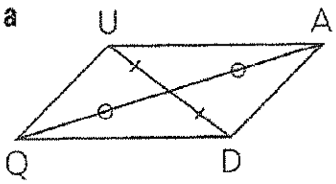
10.2.D1 PARALLELOGRAMS

- Parallelograms
  - A parallelogram is a quadrilateral with both pairs of opposite sides parallel. (definition)
  - Parallelogram/Congruent-Parallel Side Theorem
    - If one pair of opposite sides of a quadrilateral is both congruent and parallel, then the quadrilateral is a parallelogram.

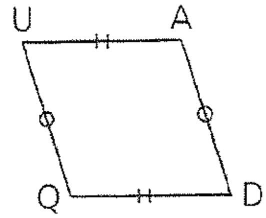
For a list of the properties of parallelograms, see page 829 of your text.

Examples

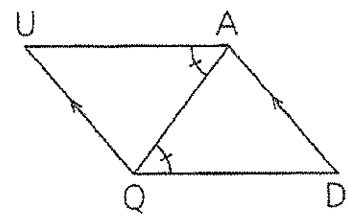
1. For each quadrilateral *QUAD*, state the property or definition that proves that *QUAD* is a parallelogram. (Refer to the "Properties of Parallelograms" on page 829 of your text.)

a


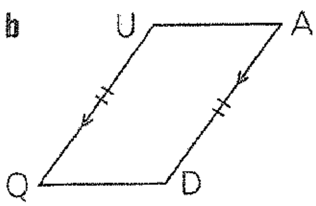
the diagonals of a //ogram bisect each other.

c


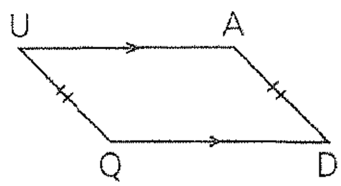
The opposite sides of a //ogram are ≅

e


$\angle UAQ \cong \angle DQU$
$\therefore \overline{UA} \parallel \overline{QD}$
Def. of //ogram

b


Parallelogram/ Congruent-Parallel Side theorem

d


Not a //ogram

2. Complete each statement about *JKLM*.

$\overline{JK} \cong$ _?_ $\overline{LM}$

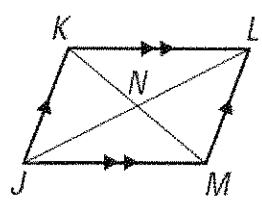
$\angle MLK \cong$ _?_ $\angle KJM$

$\angle JKL \cong$ _?_ $\angle LMJ$

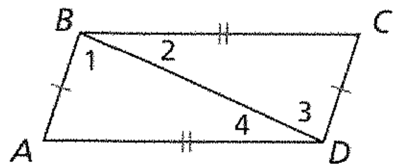
$\overline{JN} \cong$ _?_ $\overline{LN}$

$\angle MNL \cong$ _?_ $\angle JNK$

$\overline{NM} \cong$ _?_ $\overline{NK}$

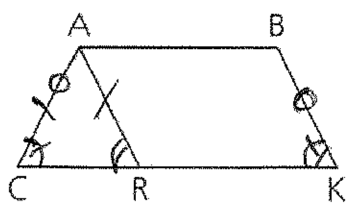


3. Given: $\overline{AB} \cong \overline{CD}$
$\overline{BC} \cong \overline{DA}$
Prove: $ABCD$ is a parallelogram



Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$	1. Given
2. $\overline{BC} \cong \overline{DA}$	2. Given
3. $\overline{BD} \cong \overline{BD}$	3. Reflexive
4. $\triangle DAB \cong \triangle BCD$	4. SSS
5. $\angle 1 \cong \angle 3$	5. CPCTC
6. $\angle 4 \cong \angle 2$	6.
7. $\overline{AB} \parallel \overline{CD}$	7. Converse of the Alt. Int. ∠s theorem
8. $\overline{BC} \parallel \overline{DA}$	8.
9. $ABCD$ is a parallelogram	9. Def. of parallelogram

4. Given: $\triangle CAR$ is isosceles w/base $\overline{CR}$
$\overline{AC} \cong \overline{BK}$
$\angle C \cong \angle K$
Prove: $BARK$ is a parallelogram

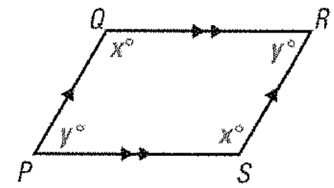


Statements	Reasons
1. $\triangle CAR$ is isosceles w/base $\overline{CR}$	1. Given
2. $\overline{AC} \cong \overline{AR}$	2. Def. of isosceles △
3. $\overline{AC} \cong \overline{BK}$	3. Given
4. $\overline{AR} \cong \overline{BK}$	4. Transitive Prop.
5. $\angle C \cong \angle ARC$	5. If △ then △
6. $\angle C \cong \angle K$	6. Given
7. $\angle ARC \cong \angle K$	7. Transitive Prop
8. $\overline{AR} \parallel \overline{BK}$	8. Converse of Corrs. ∠s Postulate
9. $BARK$ is a parallelogram	9. Parallelogram / Congruent-Parallel Side theorem

10.2.D2 PARALLELOGRAMS

- For a list of the properties of parallelograms, see page 829 of your text.
- Add to your properties list: *Consecutive angles of a parallelogram are supplementary.*

$$x + y = 180°$$



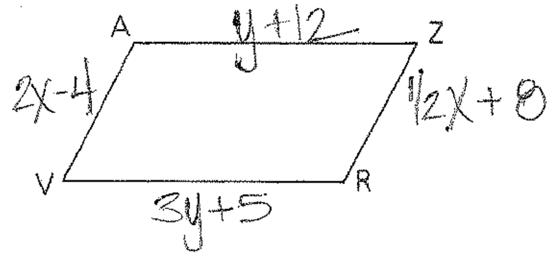
Examples

1. *VRZA* is a ▱

Given:
$AV = 2x - 4$ $= 12$
$VR = 3y + 5$ $= 15.5$
$RZ = \frac{1}{2}x + 8$
$ZA = y + 12$

P= 55 units

Find: The values of *x* and *y* and the perimeter of *VRZA*



What property of parallelograms are you going to use?

* opp. sides are ≅

$2x-4=\frac{1}{2}x+8$
$1.5x=12$
$x=8$

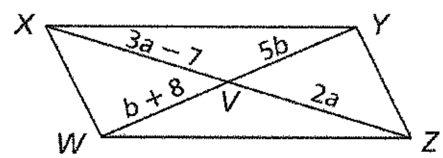
$3y+5=y+12$
$2y=7$
$y=3.5$

2. *WXYZ* is a ▱. Find:

a. a = 7
b. b = 2
c. WV = 10
d. YW = 20
e. XZ = 28
f. ZV = 14

What property of parallelograms are you going to use?

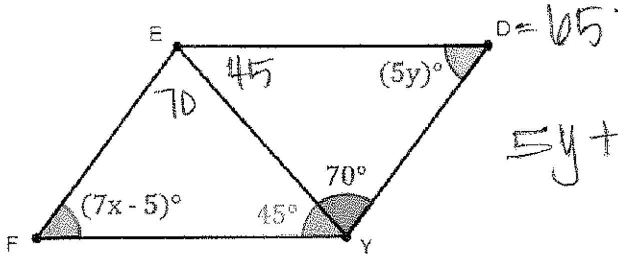
* the diagonals bisect each other



$3a-7=2a$
$+7=+a$

$b+8=5b$
$8=4b$
$2=b$

3. *FEDY* is a ▱. Find the value of each variable.



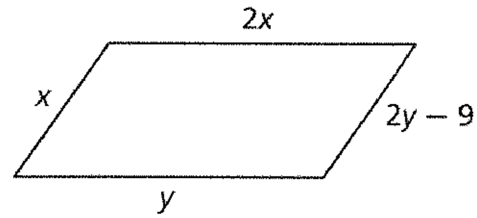
What property of parallelograms are you going to use?

* opp. ∠s are ≅

$5y+115=180$
$5y=65$ ← △ Sum theorem
$y=13$

$7x-5=65$
$7x=70$
$x=10$

4. For the given parallelogram, set up and solve a system of equations to find the value of the variables.



What property of parallelograms are you going to use?

* opp. sides are ≅

$x = 2y - 9$
$y = 2x$

$x = 2(2x) - 9$
$x = 4x - 9$
$-3x = -9$
$x = 3$ $\quad y = 6$

10.1 SQUARES & RECTANGLES

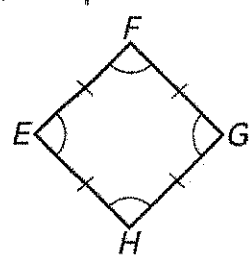
- Squares
  - A square is a quadrilateral with four right angles and all sides congruent. (definition)
    - Area: $A = s^2$
- Rectangles
  - A rectangle is a quadrilateral with opposite sides congruent and with four right angles. (definition)
    - Area: $A = bh = \ell w$

For a list of the properties of squares and rectangles, see page 828 of your text.

Examples

1. Given: $EFGH$ is a square with a perimeter of 36
   $EH = x + 6$
   $\angle F = 2y - 4$

   side = 9



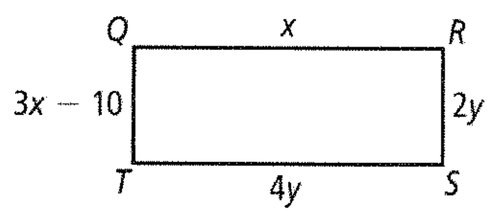
   Find: x & y
   The area of square $EFGH$

   $EH = 9$
   $x + 6 = 9$
   $x = 3$

   $m\angle F = 90°$
   $2y - 4 = 90$
   $2y = 94$
   $y = 47$

   $A = s^2$
   $A = 9^2 = 81$ units^2

2. Given: Rectangle $QRST$



   a. Set up and solve a system of equations to find the value of the variables.
   b. Find the rectangle's base and height. 4 2
   c. What is the perimeter and area of rectangle $QRST$?

   12 units 8 units^2

   $3x - 10 = 2y$
   $x = 4y$

   $3(4y) - 10 = 2y$
   $12y - 10 = 2y$
   $-10 = -10y$
   $1 = y$ $x = 4$

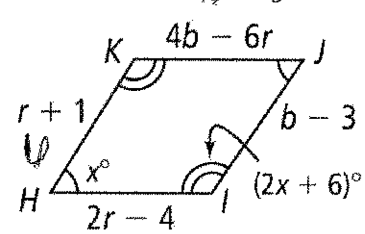
10.2 & 10.3 RHOMBI & KITES

- Rhombi
  - A rhombus is a quadrilateral with all sides congruent. (definition)
    - Area: $A = d_1 d_2$
      - Add to your properties list: *Consecutive angles are supplementary.*
- Kites
  - A kite is a quadrilateral with two pairs of consecutive congruent sides with opposite sides that are not congruent. (definition)
    - Area: $A = d_1 d_2$

For a list of the properties of rhombi and kites, see page 830 of your text.

Examples

4b-30



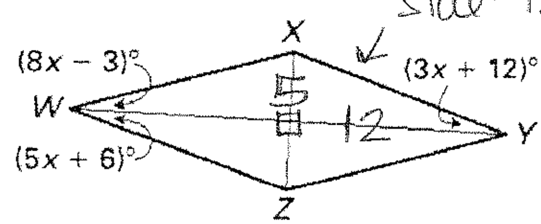
1. Given: Rhombus *HIJK* r=5, b=9, X=58
   a. Find the value of the variables.
   b. What is the perimeter of rhombus *HIJK*? 24
   c. Find $m\angle J$ & $m\angle K$. m∠J = 58° m∠K = 122°

2r-4 = r+1
r = 5

4b-30 = 6
4b = 36
b = 9

X+2X+6 = 180
3X = 174
X = 58

side = 13



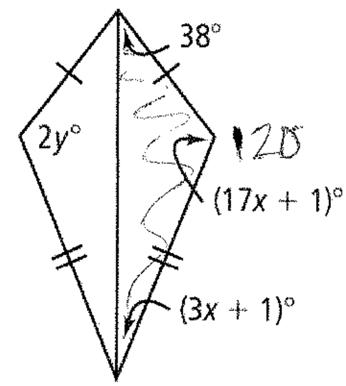
2. Given: Rhombus *WXYZ*
   $XZ = 10$ & $WY = 24$
   a. Find the value of *x*. X=3
   b. Find the area of rhombus *WXYZ*. 120 units²
   c. Find the perimeter of rhombus *WXYZ*. 52

8X-3 = 5X+6
3X = 9
X = 3

A = 1/2·10·24
A = 120 units²

* diagonals bisect the vertex ∠s

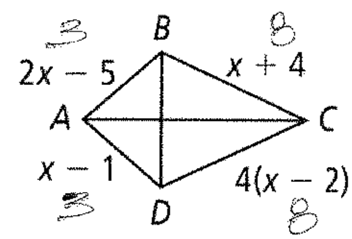
3. Find the value of the variables in the kite.



$17x+1+3x+1+38=180$
$20x+40=180$
$20x=140$
$x=7$

$2y=120$
$y=60$

4. Given: Kite *ABCD*
Find the value of *x* and the perimeter of *ABCD*.



$2x-5=x-1$
$x=4$

P = 22 units

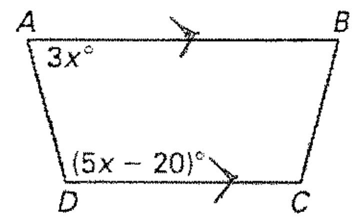
10.3.D2 TRAPEZOIDS

- Trapezoids
  - A trapezoid is a quadrilateral with exactly one pair of parallel sides. (definition)
    - Add to your properties list: *Consecutive non-base angles are supplementary.*
  - An isosceles trapezoid is a trapezoid with congruent non-parallel sides. (definition)
    - Area: $A = \frac{1}{2}h(b_1 + b_2)$

For a list of the properties of <u>*isosceles*</u> *trapezoids,* *see page 831 of your text.*

Examples

1. Given: $ABCD$ is a trapezoid.

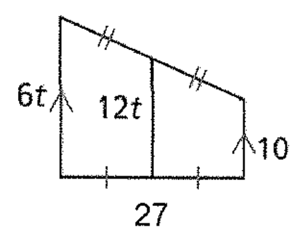


   d. Find the value of x. $x = 25$

   e. Find $m\angle A$ & $m\angle D$. $m\angle A = 75°$ $m\angle D = 105°$

$3x + 5x - 20 = 180$
$8x - 20 = 180$
$8x = 200$
$x = 25$

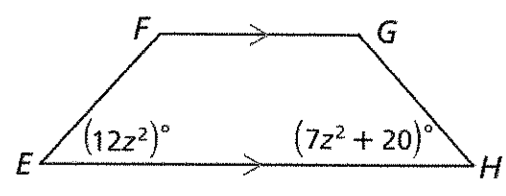
2. Find the length of the midsegment of the trapezoid. $\frac{20}{3}$
   What is the trapezoid's area? 180 units^2



$12t = \frac{1}{2}(6t + 10)$
$12t = 3t + 5$
$9t = 5$
$t = 5/9$ midseg $= 12 \cdot \frac{5}{9} = \frac{20}{3}$

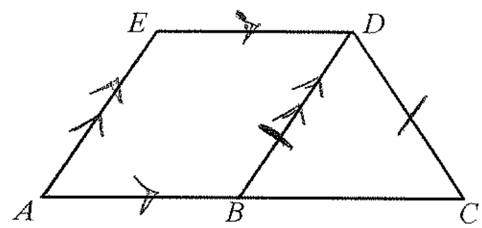
$A = \frac{1}{2} \cdot 27 \left(\frac{10}{3} + 10\right)$
$A = 180$

3. Find the value of z so that trapezoid $EFGH$ is isosceles.



$12z^2 = 7z^2 + 20$
$5z^2 = 20$
$z^2 = 4$
$z^2 = \pm 2$

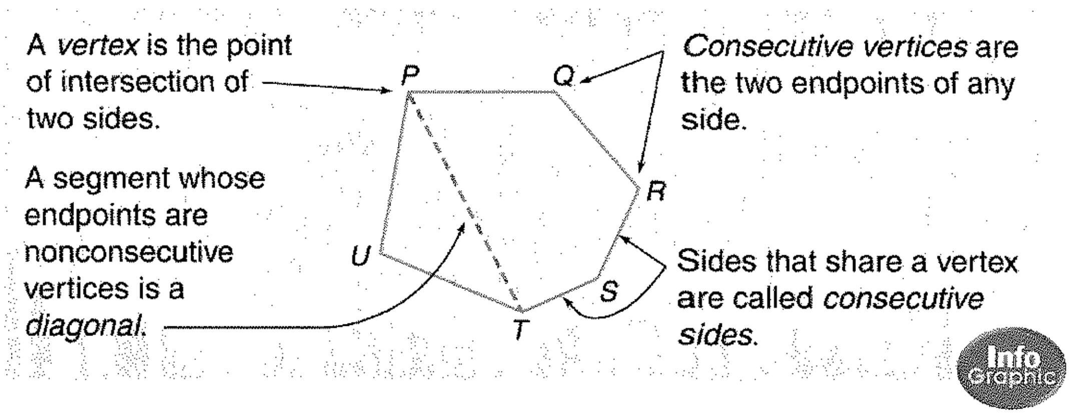
4. Given: $ABDE$ is a parallelogram
$\triangle BCD$ is isosceles with base $\overline{BC}$
Prove: $ACDE$ is an isosceles trapezoid



Statements	Reasons
1. ABDE is a //ogram	1. Given
2. $\overline{EA} \cong \overline{BD}$	2. Opp. sides of a //ogram are $\cong$
3. $\overline{ED} \parallel \overline{AB}$	3. Def. of // ogram
4. $\triangle BCD$ is isosceles	4. Given
5. $\overline{DB} \cong \overline{DC}$	5. Def. of isosceles Δ
6. $\overline{EA} \cong \overline{DC}$	6. Transitive Prop.
7. ACDE is an isosceles trapezoid	7. Def. of isosceles trapezoid

10.4 & 10.5 Interior & Exterior Angles of Polygons

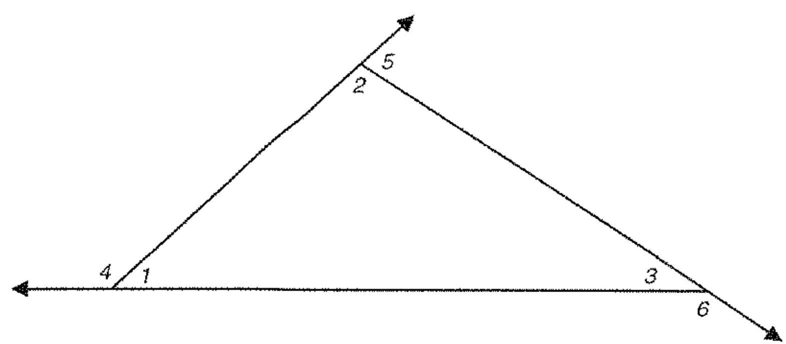
- Polygons



- Interior Angles of Polygons
  - ➢ The sum of the measures of the interior angles of a polygon with n sides is $180(n-2)$

- Exterior Angles of Polygons

Use the figure to answer each question.

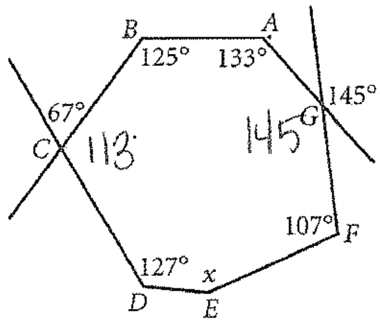


1. What is the sum of the measures of $\angle 1$ & $\angle 4$? Explain your reasoning.
   180° Linear Pair, therefore Supp.
2. What is the sum of the measures of $\angle 2$ & $\angle 5$? 180°
3. What is the sum of the measures of $\angle 3$ & $\angle 6$? 180°
4. What is the sum of the measures of $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$, and $\angle 6$? Explain your reasoning.
   $540 = 180 \cdot 3$
5. What is the sum of the measures of $\angle 1$, $\angle 2$, and $\angle 3$? Explain your reasoning.
   180 $\triangle$ Sum theorem
6. What is the difference of the sum of the measures of $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$, and $\angle 6$ and the sum of the measures of $\angle 1$, $\angle 2$, and $\angle 3$? What does this demonstrate? all the ext. $\angle$S
   $540 - 180 = 360$ $m\angle 4 + m\angle 5 + m\angle 6 = 360$
7. What is the sum of the exterior angles of any polygon? 360°

Examples

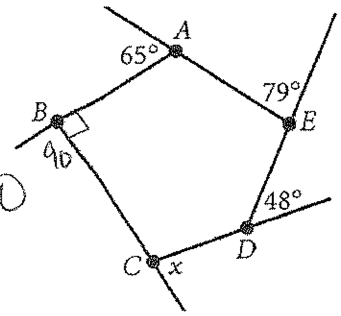
Find the value of x in each convex polygon.

1.



$180(7-2)$
900
$x+750=900$
$x=150$

2.

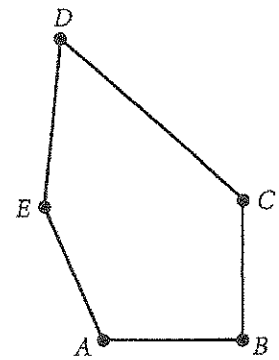


$x+282=360$
$x=78$

3. Given: $m\angle A = 4x + 7$, $m\angle B = 4x - 18$, $m\angle C = 5(x-1)$, $m\angle D = 2x + 1$, & $m\angle E = 7x - 39$

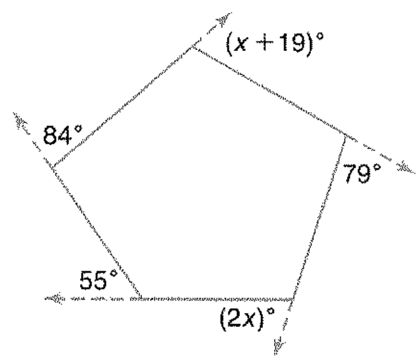
$5x-5$

Set up and solve an equation and find the value of x.



$180(5-2)=540$
$22x-54=540$
$22x=594$
$x=27$

4. Set up and solve an equation to find the value of x.



$3x+237=360$
$3x=123$
$x=41$

5. If a regular polygon has 30 sides, what is the measure of (a) each interior angle? And (b) each exterior angle?

$\frac{180(30-2)}{30}=168°$

$\frac{360}{30}=12$

6. If the measure of each exterior angle of a regular polygon is 18°, how many sides does the polygon have?

$\frac{360}{n}=18$

$n=20$