Name: ______________________________

Unit 5: Systems of Linear Equations & Inequalities

5-1 Solving Systems of Equations by Graphing

OBJECTIVE #1 – SOLVE SYSTEMS OF EQUATIONS BY GRAPHING

✤ System of equations—Two or more equations with the same variables

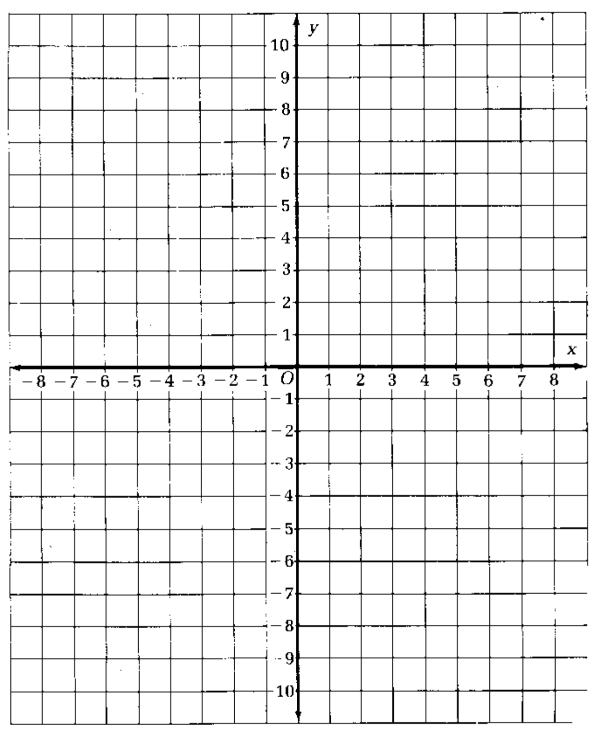
- A solution to the system is an ordered pair that represents a point on both lines and satisfies both equations.
  - Can be solved by graphing each equation on the same coordinate plane
    - The point of intersection represents the solution

How to Graph a Line

1. Rewrite the equation in slope-intercept form: $y = mx + b$
2. Start at b on the y-axis; m represents the slope: if positive, go up; if negative, go down (the number of units in the numerator); & always go right (the number of units in the denominator).
3. Be aware of special lines:
   a. x = #: vertical line through x
   b. y = #: horizontal line through y

Example: Solving a System

1. $y = 2x + 9$
   $y = -x + 3$



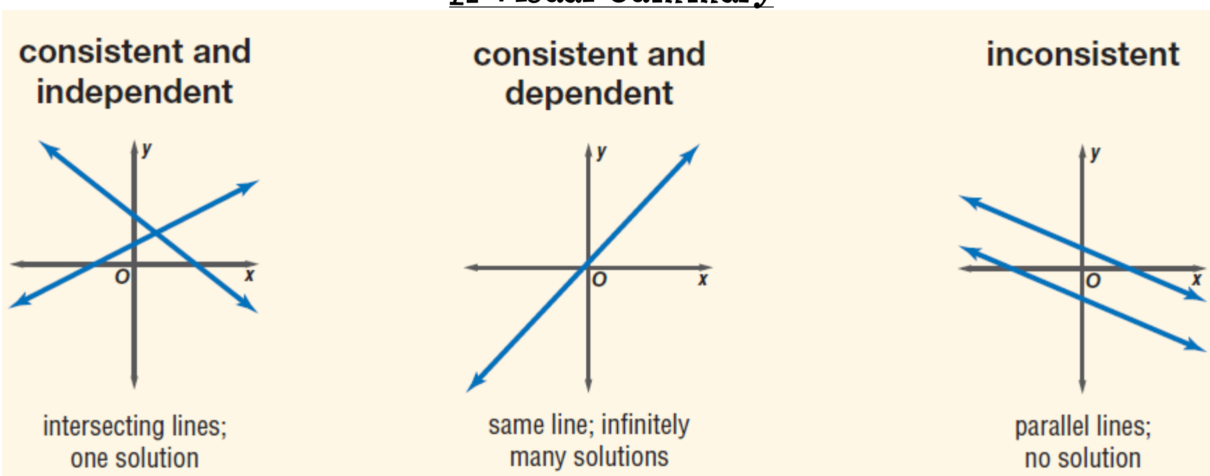
When using a graph to find a solution, always check the ordered pair in both of the original equations.

OBJECTIVE #2 – DETERMINE WHETHER A SYSTEM OF LINEAR EQUATIONS IS CONSISTENT & INDEPENDENT, CONSISTENT & DEPENDENT OR INCONSISTENT

✻For any two lines in a plane, the lines must: intersect, be coincident, or be parallel

LINES…	INTERSECT	COINCIDE	ARE PARALLEL
Name (of system)	Consistent & Independent	Consistent & Dependent	Inconsistent
Slopes	Different	Same slopes; same intercepts	Same slope; different intercepts
# of Solutions	One	Infinite	None

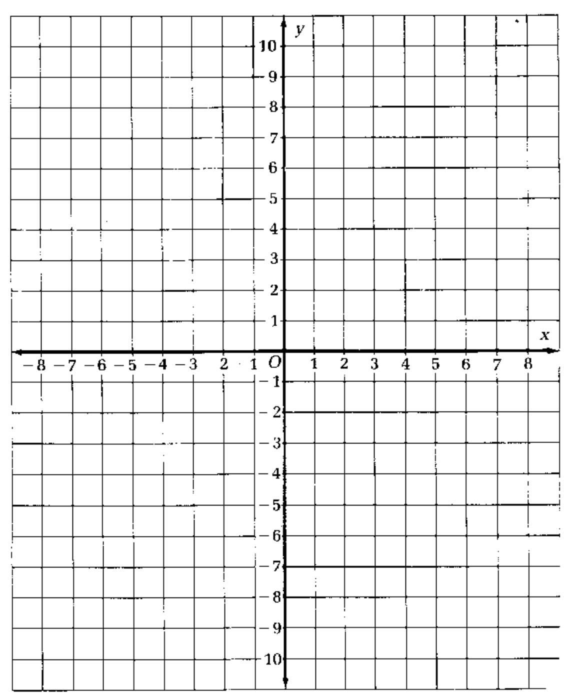
A Visual Summary



Examples: Classifying a System of Equations

Graph each system of equations & state its solution and then state whether the system is consistent & independent, consistent & dependent, or inconsistent.

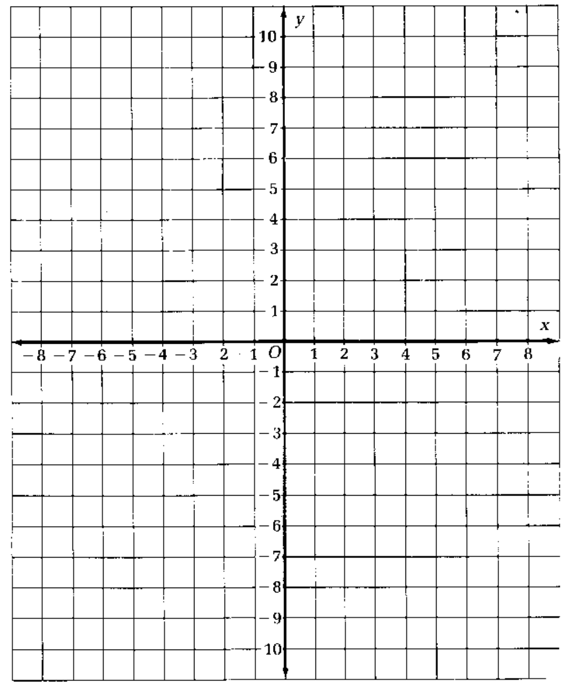
2. $x + y = 5$
$3x - 2y = 20$



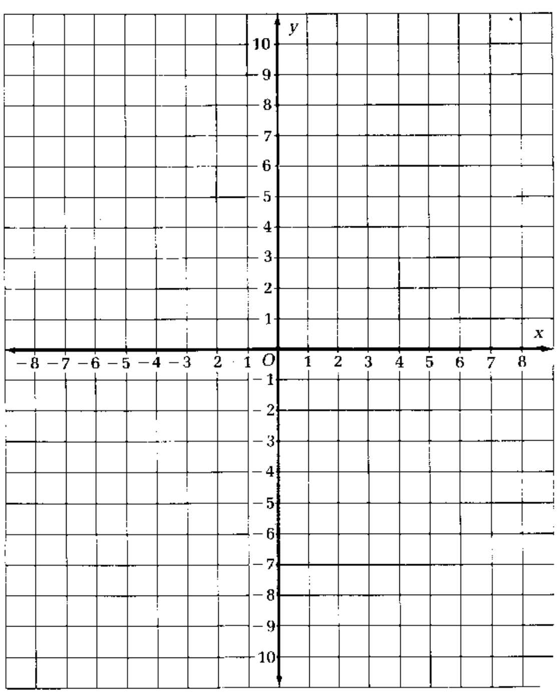
Examples: Classifying a System of Equations

Graph each system of equations & state its solution and then state whether the system is consistent & independent, consistent & dependent, or inconsistent.

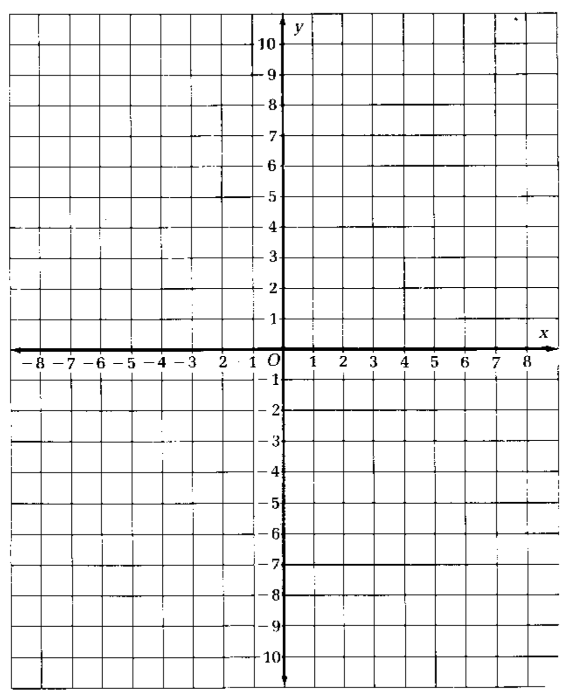
3. $y = -3x - 12$
$2x + 3y = -15$



4. $y = -3x + 5$
$9x + 3y = 15$



5. $y = 2x + 3$
$2x - y + 7 = 0$

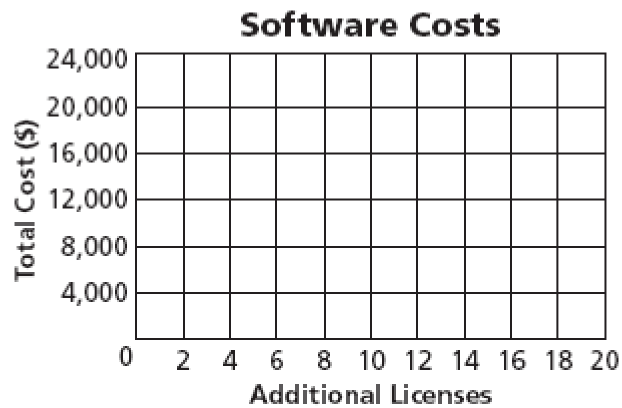


✤Business Application: "Breaking Even"

- Systems are used in business to determine the break-even point – the point at which the income equals the cost

Example:
Location Mapping needs new software. Software A costs \$12,000 plus \$500 per additional site license. Software B costs \$4000 plus \$2000 per additional site license.

a. Write two equations that represent the cost of each software license.



b. Graph the equations.

c. Estimate the break-even point of the software costs.

d. If Location Mapping plans to buy 10 additional site licenses, which software will cost less?

5-2 Solving Systems of Equations Algebraically

OBJECTIVE – SOLVE SYSTEMS OF LINEAR EQUATIONS BY USING ELIMINATION

The Elimination Method

- Use when the coefficients of one of the variables in both equations are additive inverses

n + (–n) = 0

- Eliminate one of the variables by adding or subtracting the equations
  - When you add two true equations, the result is a new equation that is also true
- It may be necessary to use multiplication to write an equivalent equation so that one of the variables has the same or opposite coefficient in both equations

Examples: Solving Systems Algebraically

1. $-8x - 9y = 9$

   $x + 9y = -9$

2. $2x + 3y = 23$

   $11x + 3y = 32$

3. $-6x - 5y = 1$

   $3x + 11y = 8$

4. $-10x + 6y = -22$

$4x - 9y = -11$

Error Analysis

Juanita & Jamal are solving the system $2x - y = 6$ & $2x + y = 10$. Who is correct? Explain your reasoning.

Juanita

$2x - y = 6$

$(-)2x + y = 10$

$0 = -4$

The statement $0 = -4$ is never true, so there is no solution.

Jamal

$2x - y = 6$

$(+)2x + y = 10$

$4x = 16$

$x = 4$

$2x - y = 6$

$2(4) - y = 6$

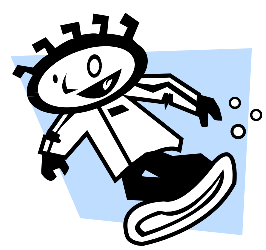
$8 - y = 6$

$y = 2$

The solution is $(4, 2)$.

Example: An Application Problem

All 28 members in Central High School's Ski & Snowboard Club went on a one-day ski trip. Members can rent skis for \$16 per day or snowboards for \$19 per day. The club paid a total of \$478 for rental equipment.



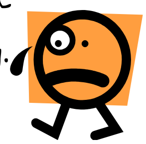
a. Identify your variables.

b. Write and solve a system of equations that represents the number of members who rented the two types of equipment.

c. How many members rented skis and how many rented snowboards? (Write your answer as a complete sentence.)

AN ALTERNATE METHOD:

Pick on the lonely guy.

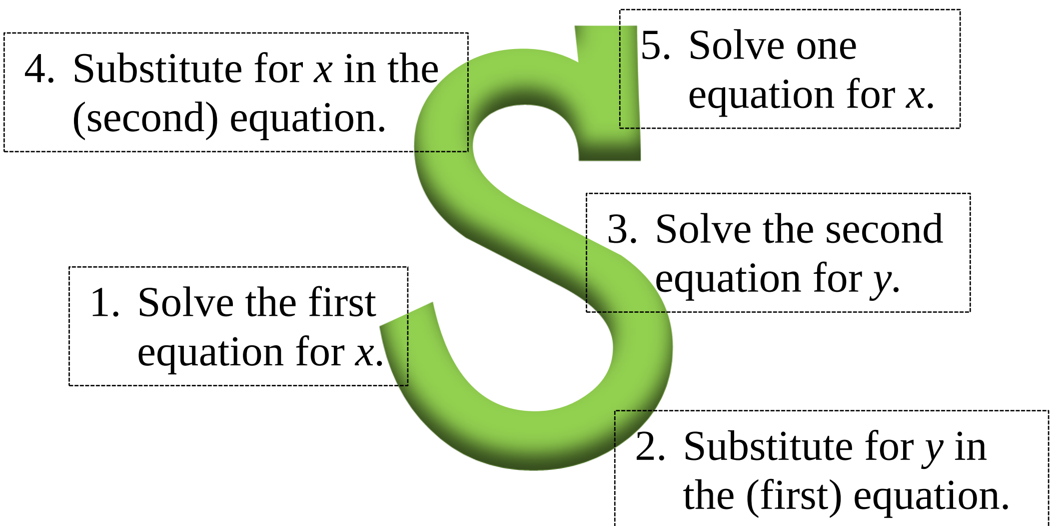


✤ The Substitution Method

Use when there is a "lonely variable"

- ➢ Solve one of the two equations for one of the variables.
- ➢ Substitute the resulting expression into the other equation
  - This results in a single equation in one variable that can be solved
  - This solution is substituted into one of the original two-variable equations & solved for the other variable

S is for Substitution:



<u>Examples: Solving Systems Algebraically</u>

1. $y = 3x + 15$
   $y = -2x + 5$

2. $y = -1$
   $-7x - 3y = 24$

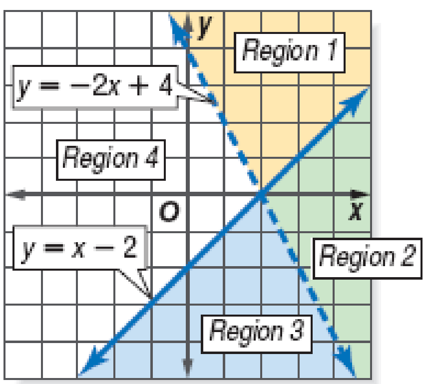
3. $10x - 2y = 26$
$y = 6x - 14$

4. $5x + y = 1$
$-2x + 9y = 9$

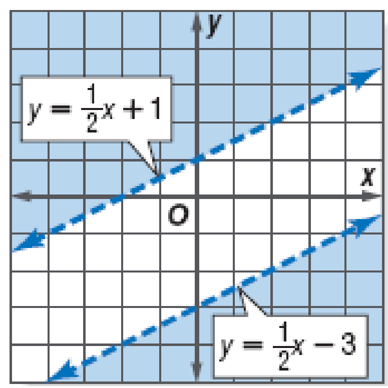
5-3 Solving Systems of Inequalities by Graphing

OBJECTIVE #1 – SOLVE SYSTEMS OF LINEAR INEQUALITIES BY GRAPHING

- Solving a system of inequalities by graphing is similar to solving a system of equations by graphing.
  - The inequalities in the system are graphed on the same coordinate plane.
    - The ordered pairs that satisfy all of the inequalities in the system are found.
      - The region that is common to the two inequalities must be determined.
      - If the two regions do not intersect, the solution is the empty set or no solution exists.



Region 2 is common to both inequalities.



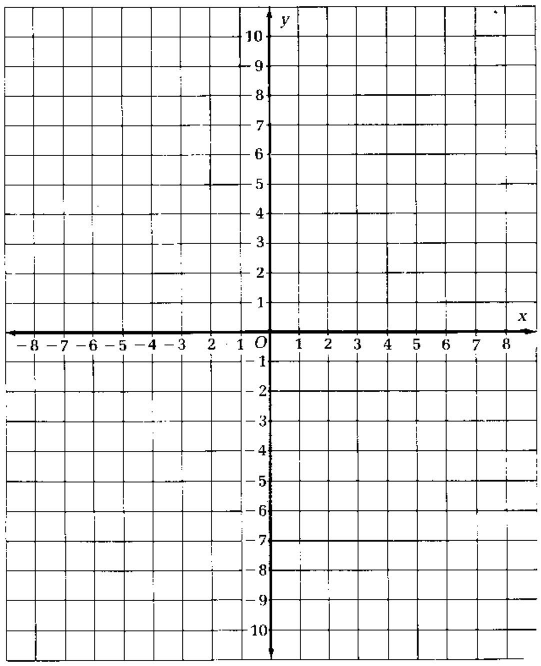
The two regions do not intersect; therefore no solution exists.

How to Graph Inequalities

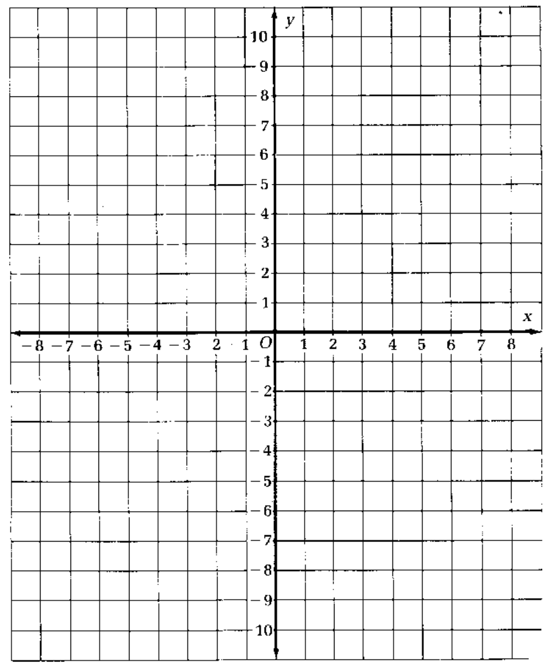
1. Graph the corresponding line; Be aware of special lines:
   a. x = #: vertical line through x
   b. y = #: horizontal line through y
2. Remember: solid if $\le$ or $\ge$ and dashed if $<$ or $>$
3. Test a point (or two); shade where true

Examples: Solve these systems of inequalities by graphing.

1. $y \geq 2x - 3$ & $y < -x + 2$



2. $y > \frac{1}{4}x + 4$ & $y < \frac{1}{4}x - 2$

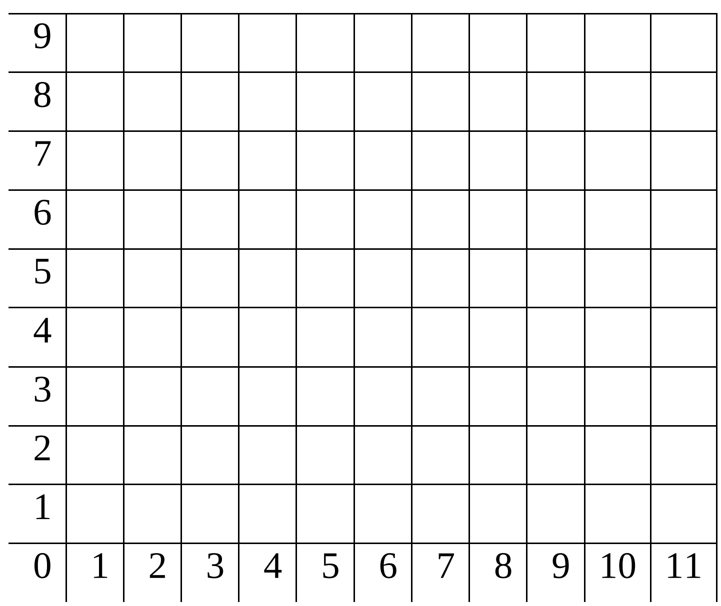


<u>A Real-World Example</u>

The most Jack can spend on bagels and muffins for the cross country team is \$28. A package of 6 bagels costs \$2.50. A package of 8 muffins costs \$3.50. He needs to buy at least 12 bagels and 24 muffins.

a) Set up and graph a system of inequalities that shows how many <u>packages</u> of each item he can purchase.

b) Give an example of three different purchases he can make.



OBJECTIVE #2 – DETERMINE THE COORDINATES OF THE VERTICES OF A REGION FORMED BY THE GRAPH OF A SYSTEM OF INEQUALITIES

Systems of 3 or More Linear Inequalities

- The graphs form a bounded polygonal region
  - The vertices of the region can be found by determining the coordinates of where the boundary lines intersect.

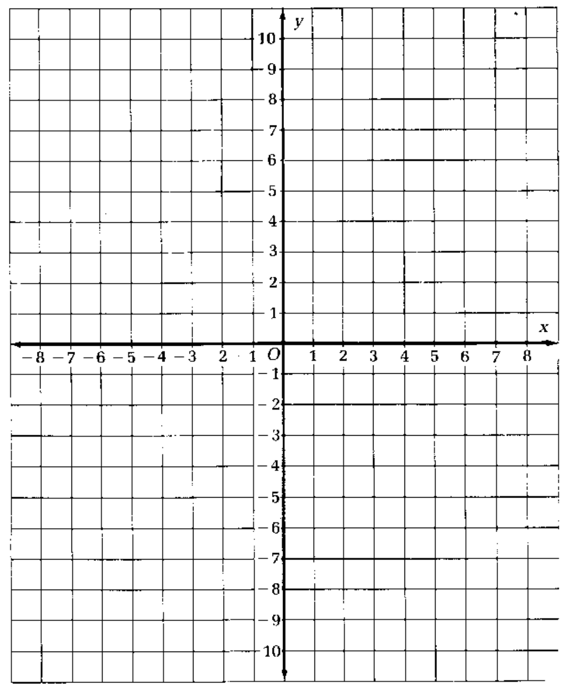
Example:

Find the coordinates of the vertices of the figure formed by:

$2x - y \geq -1$

$x + y \leq 4$

$x + 4y \geq 4$



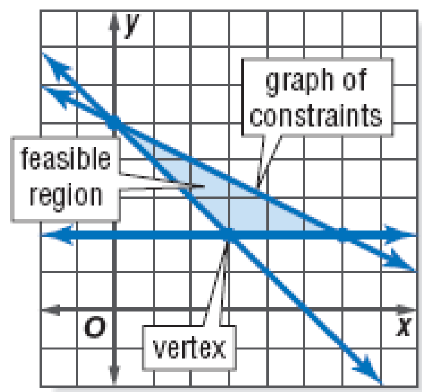
5-4 Linear Programming

OBJECTIVE #1 – FIND THE MAXIMUM & MINIMUM VALUES OF A FUNCTION OVER A REGION.

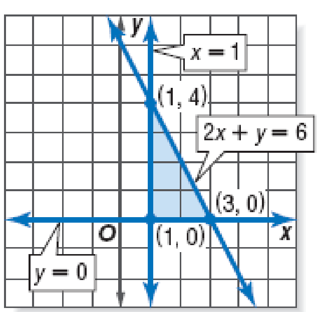
- Solving a linear programming problem adds two skills to the skills learned in the previous lesson.
  - After the system of inequalities has been graphed and the coordinates of the vertices of the feasible region determined, a given function must be evaluated using the vertex coordinates.
    - The solution to the problem is the point that maximizes or minimizes the function.

- Linear programming—A procedure for finding the maximum or the minimum value of a function in two variables, subject to given conditions on the variables, called constraints.

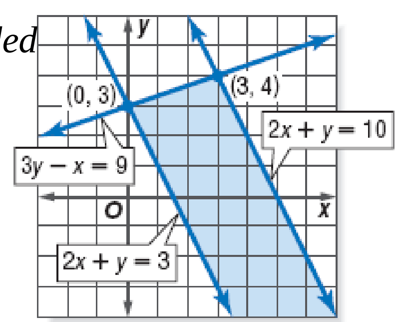
- In a linear programming problem…
  - Each of the inequalities in the system is called a constraint.
  - The region that represents the system's solution is called the feasible region.
  - The intersections of pairs of boundary lines are referred to as the vertices of the feasible region.
    - A bounded feasible region is one whose outline is a polygon.
    - An unbounded feasible is one whose boundary is not closed.



A bounded region



An unbounded region

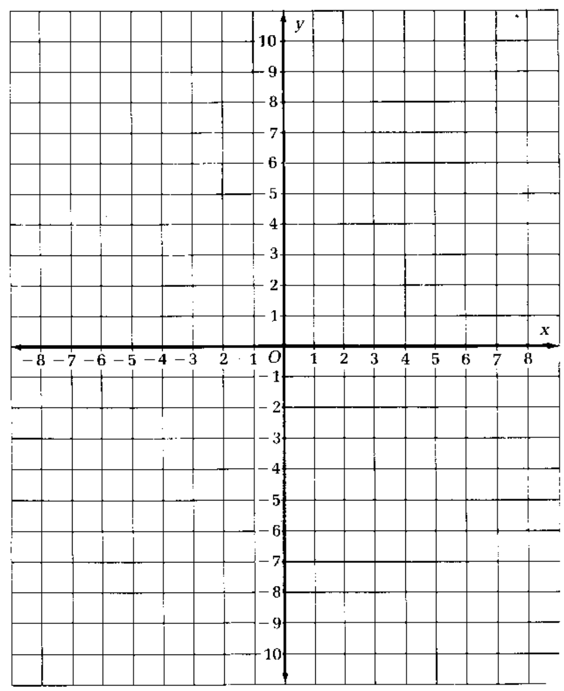


Common Misconception: Do not assume that there is no minimum value if the feasible region is unbounded below the line, or that there is no maximum value if the feasible region is unbounded above the line.

Examples

Graph the following system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function of this region.

1. $x \geq 0$
   $0 \leq y \leq 5$
   $x + y \leq 7$
   $3y \geq x - 3$
   $f(x, y) = 5x - 3y$



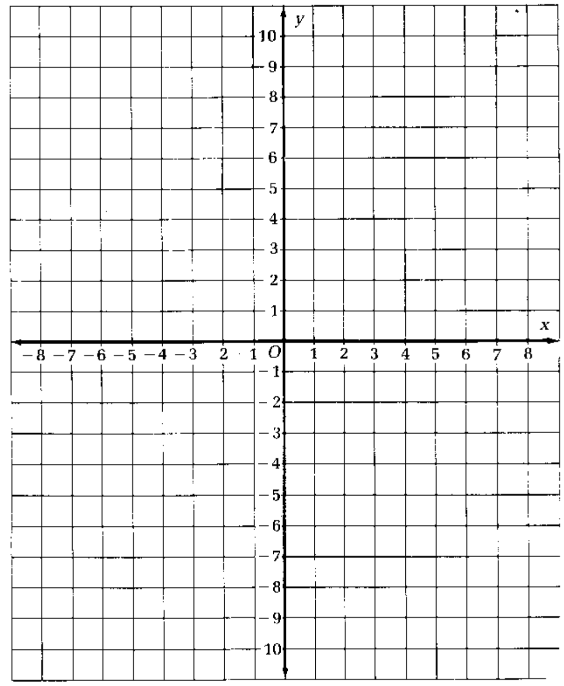
Step 1: Graphed the inequalities.

Step 2: Identify the vertices.

Step 3: Substitute the coordinates of the vertices into the function.
$f(x, y) = 5x - 3y$

Step 4: Identify the maximum & minimum values of $f(x, y)$.

2. $-x + 2y \le 2$
   $x - 2y \le 4$
   $x + y \ge -2$
   $f(x, y) = 2x + 3y$



OBJECTIVE #2 – SOLVE REAL WORLD PROBLEMS VIA LINEAR PROGRAMMING

Many practical problems can be solved by linear programming. These problems are of such a nature that certain constraints exist for the variables, and some function of these variables must be maximized or minimized. Use the following method to solve linear programming problems.

$<$	$>$	$\le$	$\ge$
▪ Is less than ▪ Is fewer than	▪ Is greater than ▪ Is more than	▪ Is less than or equal to ▪ Is at most ▪ Is no more than ▪ The greatest value of… ▪ The maximum value of…	▪ Is greater than or equal to ▪ Is at least ▪ Is no less than ▪ The least value of… ▪ The minimum value of…

<u>Linear Programming Problem Solving Procedure</u>

1. Define variables.
2. Write system of inequalities.
3. Graph the system of inequalities.
4. Find the coordinates of the vertices of the feasible region.
5. Write a linear function to be maximized or minimized.
6. Substitute the coordinates of the vertices into the function.
7. Select the greatest or least result. Answer the problem.

Example

The available parking area of a parking lot is 600 square meters. A car requires 6 square meters and a bus requires 30 square meters of space. The attendant can handle no more than 60 vehicles. If a car is charged $2.50 and a bus $7.50, how many of each should be accepted to maximize income? (Let c = the number of cars accepted. Let b = the number of buses accepted.)

a. Write an expression to represent the total income.

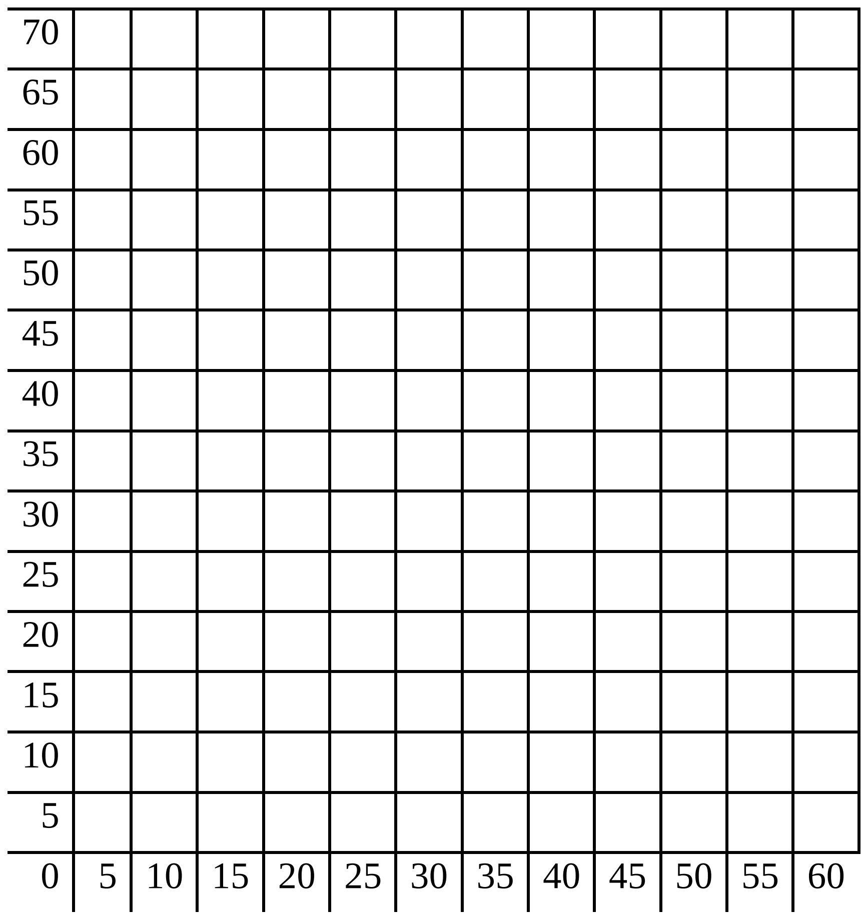
b. Write an inequality to represent the total number of cars & buses.

c. Write an inequality to represent the amount of space required for the cars and buses.

d. Graph the system of inequalities. (Show only the first quadrant since b & c cannot be negative.)

e. Identify & name the vertices of the polygon.

f. Evaluate the expression from part a for each vertex.



g. To maximize income, how many cars & how many buses should be accepted?

h. What is the maximum income?