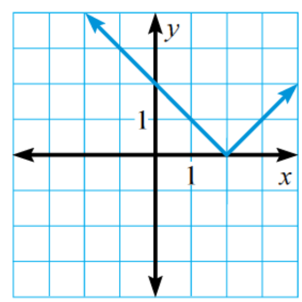
EXIT PROBLEM

Match the equation of the absolute value function to its graph.

1. $y = |x-2|$
2. $y = |x| - 2$
3. $y = |x+2|$

A.


B.


C.


3.8 Graphing Transformations & the Absolute Value Function

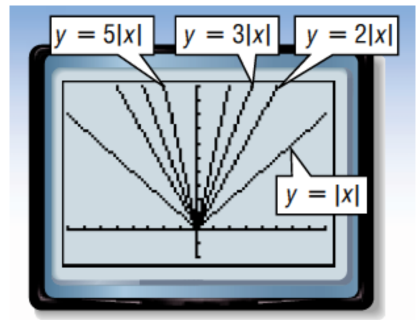
Objectives:

- Develop the parent function for an absolute value function
- Identify the effect on the graph of the absolute value function replacing $f(x)$ by $f(x)+k$, $kf(x)$, $f(kx)$, and $f(x+k)$ for specific values of k (both positive or negative)
- Graph absolute value functions using graphing transformations

GRAPHING CALCULATOR LAB

Family of Absolute Value Graphs

The calculator screen shows the graphs of $y = |x|$, $y = 2|x|$, $y = 3|x|$, and $y = 5|x|$.



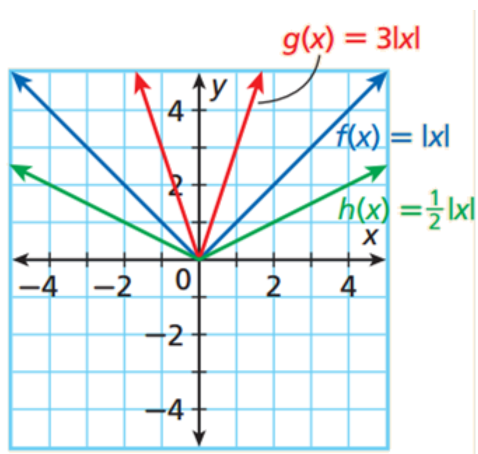
[−8, 8] scl: 1 by [−2, 10] scl: 1

1. What do these graphs have in common?

2. Describe how the graph of $y = a|x|$ changes as a increases. Assume $a > 0$.

3. What is true about the graph of $y = a|x|$ when $a < 0$?

What do you observe when $|a| > 1$ or $|a| < 1$?



TODAY'S TRANSFORMATION ~ Vertical Size Changes

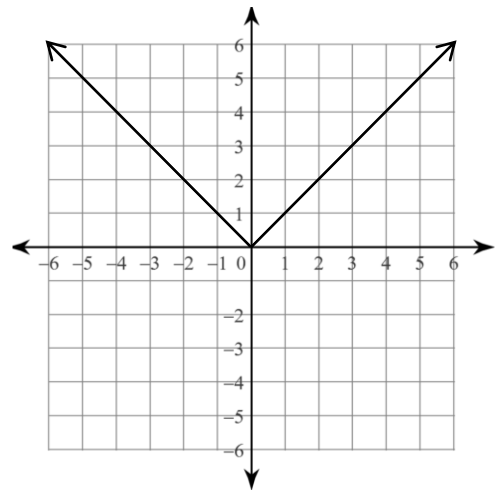
$$f(x) = a|x|$$

- Size Changes: Stretches & Compressions
  - Let a be a positive real number. Then the following transformations result in VERTICAL size changes of the graph of $y = |x|$
    - $y = a|x|$
      - A stretch by a factor of a if $a > 1$
      - A compression by a factor of a if $0 < a < 1$

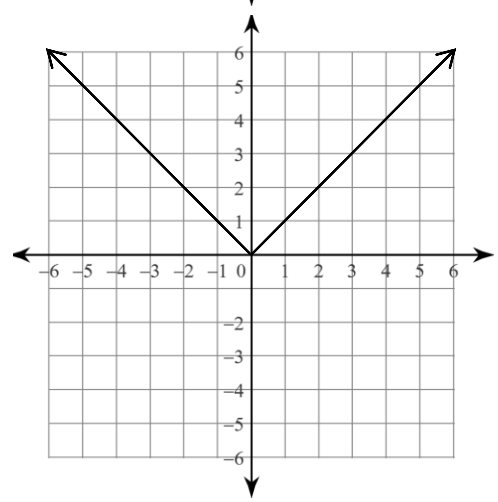
EXAMPLES

Describe the graphing transformation made to the graph of $y = |x|$ and then graph.

4. $y = 2|x|$



5. $y = \frac{1}{3}|x|$

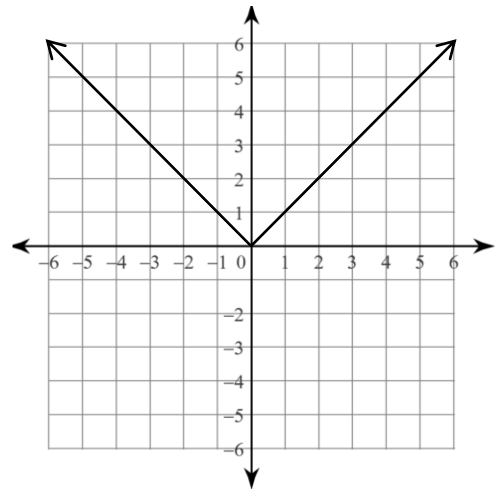


EXAMPLES

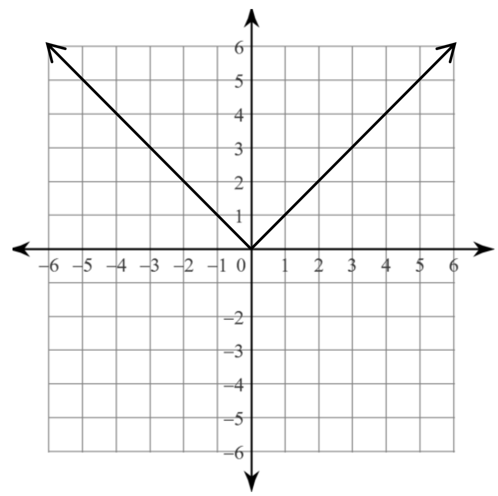
Describe the graphing transformations made to the graph of $y = |x|$ and then graph.

Hint: There is a size change <u>and</u> a translation.

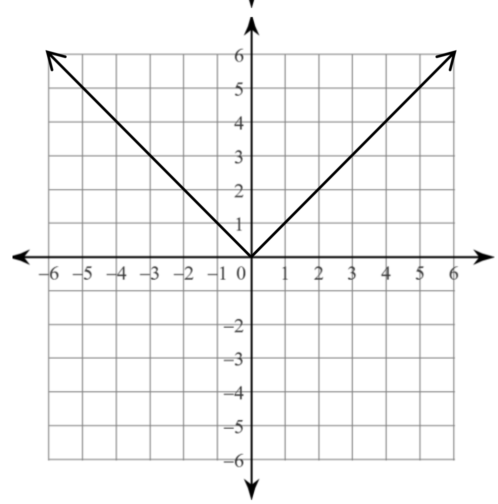
6. $y = 3|x + 2|$



7. $y = \frac{2}{3}|x| + 3$



8. $y = 2|x - 1| - 4$

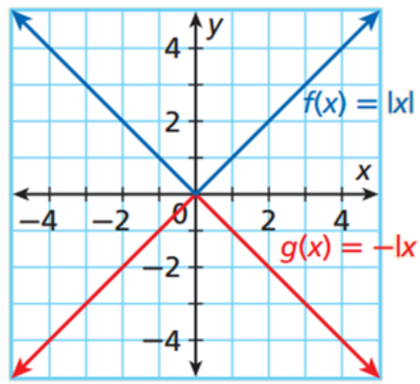


3.8 Graphing Transformations & the Absolute Value Function

<u>Objectives</u>:

- Develop the parent function for an absolute value function
- Identify the effect on the graph of the absolute value function replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x+k)$ for specific values of k (both positive or negative)
- Graph absolute value functions using graphing transformations

What do you observe when $a > 0$ or $a < 0$?



TODAY'S TRANSFORMATION ~ Reflections Across Axes

$$f(x) = \pm a|x|$$

❖ Reflections Across Axes (flips)
- ➢ The following transformations result in reflections of the graph $y = f(x)$
  - $y = -f(x)$: a reflection across the x-axis
  - $y = f(-x)$: a reflection across the y-axis

SCHULTZ SAYS ~

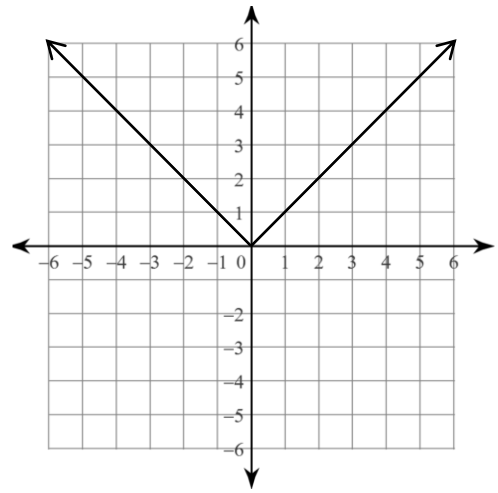
When performing multiple transformations, graph them in the following order: translations (slides), reflections (flips), and then size changes.

EXAMPLES

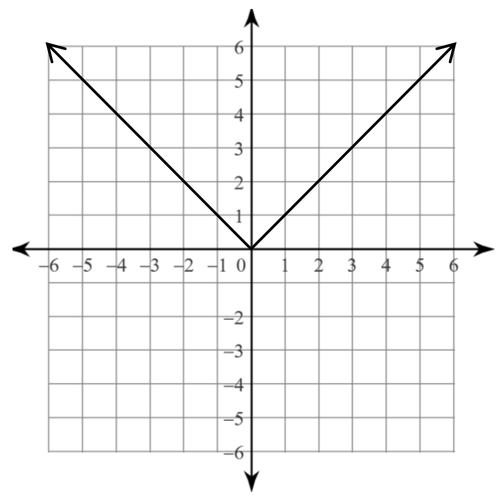
Describe the graphing transformations made to the graph of $y = |x|$ and then graph.

Hint: There are <u>two</u> transformations in each.

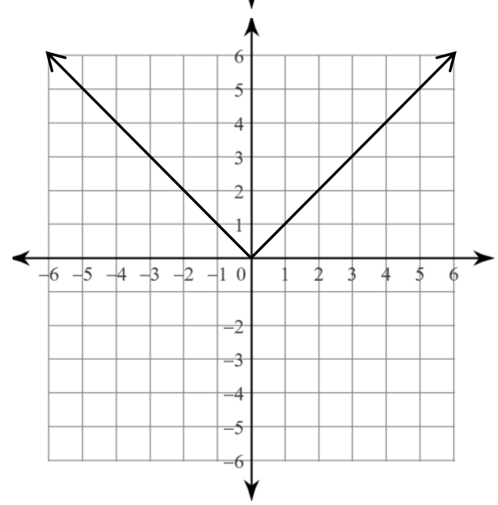
1. $y = -|x + 4|$



2. $y = -|x| + 3$



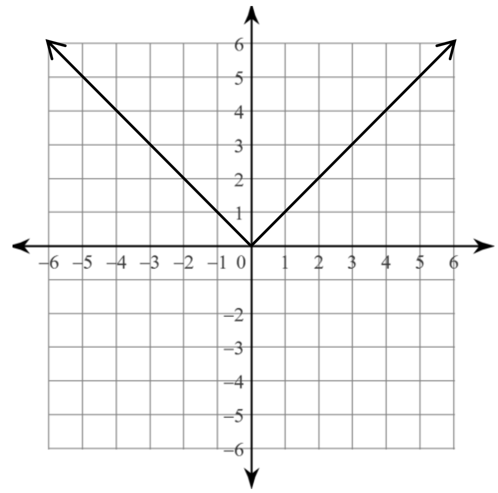
3. $y = |-(x - 4)|$



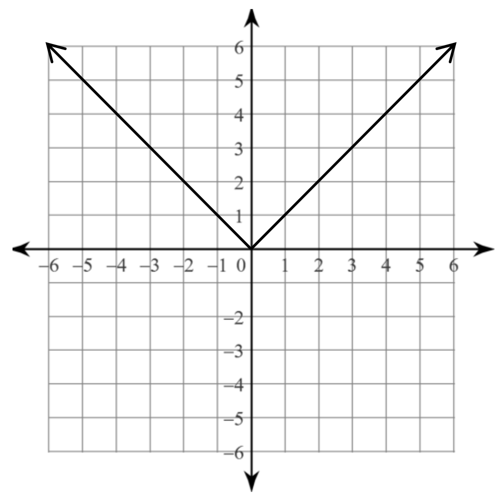
EXAMPLES

Describe the graphing transformations made to the graph of $y = |x|$ and then graph.
Hint: There are three or four transformations in each.

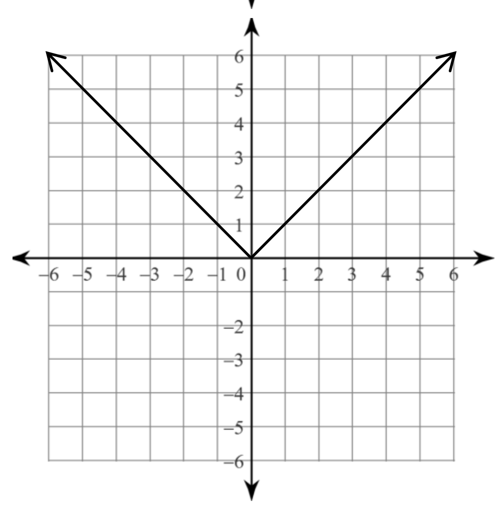
4. $y = -\frac{3}{4}|x - 2|$



5. $y = 2|-x| + 1$



6. $y = -\frac{1}{2}|x + 2| - 4$



3.8 Writing Equations of Absolute Value Functions

Objective:

- Given the graph of an absolute value function, identify the changes made to the parent function y = |x| and write the equation of the graph shown

❖ Write an equation of the graph shown.

- ➢ Identify the vertex: (h, k)
- ➢ Select another point on the graph: (x, y)
- ➢ Find the value of a by substituting h, k, x, and y into the absolute value equation:

$$y = a|x - h| + k$$

- ➢ Write the equation.

Vertex: $(h, k) = (0, -3)$

Point: $(x, y) = (2, 1)$

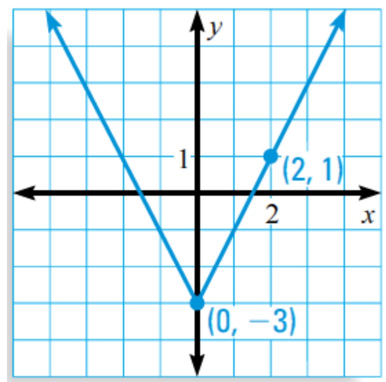
Substitute: $1 = a|2 - 0| + (-3)$

$1 = a|2| - 3$

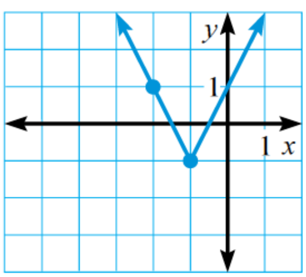
$4 = a \cdot 2$

$2 = a$

Equation: $y = 2|x| - 3$



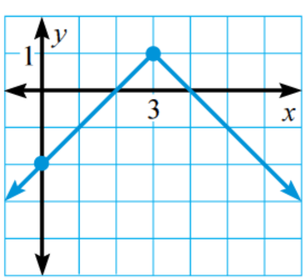
1.



Vertex	h	k
Point	x	y

$y = a|x - h| + k$

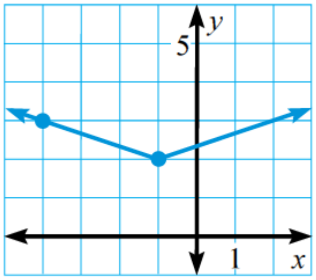
2.



Vertex	h	k
Point	x	y

$y = a|x - h| + k$

3.



Vertex	h	k
Point	x	y

$y = a|x - h| + k$

3.9 Piecewise Functions

<u>Objectives</u>:

- Graph piecewise functions

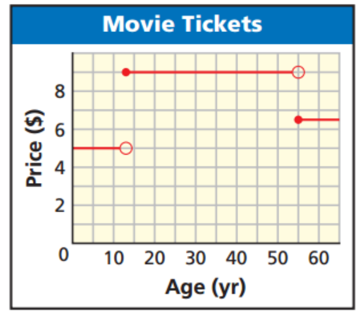
❖ What is a piecewise function?

➢ A function that is a combination of one or more functions.

- The rule for a piecewise function is different for different parts/pieces of the domain.
  - Example:

    Movie ticket prices are often different for different age groups. So the function for movie ticket prices would assign a different value (ticket price) for each domain interval (age group).

Movie Tickets	
Age	**Price ($)**
0–12	5.00
13–54	9.00
55+	6.50



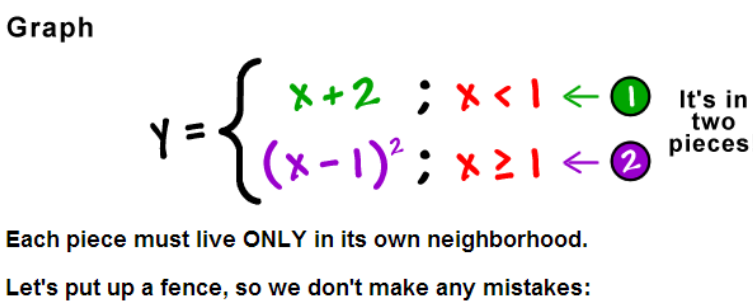
EXAMPLES:

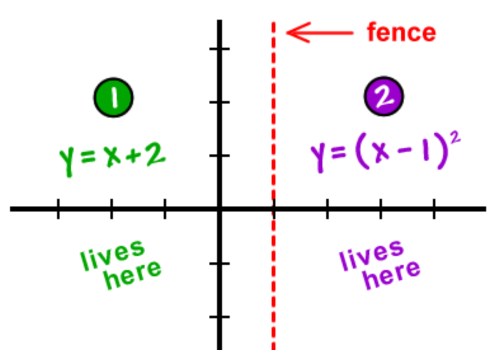
1. Evaluate $f(x)$ when **(a)** $x = 0$, **(b)** $x = 2$, and **(c)** $x = 4$.

$$f(x) = \begin{cases} x + 2, & x < 2 \\ 2x + 1, & x \geq 2 \end{cases}$$

(a) $f(0)$ (b) $f(2)$ (c) $f(4)$

Graphing Piecewise Functions



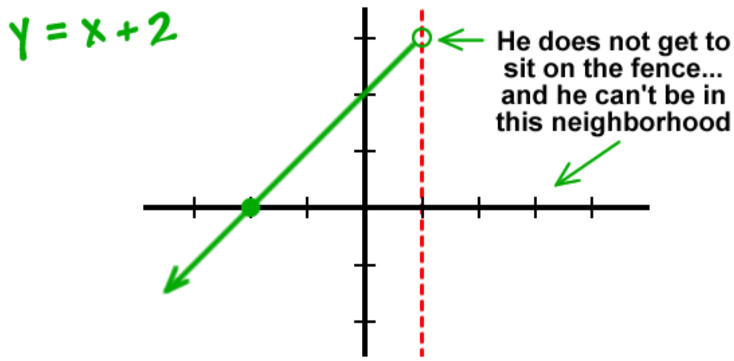


Now, we just need to figure out who the fence owner is...

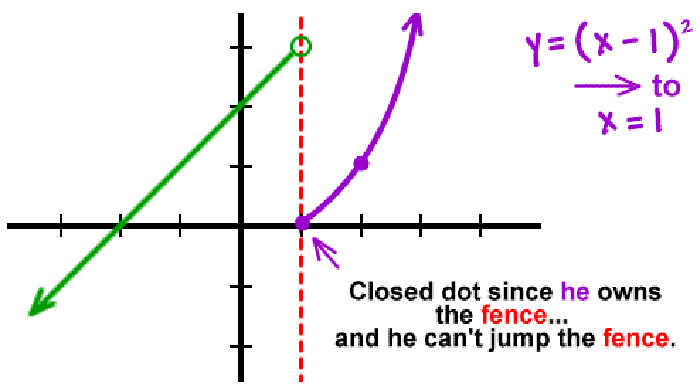
$$y = \begin{cases} x+2 & ; \; x < 1 \\ (x-1)^2 & ; \; x \geq 1 \end{cases}$$

This guy has the "=", so he gets to live ON the fence.

Let's graph part 1 :



Now, let's graph part 2 :



Student to Student

Graphing Piecewise Functions

Mateo Morales
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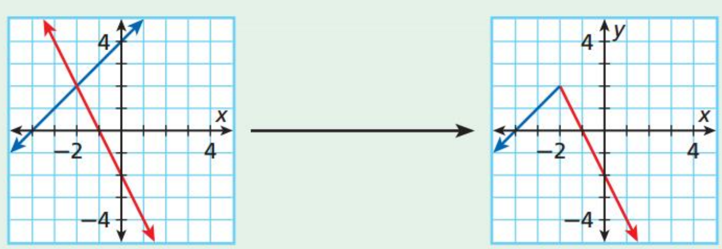
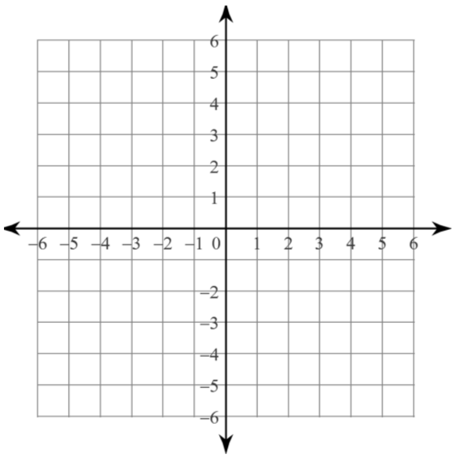
When I graph a piecewise function, I like to graph each piece like it's a separate function. Then I go back and erase the parts that are outside of the restricted domain.

Example: $f(x) = \begin{cases} x+4 & \text{if } x < -2 \\ -2x & \text{if } x \geq -2 \end{cases}$

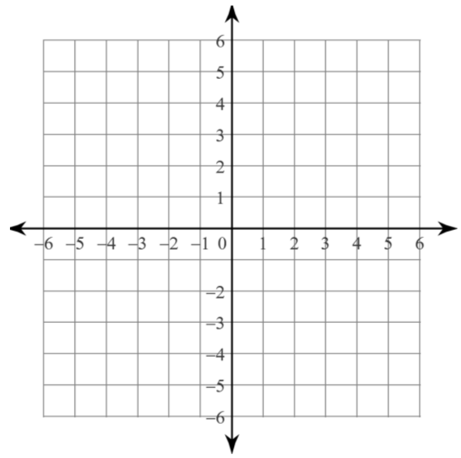


Graph the functions.

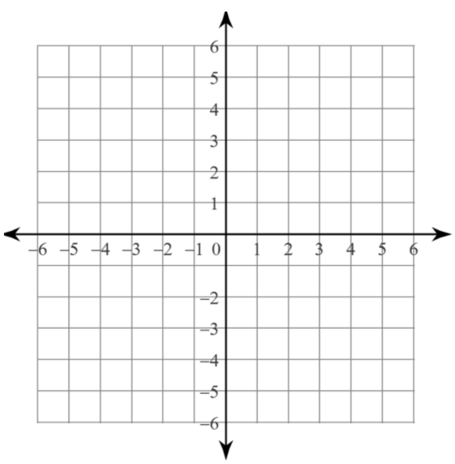
2. $f(x) = \begin{cases} 2x, & x < -1 \\ 4, & x \geq -1 \end{cases}$



3. $f(x) = \begin{cases} 2x + 1, & x < 1 \\ -x + 4, & x \geq 1 \end{cases}$



4. $f(x) = \begin{cases} 2x - 1, & -2 \leq x < 1 \\ -x + 3, & 1 \leq x \leq 4 \end{cases}$



5. $f(x) = \begin{cases} x + 2, & x < -1 \\ 3x, & -1 \leq x < 2 \\ -x + 1, & x \geq 2 \end{cases}$