1.1 Modeling & Equation Solving

OBJECTIVES: USE NUMERICAL, ALGEBRAIC, AND GRAPHICAL MODELS TO SOLVE PROBLEMS & TRANSLATE FROM ONE MODEL TO ANOTHER

Modeling

- A mathematical model is…

Numerical Models

- Numbers – or data – are analyzed to gain insights into phenomena

Example 1 - A Numerical Model

Refer to the data in Table 1.6 showing the percentage of the female & male populations in the United States employed in the civilian work force in selected years from 1954 to 2004.

a) According to the numerical model, what had been the trend in females joining the work force since 1954?

b) In what 5-year interval did the percentage of women who were employed changed the most?

Table 1.6 Employment Statistics

Year	Female	Male
1954	32.3	83.5
1959	35.1	82.3
1964	36.9	80.9
1969	41.1	81.1
1974	42.8	77.9
1979	47.7	76.5
1984	50.1	73.2
1989	54.9	74.5
1994	56.2	72.6
1999	58.5	74.0
2004	57.4	71.9

Source: www.bls.gov

Algebraic Models

- Formulas are used to relate variable quantities associated with the phenomena being studied
  - Useful as a predictor of behavior (compared to a numerical model)

Example 2: An Algebraic Model

Model the data (from Table 1.6) algebraically with linear equations of the form $y = mx + b$. Write one equation for the women's data and another for the men's data. Use the 1954 & 1999 ordered pairs to compute the slopes.

- Graphical Models
  - A visible representation of a numerical model or an algebraic model that gives insight into the relationships between variable quantities

Example 3: A Graphical Model

Model the data (from Table 1.6) graphically with two scatter plots on separate graphs, one showing the percentage of women employed as a function of time and the other showing the same for men. (Measure time in years since 1954; $x = 0$; 1959, $x = 5$…) Superimpose the linear equation found in Example 2 on the scatter plot. How well does the equation fit the data?

- Equation Solving
  - Quadratic Equations
    - Zero Factor Property
      - A product of real numbers is zero if and only if at least one of the factors in the product is zero.
    - There are four basic ways to solve quadratic equations algebraically
      1. Factoring
      2. Extracting Square Roots
      3. Completing the Square
      4. Using the Quadratic Formula

Refer to your Section P.5 notes for examples of each method.

- Equation Solving – A Fundamental Connection
  - If a is a real number that solves the equation $f(x) = 0$, then these three statements are equivalent:
    - The number a is a ______________________________ of the equation: $f(x) = 0$
    - The number a is a ______________ of $y = f(x)$
    - The number a is an __________________ of the graph of $y = f(x)$; the x-intercept $\rightarrow (a, 0)$

Examples: Solving an Equation Algebraically

4. $2x^3 = 13x^2 - 20x$
5. $x + \sqrt{x} = 6$
6. $4 + \sqrt{x + 2} = x$

Example 7: Solving an Equation Graphically

Solve by finding x-intercept(s) or point(s) of intersection: $\sqrt{x+6} = 6 - 2\sqrt{5-x}$

A Problem Solving Process

1. Understand the problem
2. Develop a mathematical model of the problem
3. Solve the mathematical model and support or confirm the solution
4. Interpret the solution in the problem setting

Examples: Applying the Problem-Solving Process

8. The engineers at an auto manufacturer pay students \$0.10 per mile plus \$20 per day to road test their new vehicles.
   a) How much did the auto manufacturer pay Dennis to drive 200 miles in one day?

   b) Erma earned \$30 test-driving a new car in one day. How far did she drive?

9. Engineers have a tank that contains 50 gallons of water. They let in 10 gallons of water per minute.
   a) How many gallons of water will be in the tank after 12 minutes?

   b) How many minutes will it take for the tank to contain 85 gallons in all?

1.2 Functions & Their Properties

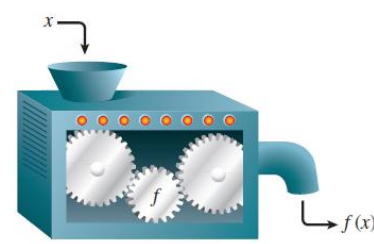
OBJECTIVES: REPRESENT FUNCTIONS NUMERICALLY, ALGEBRAICALLY AND GRAPHICALLY; DETERMINE THE DOMAIN AND RANGE FOR FUNCTIONS; AND ANALYZE FUNCTION CHARACTERISTICS

Functions (according to the Common Core)

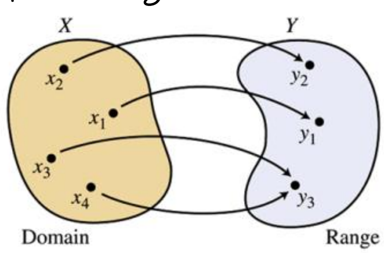
- Understand that a function from one set (called the _______________) to another set (called the _____________) assigns to each element of the _______________ exactly one element of the _____________.
  - If f is a function and x is an element of its domain, then $f(x)$ denotes the _____________ of f corresponding to the _____________ x.

OBJECTIVE #1: REPRESENT FUNCTIONS NUMERICALLY, ALGEBRAICALLY AND GRAPHICALLY

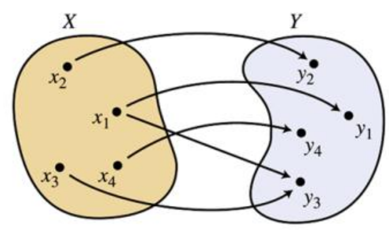
- Function Definition & Notation
  - A function from a set D to a set R is a rule that assigns to element in D a unique element of R
    - Domain—the set D of all inputs; the variable ____ (AKA _____________ variable)
    - Range—the set R of all outputs, the variable ____ (AKA _____________ variable)
  - Function Notation: $y = f(x)$



- Functions & Their Representation
  - The "machine" concept – values of the domain x are fed into a machine (the function) to produce range values y
  - A function can also be viewed as a mapping of the elements from the domain onto elements of the range



A function

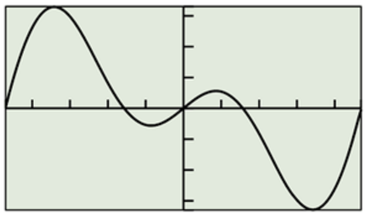


Not a function

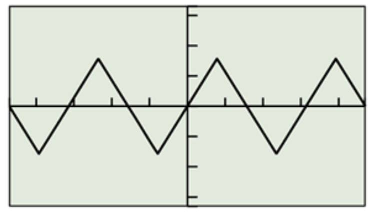
  - Graphically
    - The graph of f is the graph of the equation ___________
    - Vertical Line Test
      -

Example 1: Seeing a Function Graphically

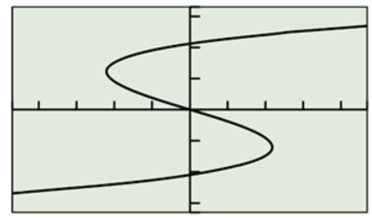
Of the three graphs shown below, which is not the graph of a function?



[–4.7, 4.7] by [–3.3, 3.3]



[–4.7, 4.7] by [–3.3, 3.3]



[–4.7, 4.7] by [–3.3, 3.3]

OBJECTIVE #2: DETERMINE THE DOMAIN & RANGE FOR FUNCTIONS

- Finding the Domain of a Function
  - Expressions w/Square Roots
    - Recall that radicals are undefined when the ______________________________.
    - To find the domain…
  - Rational Expressions
    - Recall that a rational expression is undefined when the ___________________ equals zero.
    - To find where this occurs…
  - Miscellaneous Expressions
    - You can use a grapher to see the domain of a function.
    - To find the domain we look for all ___________________ that correspond to points on the graph.

Examples: Finding the Domain of a Function

2. $f(x) = \dfrac{3x - 1}{x^2 - 5x}$

3. $g(x) = \dfrac{\sqrt{4 - x}}{(x + 1)(x^2 + 1)}$

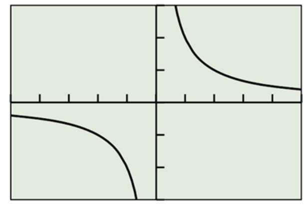
4. $h(x) = x^2 + 1$

- Finding the Range of a Function
  - You can use a grapher to see the range of a function.

- To find the range we look for all ____________________ that correspond to points on the graph.

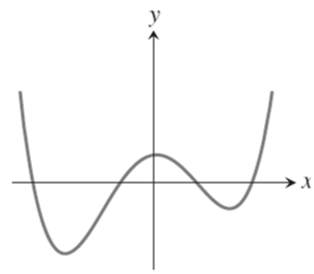
<u>Example 5: Finding the Range of the Function</u>

$$f(x) = \frac{2}{x}$$



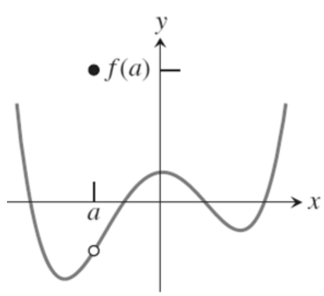
[–5, 5] by [–3, 3]

- **Continuity**
  - A function is continuous at a point if…

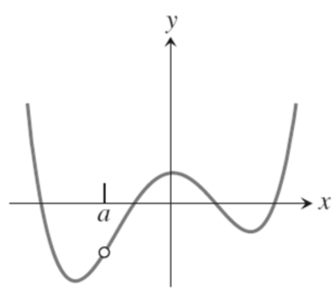


Continuous at all x

- **Discontinuity**
  - Removable Discontinuities
    - Continuous everywhere except for the "____________" at $x = a$
      - Removable because it can be "patched" by ____________________________ so as to "plug" the hole.

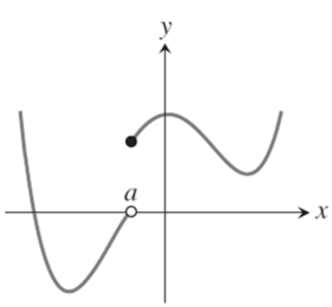


Removable discontinuity

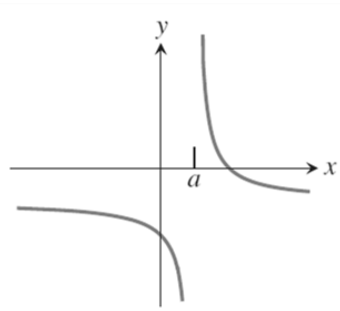


Removable discontinuity

  - Non-removable Discontinuities
    - Jump Discontinuity
      - Impossible to plug a "gap" with a single point
    - Infinite Discontinuity



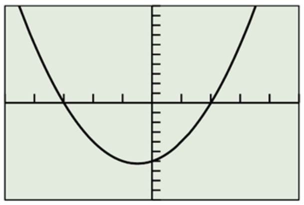
Jump discontinuity



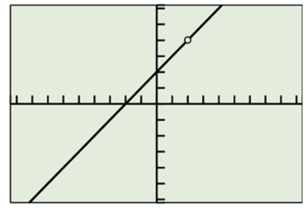
Infinite discontinuity

Examples: Identifying Points of Discontinuity

Judging from the graphs, which of the following figures shows functions that are discontinuous at $x = 2$? Are any of the discontinuities removable?

6. 

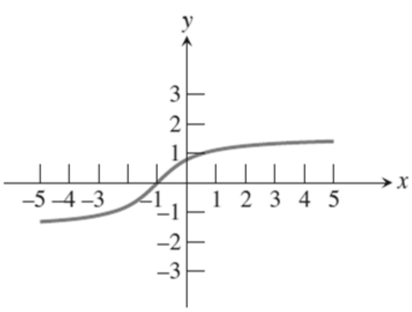
[–5, 5] by [–10, 10]

7. 

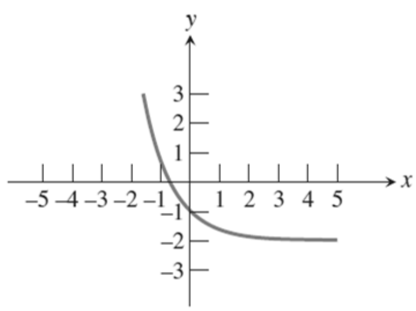
[–9.4, 9.4] by [–6.2, 6.2]

OBJECTIVE #3: ANALYZE FUNCTION CHARACTERISTICS SUCH AS EXTREME VALUES, SYMMETRY, ASYMPTOTES, END BEHAVIOR & MORE

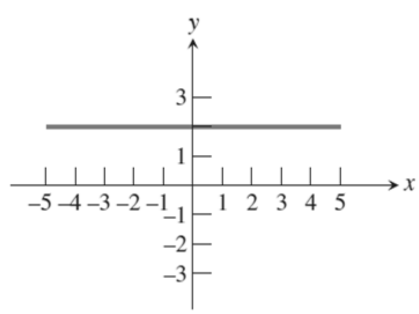
- Increasing, Decreasing & Constant Functions on an Interval
  - Increasing—if for any two points in the interval, a positive change in x results in a ___________________ change in $f(x)$.
  - Decreasing—if for any two points in the interval, a positive change in x results in a ___________________ change in $f(x)$.
  - Constant—if for any two points in the interval, a positive change in x results in a ___________________ change in $f(x)$.



Increasing



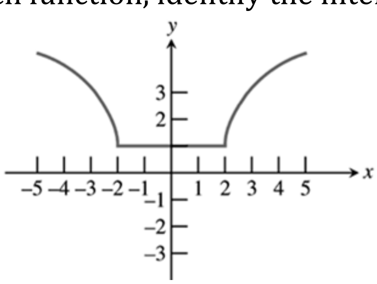
Decreasing



Constant

Examples: Analyzing a Function for Increasing-Decreasing Behavior

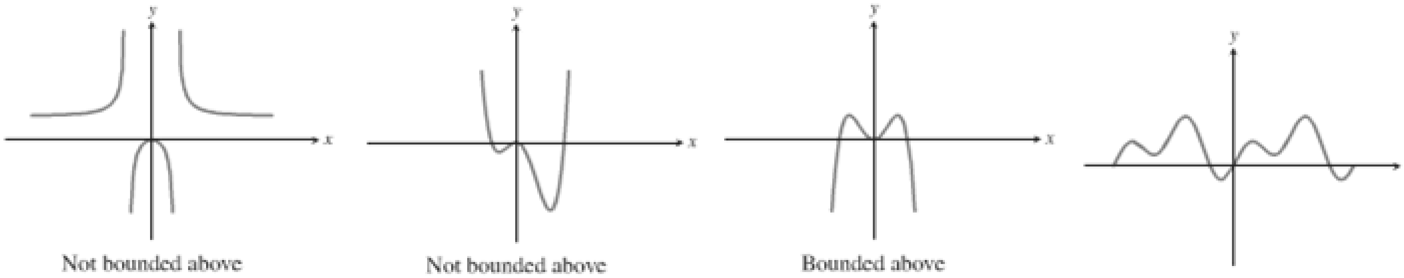
For each function, identify the intervals on which it is increasing, decreasing, and/or constant.

8. 

9. $f(x) = x^2 - 2x - 2$

10. $h(x) = \dfrac{x^3}{4 - x^2}$

Boundedness



Not bounded above
Not bounded below

Not bounded above
Bounded below

Bounded above
Not bounded below

Bounded

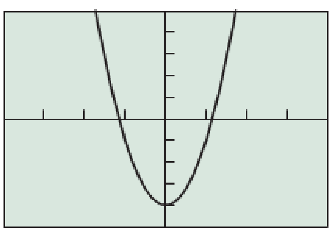
Lower Bound, Upper Bound & Bounded

- Bounded below: some number b that is less than or equal to every number of the range of f
  - Any such number b is called a lower bound.
- Bounded above: some number B that is greater than or equal to every number of the range of f
  - Any such number B is called an upper bound.
- Bounded:

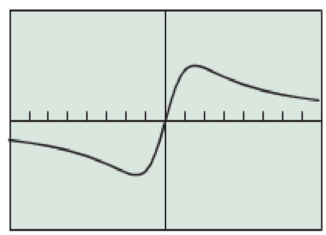
Examples: Checking Boundedness

Identify each of these functions as bounded below, bounded above, or bounded.

11. $w(x) = 3x^2 - 4$



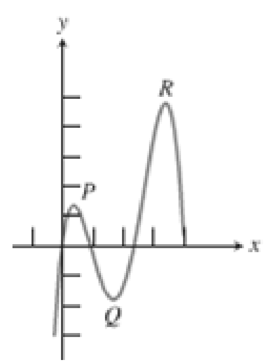
12. $p(x) = \dfrac{x}{1 + x^2}$



Local & Absolute Extrema

Many graphs are characterized by peaks and valleys where they change from increasing to decreasing and vice versa.

- Local Maximum (of a function)—A value $f(c)$; if $f(c)$; is greater than or equal to all range values of f, then $f(c)$; is the maximum.
  - At the local max, the function goes from ________________ to __________________
- Local Minimum (of a function)—A value $f(c)$; if $f(c)$; is less than or equal to all range values of f, then $f(c)$; is the minimum.



Local max: _____

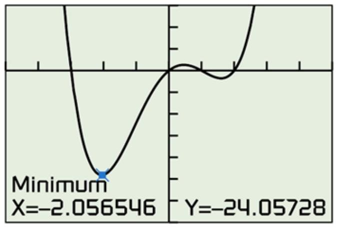
Local min: _____

- At the local min, the function goes from ______________ to ______________

Local extrema are also called <u>relative extrema.</u>

<u>Examples: Identifying Local Extrema</u>

13. The graph of the function is given: $f(x) = x^4 - 7x^2 + 6x$
    Find the local maxima or local minima of $f(x)$. Find the values of x where each local maximum and local minimum occurs.



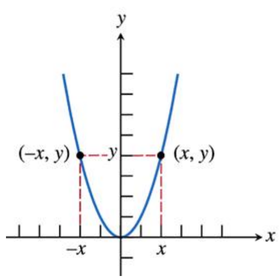
14. Use a grapher to find all local extrema & the values of x where they occur of the function: $g(x) = x^2\sqrt{x+4}$ (Round to 2 decimal places.)

Symmetry

- Three particular types of symmetry:
  - Symmetric w/respect to the y-axis, x-axis and the origin
- Three ways of viewing symmetry: graphically, numerically, algebraically

Symmetric with respect to the y-axis: $f(x) = x^2$

Graphically



Numerically

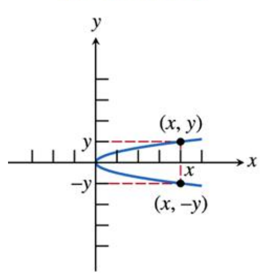
x	$f(x)$
−3	9
−2	4
−1	1
1	1
2	4
3	9

Algebraically

For all x in the domain of f,

Symmetric with respect to the x-axis: $x = y^2$

Graphically



Numerically

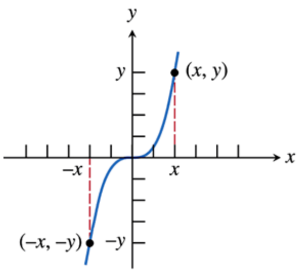
x	y
9	−3
4	−2
1	−1
1	1
4	2
9	3

Algebraically

Graphs with this kind of symmetry are not functions (except the zero function), but we can say that $(x, -y)$ is on the graph whenever (x, y) is on the graph.

Symmetric with respect to the origin: $f(x) = x^3$

Graphically



Numerically

x	y
-3	-27
-2	-8
-1	-1
1	1
2	8
3	27

Algebraically

For all x in the domain of f,

<u>Examples: Checking Functions for Symmetry</u>

State whether each of the following functions is odd, even, or neither. Solve algebraically; confirm graphically.

15. $f(x) = 2x^2 + 1$ 16. $g(x) = x^2 - 2x + 3$ 17. $h(x) = \dfrac{x}{x^2 - 1}$

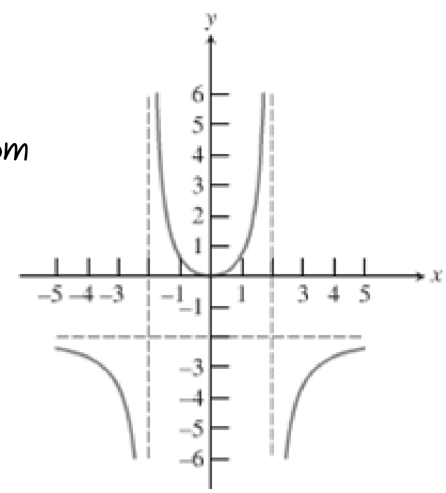
Asymptotes

- A Visual Introduction
  - Where does the graph appear to flatten out to the right & left?
    - What (invisible) horizontal line does it approach?
  - Where does the graph appear to flatten out as it goes off the top & bottom of the screen?
    - What (invisible) vertical line does it approach?

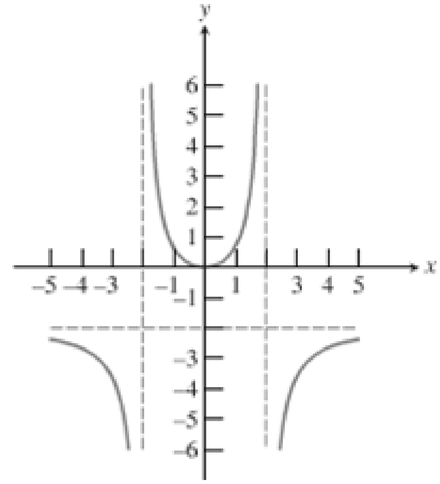
- Vertical Asymptotes
  - The line __________
  - $f(x)$ approaches a limit of $\pm\infty$ as x approaches a from either direction

$$\lim_{x \to a^-} f(x) = \pm\infty \text{ or } \lim_{x \to a^+} f(x) = \pm\infty$$

What are the vertical asymptotes – and their corresponding limits – of the graph shown?



- To find vertical asymptotes algebraically…



- Horizontal Asymptotes
  - The line __________
  - $f(x)$ approaches a limit of b as x approaches $\pm\infty$

$$\lim_{x\to-\infty} f(x) = b \text{ or } \lim_{x\to\infty} f(x) = b$$

What are the horizontal asymptotes – and their corresponding limits – of the graph shown?

  - To find horizontal asymptotes algebraically,
    - If N° < D°, there is a horizontal asymptote at __________. Use the graph to determine whether x is approaching ∞ or $-\infty$.
    - If N° = D°, there is a horizontal asymptote at ____________________________

<u>Examples: Identifying the Asymptotes of a Graph</u>

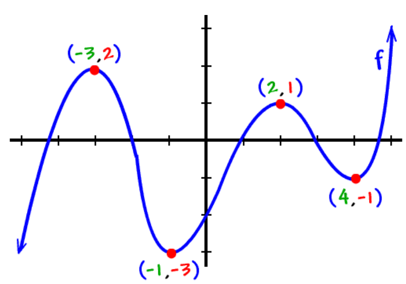
Find all horizontal or vertical asymptotes of the function.

18. $f(x) = \dfrac{2}{x^2 - 4}$ 19. $g(x) = \dfrac{x+2}{3-x}$ 20. $h(x) = \dfrac{x}{x^2 - 3x - 4}$

End Behavior

- A horizontal asymptote gives one kind of end behavior for a function because it shows how the function behaves as it goes off toward either "end" of the x-axis

<u>Example 21: Analyzing End Behavior</u>



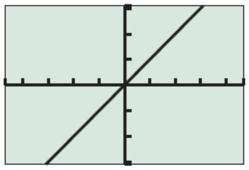
$$\lim_{x\to-\infty} f(x) =$$

$$\lim_{x\to\infty} f(x) =$$

1.3 Twelve Basic Functions

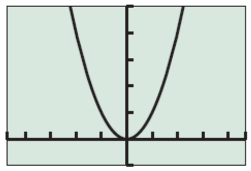
OBJECTIVES: RECOGNIZE GRAPHS OF 12 BASIC FUNCTIONS; DETERMINE THE DOMAINS OF FUNCTIONS RELATED TO THE 12 BASIC FUNCTIONS; COMBINE THE 12 BASIC FUNCTIONS IN VARIOUS WAYS TO CREATE NEW FUNCTIONS.

Graphs of the Twelve Basic Functions



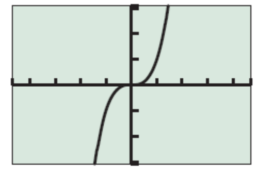
[–4.7, 4.7] by [–3.1, 3.1]

Identity Function
$f(x) = x$
Domain $= (-\infty, \infty)$
Range $= (-\infty, \infty)$



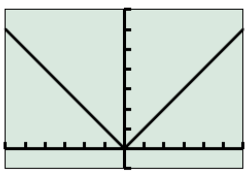
[–4.7, 4.7] by [–1, 5]

Squaring Function
$f(x) = x^2$
Domain $= (-\infty, \infty)$
Range $= [0, \infty)$



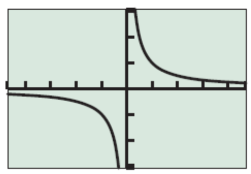
[–4.7, 4.7] by [–3.1, 3.1]

Cubing Function
$f(x) = x^3$
Domain $= (-\infty, \infty)$
Range $= (-\infty, \infty)$



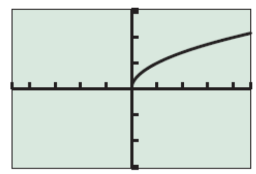
[–6, 6] by [–1, 7]

Absolute Value Function
$f(x) = |x| = \text{abs}(x)$
Domain $= (-\infty, \infty)$
Range $= [0, \infty)$



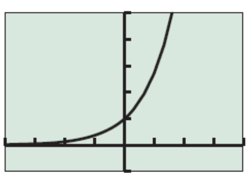
[–4.7, 4.7] by [–3.1, 3.1]

Reciprocal Function
$f(x) = \dfrac{1}{x}$
Domain $= (-\infty, 0) \cup (0, \infty)$
Range $= (-\infty, 0) \cup (0, \infty)$



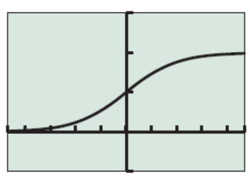
[–4.7, 4.7] by [–3.1, 3.1]

Square Root Function
$f(x) = \sqrt{x}$
Domain $= [0, \infty)$
Range $= [0, \infty)$



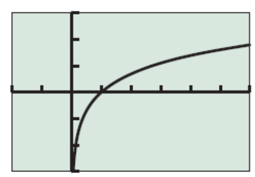
[–4, 4] by [–1, 5]

Exponential Function
$f(x) = e^x$
Domain $= (-\infty, \infty)$
Range $= (0, \infty)$



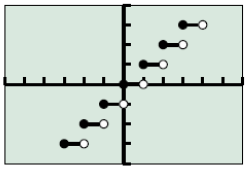
[–4.7, 4.7] by [–0.5, 1.5]

Logistic Function
$f(x) = \dfrac{1}{1 + e^{-x}}$
Domain $= (-\infty, \infty)$
Range $= (0, 1)$



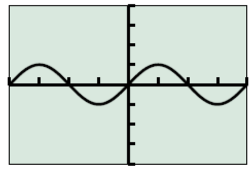
[–2, 6] by [–3, 3]

Natural Logarithmic Function
$f(x) = \ln x$
Domain $= (0, \infty)$
Range $= (-\infty, \infty)$



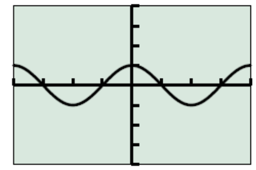
[–6, 6] by [–4, 4]

Greatest Integer Function
$f(x) = \text{int}(x)$
Domain $= (-\infty, \infty)$
Range = all integers



[–2π, 2π] by [–4, 4]

Sine Function
$f(x) = \sin(x)$
Domain $= (-\infty, \infty)$
Range $= [-1, 1]$



[–2π, 2π] by [–4, 4]

Cosine Function
$f(x) = \cos(x)$
Domain $= (-\infty, \infty)$
Range $= [-1, 1]$

- Looking for Domains
  - Nine of the functions have the set of all real numbers as its domain. Which three do not?
  - One of the functions has the set of all real numbers except 0 as its domain. Which function is it and why isn't zero in its domain?
  - Which two functions have no negative numbers in their domain? Of these two, which one is defined at zero?
- Looking for Continuity
  - Only two of the twelve functions have points of discontinuity. Are these points in the domain of the function?
- Looking for Boundedness
  - Only three of the twelve basic functions are bounded (above & below). Which three?
- Looking for Symmetry
  - Three of the twelve basic functions are even. Which are they?
- Looking for Asymptotes
  - Two of the basic functions have vertical asymptotes at x = 0. Which two?
  - Three of the basic functions have horizontal asymptotes at y = 0? Which three?

Examples: Analyzing a Function Graphically

Use a grapher to produce a graph of the function. Then determine the domain and range of the function by looking at its graph. (Express in interval notation.)

1. $g(x) = \ln(x + 5)$
2. $h(x) = |x| - 5$

Graph the function using a graphing utility. Then answer the following questions:

a. On what interval is the function increasing? decreasing?
b. Is the function even, odd or neither?
c. Does the function have any extrema? If so, what?

3. $h(x) = |x + 1| + |x - 2|$
4. $k(x) = x^3 - x^2 - 2x$

1.4 BUILDING FUNCTIONS FROM FUNCTIONS

OBJECTIVES: BUILD NEW FUNCTIONS FROM BASIC FUNCTIONS BY ADDING, SUBTRACTING, MULTIPLYING, DIVIDING & COMPOSING FUNCTIONS

Combining Functions Algebraically

- Let f & g be two functions with intersecting domains. Then for all values of x in the intersection, the algebraic combinations of f & g are defined by the following rules:
  - Sum: $(f + g)(x) =$ ________________
  - Difference: $(f - g)(x) =$ ________________
  - Product: $(fg)(x) =$ ________________
  - Quotient: $\left(\frac{f}{g}\right)(x) =$________, provided that $g(x) \neq 0$
- In each case, the domain of the new function consists of…
  - As noted, the zeros of the denominator are excluded from the domain of the quotient.

<u>Examples: Defining New Functions Algebraically</u>

1. If $f(x) = 4 + x$ and $g(x) = 2 - 3x$, find formulas for $f + g$, $f - g$, and fg. State the domain of each.

$f + g =$ ______________ Domain: ______________

$f - g =$ ______________ Domain: ______________

$fg =$ ______________ Domain: ______________

2. Let $f(x) = x^2$ and $g(x) = \sqrt{1 - x^2}$.
   - Find formulas of the functions…

$f + g =$ ______________ $fg =$ ______________

$f - g =$ ______________ $f/g =$ ______________

Composition of Functions

The function $sin(x^2)$ is not formed by multiplying $sine$ & x^2. They are merely combined in order. This is known as <u>function composition.</u>

- Let f & g be two functions such that the domain of f intersects the range of g.
  - <u>Composition f of g</u> —

<u>Examples: Composing Functions</u>

Find $(f \circ g)(3) = f(g(3))$. and $(g \circ f)(-2) = g(f(-2))$.

3. $f(x) = 2x - 3; g(x) = x + 1$

4. $f(x) = x^2 + 4; g(x) = \sqrt{x + 1}$

Find $(f \circ g)(x)$ and $(g \circ f)(x)$.

5. $f(x) = 2x; g(x) = x^2$

6. $f(x) = 2^x; g(x) = x^2$

7. $f(x) = \frac{1}{2x}; g(x) = \frac{1}{3x}$

8. $f(x) = x^2 - 2; g(x) = \sqrt{x+1}$

Decomposing Functions

- Start with $h(x) = f(g(x))$ & find functions f & g.

Examples: Decomposing Functions

For each function h, find functions f & g such that $h(x) = f(g(x))$. (There may be more than one possible decomposition.)

9. $h(x) = 2(x^2 + 5)^2$

10. $h(x) = -2(x^2 - 1)^3 + x^2 - 1$

11. $h(x) = \sqrt{x^2 - 5x}$

12. $h(x) = \sin\sqrt{x}$

Relations

- Relation—
  - If the relation happens to relate a single value of y to each value of x, then the relation is also a ______________.

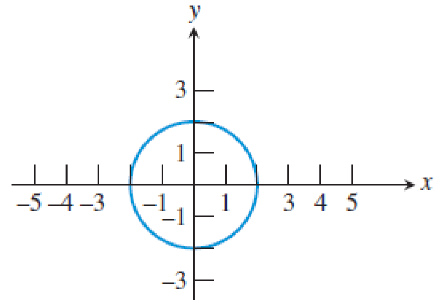
Example 12: Verifying Pairs in a Relation

Which of the ordered pairs (1, 1), (3, 4) and (3, -1) are in the relation given by $3x + 4y = 5$?

Implicitly Defined Functions

- A relation, that is not a function, can be split into two equations that do define functions.

FIGURE 1.59 A circle of radius 2 centered at the origin. This set of ordered pairs (x, y) defines a *relation* that is not a function, because the graph fails the vertical line test.



The circle in Figure 1.59 has the equation: $x^2 + y^2 = 4$

$y^2 = 4 - x^2 \rightarrow \sqrt{y^2} = \sqrt{4 - x^2} \rightarrow\ y =\ \pm\sqrt{4 - x^2}$

$$y = \sqrt{4 - x^2}\ \ \&\ \ y = -\sqrt{4 - x^2}$$

Example 13: Defining Functions Implicitly

Find two functions, defined implicitly, for the relation: $3x^2 - y^2 = 25$

1.5 INVERSE RELATIONS & INVERSE FUNCTIONS

OBJECTIVE #1: DEFINE FUNCTIONS & RELATIONS PARAMETRICALLY

- Relations Defined Parametrically
  - Some functions & graphs can best be defined parametrically.
    -

- Consider the set of all ordered pairs (x, y) defined by the equations

  $$x = t - 1\ \ \&\ \ y = t^2 + 2$$

  Find an algebraic relationship between x & y by eliminating the parameter t. (Use substitution.)

Examples: Defining a Function Parametrically

1. Suppose $x = t^3 - 4t$ and $y = \sqrt{t + 1}$. Find the (x, y) pair if $t = 3$.

2. Suppose $x = 2t$ and $y = t - 2$. Find a direct algebraic relationship between x and y.

OBJECTIVE #2: FIND INVERSES OF FUNCTIONS & RELATIONS

- Inverse Relations
  - Inverse relation—

  - The ordered pair (a, b) is in a relation if & only if the ordered pair __________ is in the inverse relation.
  - Horizontal Line Test
    -

  - What do you call a function that passes both the vertical and horizontal line tests?
    - One-to-one:

- Inverse Functions
  - If f is a one-to-one function with domain D and range R, then the inverse function of f, denoted f^{-1}, is the function with domain R and range D defined by:

    ____

    - The domain of the inverse is the ___________ of the original function.

- How to Find an Inverse Function Algebraically
  - Given a formula for a function f, proceed as follows to find a formula for f^{-1}.
    1. Determine that there is a function f^{-1} by checking that f is one-to-one. State any restrictions on the domain of f.
    2. ______________________ in the formula $y = f(x)$.
    3. Solve for _____ to get the formula $y = f^{-1}(x)$. State any restrictions on the domain of f^{-1}.

Examples: Finding an Inverse Function Algebraically

Find a formula for $f^{-1}(x)$. Give the domain of f^{-1}, including any restrictions "inherited" from f.

3. $f(x) = \dfrac{2x-3}{x+1}$

4. $f(x) = \sqrt[3]{x+5}$

- Verifying Inverse Functions
  - There is a natural connection between inverses & function composition that gives further insight into what an inverse actually does:

    It "undoes" the action of the original function.

- The Inverse Composition Rule
  - A function f is one-to-one with inverse function g if & only if:
    - $f(g(x)) = x$ for every x in the domain of g, and
    - $g(f(x)) = x$ for every x in the domain of f.

Example 5: Verifying Inverse Functions

Show algebraically that $f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x-1}$ are inverse functions.

~~~~~~

1.6 Graphical Transformations

OBJECTIVE: REPRESENT TRANSLATIONS (SHIFTS), REFLECTIONS (FLIPS) & SIZE CHANGES OF FUNCTIONS GRAPHICALLY

Identify the effect on the graph of replacing $f(x)$ by $f(x)+k, kf(x), f(kx)$ and $f(x+k)$ for specific values of k; find the value of k given the graphs.

- Transformations
  - Transformations –
    - Rigid transformations leave the ____: horizontal & vertical translations and reflections
    - Non-rigid transformations _________________________________: horizontal & vertical stretches and shrinks

- Graphing Transformations

  All functions can be written in such a way:

$$a\,f\big(b(x+c)\big)+d$$

- Reflections Across Axes (flips)
  - The following transformations result in reflections of the graph $y = f(x)$
    - _____________: a reflection across the x-axis
    - _____________: a reflection across the y-axis
~~~~~~

$$a\,f\big(b(x+c)\big)+d$$

- Size Changes: Stretches & Shrinks
  - Let a be a positive real number. Then the following transformations result in VERTICAL size changes of the graph of $y = f(x)$.
    -
      - A stretch by a factor of a if ___________
      - A compression by a factor of a if _____________
  - Let b be a positive real number. Then the following transformations result in HORIZONTAL size changes of the graph of $y = f(x)$.
    -
      - A stretch by a factor of b if ___________
      - A compression by a factor of b if _____________
- Translations (shifts)
  - Let c be a positive real number. Then the following transformations result in HORIZONTAL translations of the graph $y = f(x)$
    - ________________ a shift right c units
    - ________________ a shift left c units
  - Let d be a positive real number. Then the following transformations result in VERTICAL translations of the graph $y = f(x)$
    - ________________ a shift up d units
    - ________________ a shift down d units

<u>Examples: Combining Transformations</u>

The graph of $y = x^2$ undergoes the following transformations. Find the equation of the graph that results.

1. a horizontal shift 2 units to the right, a vertical shrink by a factor of ½, a vertical translation 2 units up

2. a horizontal shift 1 unit to the right, a vertical stretch by a factor of 3, a vertical translation 1 unit down

3. a horizontal stretch by a factor of 2, a vertical translation 3 units up

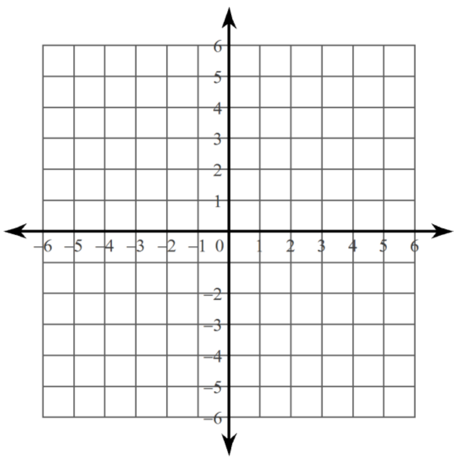
Examples: Graphing Transformations

Describe how to transform the graph of the function and then sketch its graph.

4. $f(x) = -3|x+4| + 5$

Transformations:

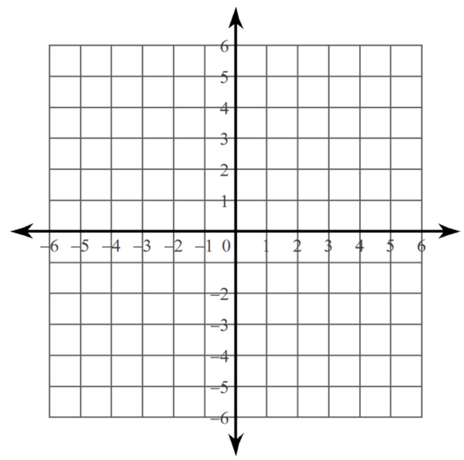
Graph:



5. $h(x) = -\sqrt{1-x}$

Transformations:

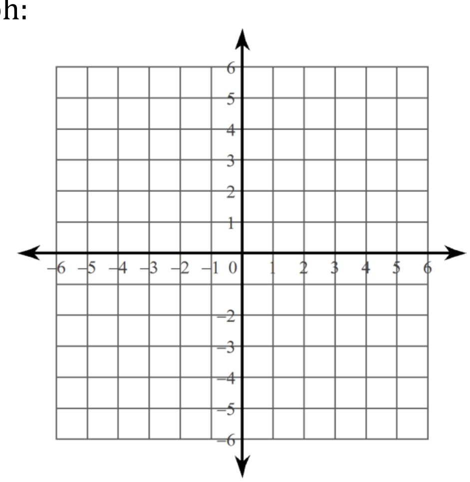
Graph:



6. $g(x) = \frac{1}{2}(x+2)^2 - 3$

Transformations:

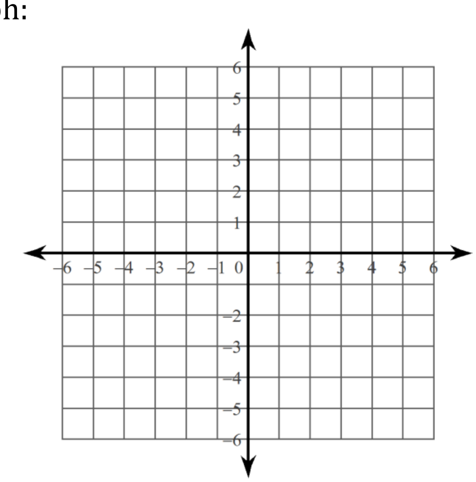
Graph:



7. $f(x) = 2(x-1)^3 - 4$

Transformations:

Graph:



1.7 Modeling with Functions

OBJECTIVES: IDENTIFY APPROPRIATE BASIC FUNCTIONS WITH WHICH TO MODEL REAL-WORLD PROBLEMS AND PRODUCE SPECIFIC FUNCTIONS TO MODEL DATA, FORMULAS, GRAPHS & VERBAL DESCRIPTIONS.

Example 1: Functions From Formulas

A traveler averaged 52 miles per hour on a 182-mile trip. How many hours were spent on the trip?

Example 2: Functions from Graphs

A square of side *x* inches is cut out of each corner of a 10 in. by 18 in. piece of cardboard and the sides are folded up to form an open-topped box.

a. Write the volume *V* of the box as a function of *x*.

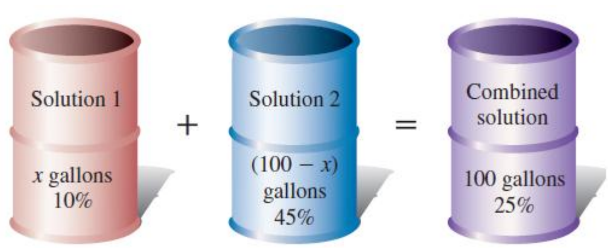
b. Find the domain of your function, taking into account the restrictions that the model imposes in *x*.

c. Using your graphing calculator to determine the dimensions of the cut-out squares that will product the box of maximum volume.

Examples: Functions from Verbal Descriptions

3. Mark received a 3.5% salary increase. His salary after the raise was $36,432. What was his salary before the raise?

4. How much 10% solution and how much 45% solution should be mixed together to make 100 gallons of 25% solutions?



Constructing a Function from Data

- Given a set of data points of the form (x, y) to construct a formula that approximates y as a function of x.
    1. Make a scatter plot of the data points.
    2. Determine, from the shape of the plot, whether the points seem to follow the graph of a familiar type of function (line, parabola, cubic, sine curve, etc.)
        i. See page 157 of your text.
    3. Transform a basic function of that type to fit the points as closely as possible – find a regression model.
- Curve-Fitting w/Technology
    - The effectiveness of a data-based model is highly dependent on the number of data points and on the way they were selected.
    - Correlation Coefficient (r) OR Coefficient of Determination (r^2 or R^2)
        - "Rule of Thumb"– ____

Example 5: Functions from Data

The imports of crude oil to the U.S. from Canada in the years 1995-2004 (in thousands of barrels per day) are given in Table 1.15.

a. Create a scatter plot of import numbers in the right-hand column (y) as a function of years since 1990 (x).

b. Find the equation of the regression line & superimpose it on the scatter plot.

c. Based on the regression line, approximately how many thousands of barrels of oil would the U.S. import from Canada in 2010?

Table 1.15 Crude Oil Imports from Canada

Year	Barrels/day × 1000
1995	1,040
1996	1,075
1997	1,198
1998	1,266
1999	1,178
2000	1,348
2001	1,356
2002	1,445
2003	1,549
2004	1,606

Source: Energy Information Administration, Petroleum Supply Monthly, as reported in The World Almanac and Book of Facts 2005.