2.1 Linear & Quadratic Functions & Modeling

- Polynomial Functions
  - Let n be a nonnegative integer and let $a_0, a_1, a_2, \ldots, a_{n-1}, a_n$ be real numbers with $a_n \neq 0$; the function given by
$$\boldsymbol{f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0}$$
is a <u>polynomial function of degree *n*</u>.
    - The ____________________ is $\boldsymbol{a_n x^n}$ because n is the largest exponent – the degree of the whole polynomial; $\boldsymbol{a_n}$ is the ____________________
    - Defined and continuous on all real numbers

- Polynomial Functions of No and Low Degree

Name	**Form**	**Degree**
Zero function	$f(x) = 0$	Undefined
Constant function	$f(x) = a \ (a \neq 0)$	0
Linear function	$f(x) = ax + b \ (a \neq 0)$	1
Quadratic function	$f(x) = ax^2 + bx + c \ (a \neq 0)$	2

<u>Skill 1: Identify Polynomial Functions</u>

Which of the following are polynomial functions? For those that are polynomial functions, state the degree and leading coefficient. For those that are not, explain why not.

<u>Polynomial?</u> <u>Degree</u> <u>Leading coefficient</u>

1) $f(x) = 6x^{-4} + 7$

2) $g(x) = 15x - 2x^4$

- Linear Functions and Their Graphs
  - <u>Linear function</u> — SEE YOUR LESSON GUIDE FOR THE DEFINITION
    - A line in the Cartesian plane is the graph of linear function if and only if it is a ____________________________________, that is, neither vertical nor horizontal.

Skill 2: Find an Equation of a Linear Function

3. Write an equation for the linear function that satisfies the given conditions: $f(-2) = -3$ & $f(2) = -4$

ට Average Rate of Change

- The average rate of change of a function $y = f(x)$ between $x = a$ and $x = b$, with $a \neq b$, given by:

- Constant Rate of Change – SEE YOUR LESSON GUIDE FOR THE DEFINITION
  - The rate of change of a linear function is the signed ratio of the corresponding line's rise over run.

$$\text{rate of change} = m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x}$$

    - This formula allows us to interpret the ____________, or rate of change, of a linear function numerically.
- The Constant Term
  - For any function f, $f(0)$ is the _________________________
    - Linear function: ____________
    - Polynomial function: ____________
    - All functions: the ___-intercept of its graph

Skill 3: Analyze Properties of Linear Functions

4. Let $f(x) = x^2$; compute the average rate of change from $x = 2$ to $x = 5$.

Skill 4: Model with Linear Functions

5. Camelot Apartments bought a $50,000 building and for tax purposes are depreciating it $2000 per year over a 25-year period using straight line depreciation.
   a. What is the rate of change of the value of the building?
   b. Write an equation for the value $v(t)$ of the building as a linear function of the time t since the building was placed in service.
   c. Evaluate $v(0)$ and $v(16)$.
   d. Solve $v(t) = 39{,}000$.

CHARACTERIZING THE NATURE OF A LINEAR FUNCTION	
Point of View	**Characterization**
Verbal	
Algebraic	
Graphical	
Analytical	

- Linear Correlation & Modeling
  - <u>Linear correlation</u> – SEE YOUR LESSON GUIDE FOR THE DEFINITION
    - Correlation is positive if the slope is positive; negative is the slope is negative
  - <u>Correlation coefficient</u> – SEE YOUR LESSON GUIDE FOR THE DEFINITION
    - Properties of the Correlation Coefficient, r: $-1 \le r \le 1$
      - When $|r| \approx 1$, there is a ____________ linear correlation.
      - When $r \approx 0$, there is _______________ linear correlation.

Correlation informs the modeling process by giving us a measure of goodness of fit.

<u>Skill 4: Model with Linear Regression</u>

6. Use the data presented in the table to write a linear model for demand (in boxes sold per week) as a function of the price per box (in dollars).

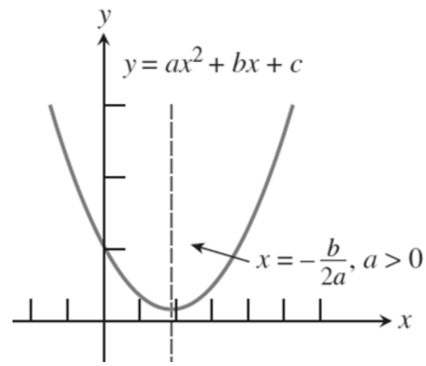
Table 2.2 Weekly Sales Data Based on Marketing Research

Price per box	Boxes sold
\$2.40	38,320
\$2.60	33,710
\$2.80	28,280
\$3.00	26,550
\$3.20	25,530
\$3.40	22,170
\$3.60	18,260

Use the model to predict weekly cereal sales if the price is dropped to \$2.00 or raised to \$4.00 per box.

Quadratic Functions & Their Graphs

- <u>Quadratic function</u> – SEE YOUR LESSON GUIDE FOR THE DEFINITION
  - The graph of any quadratic function is an upward or downward opening parabola.
    - <u>Axis</u> – the line of symmetry for a parabola
    - <u>Vertex</u> – SEE YOUR LESSON GUIDE FOR THE DEFINITION
      - It's always the lowest or highest point of the parabola.



- Vertex Form of a Quadratic Function
  - Any quadratic function $f(x) = ax^2 + bx + c, a \neq 0$, can be written in the <u>vertex form</u>:
    - With vertex: (h, k) & axis: $x = h$, where $h = -b/(2a)$ and $k = f(h)$
    - If $a > 0$, the parabola opens upward; downward if $a < 0$

<u>Skill 5: Analyze Properties of Quadratic Functions</u>

7. Find the vertex and the axis of the graph of the function: $h(x) = 5x^2 + 4 - 6x$

<u>Skill 6: Find Equations of Quadratic Functions</u>

8. Complete the square to rewrite the equation $f(x) = 2x^2 + 6x + 7$ in vertex form.

9. Write an equation, in vertex and standard form, for the quadratic function whose graph's vertex is (1, 3) and passes through the point (0, 5).

CHARACTERIZING THE NATURE OF A QUADRATIC FUNCTION	
Point of View	**Characterization**
Verbal	
Algebraic	
Graphical	

- Applications Involving Quadratic Functions
  - Maximum Value
  - Economics: Revenue
    - Revenue is found by multiplying the price per unit, x, by the number of units sold, y: $\boldsymbol{R(x) = xy}$

<u>Skill 7: Model with Quadratic Functions</u>

10. A rectangular garden has dimensions of 20 feet by 17 feet. A gravel path of equal width is to be built around the garden. How wide can the path be if there is enough gravel for 258 square feet?

The revenue is the product of the price per unit and the number of units produced.

$$R(x) = (\text{price per unit})(\text{number of units produced})$$

11. The Sweet Drip Beverage Company sells cans of soda pop in machines. It finds that the sales average 26,000 cans per month when the cans sell for $0.50 each. For each nickel increase in the price, the sales per month drop by 1,000 cans.
    a. Determine a function $R(x)$ that models the total revenue realized by Sweet Drip, where x is the number of $0.05 increases in the price of a can.
    b. Graph $R(x)$ so that it shows a maximum for $R(x)$.
    c. How much should Sweet Drip charge per can to realize the maximum revenue?
    d. What is the maximum revenue?
    e. What is the new sales average of the number cans per month?

Applications Involving Quadratic Functions (continued)

- <u>Projectile Motion</u> –
  - Vertical Free-Fall Motion
    - The height s and vertical velocity v of an object in free fall are given by:
      - t: time (in seconds)
      - g: acceleration due to gravity; $g \approx 32\ ft/sec^2 \approx 9.8\ m/sec^2$
      - v_0: initial vertical velocity
      - s_0: initial height (in feet or meters)

12. At the Bakersville Fourth of July celebration, fireworks are shot by remote control into the air from a pit that is 10 feet below the earth's surface.
    a. Find an equation that models the height of an aerial bomb t seconds after it is shot upwards with an initial velocity of 80 ft/sec. Graph this equation.
    b. What is the maximum height above ground level that the aerial bomb will reach? How many seconds will it take to reach that height?

13. Using quadratic regression on the data in Table 2.7, predict the year when the number of patent applications will reach 450,000. Let x = 0 stand for 1980, x = 10 for 1990 and so forth.

Table 2.7 U.S. Patent Applications

Year	Applications (thousands)
1980	113.0
1990	176.7
1995	228.8
1998	261.4
1999	289.5
2000	315.8
2001	346.6
2002	357.5
2003	367.0

Source: U.S. Census Bureau, Statistical Abstract of the United States, 2004–2005 (124th ed., Washington, D.C., 2004).

2.2 Power Functions with Modeling

- Power Functions and Variation
  - Power function – SEE YOUR LESSON GUIDE FOR THE DEFINITION
    - We say $f(x)$ varies as the a^{th} power of x, or $f(x)$ is proportional to the a^{th} power of x.
      - Statements of direct variation: __________
      - Statements of inverse variation: __________

Unless the word "inversely" is used in a variation statement, the variation is assumed to be direct.

  - Writing a Power Function Formula

Direct Variation	Inverse Variation
"y varies directly as x"	"y varies inversely as x"

- Monomial Functions and Their Graphs
  - Monomial function — SEE YOUR LESSON GUIDE FOR THE DEFINITION
    - Any function that can be written as $f(x) = k$ or $f(x) = k\,x^n$ where k is a constant and n is a _______________ integer
      - $f(x) = x^n$ is even if n is even & odd if n is odd

Skill 1: Analyze Properties of Power Functions

Determine whether the function is a power function. For those that are power functions, state the power and constant of variation.

1. $f(x) = \dfrac{1}{2}x^5$
2. $f(x) = 3 \cdot 2^x$

Determine whether the function is a monomial function. For those that are monomial functions state the degree and leading coefficient. For those that are not explain why not.

3. $f(x) = -4$

4. $y = 6x^{-7}$

<u>Skill 2: Write a Power Function Formula</u>

Write the statement as a power function equation. Use k for the constant of variation if one is not given.

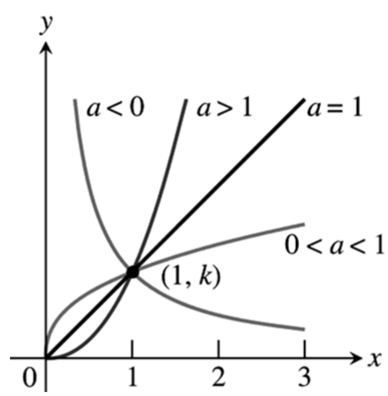
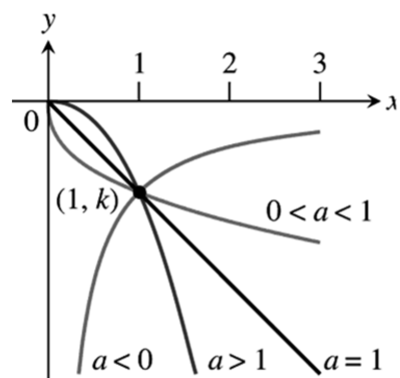
5. The current I in an electrical circuit is inversely proportional to the resistance R, with constant of variation V.

6. The energy E produced in a nuclear reaction is proportional to the mass m, with the constant of variation being c^2, the square of the speed of light.

Graphs of Power Functions

- There are <u>four</u> possible shapes for general power functions of the form $f(x) = kx^a$, for $x \geq 0$
  - The graph contains the point __________
    - If $a > 0$, the graph passes through __________
    - If $a < 0$, the graph is ______________________________
- For any power function, one of the following occurs when $x < 0$:
  - f is undefined for $x < 0$
  - f is an even function
  - f is an odd function

See your Lesson 1.2 notes on how we determine even/odd functions algebraically.

$k > 0$	$k < 0$
	

Skill 3: Analyze/Describe & Graph Power Functions

State the values of the constants k and a. Describe the portion of the curve that lies in Quadrant I or IV. Determine whether f is even, odd, or undefined for $x < 0$. Describe the rest of the curve if any. Graph the function to see whether it matches the description.

	K	a	Q1 or Q4? Description	Even, Odd or Undefined?	Further Description
7. $f(x) = 3x^{\frac{1}{4}}$					
8. $f(x) = -2x^{4/3}$					

Skill 4: Model with Power Functions

9. The volume of an enclosed gas (at a constant pressure) varies directly as the absolute temperature. If the pressure of a 3.46-L sample of neon gas at 302°K is 0.926 atm, what would the volume be at a temperature of 338°K if the pressure does not change?

10. Use the data in Table 2.10 to obtain a power function model for orbital period as a function of average distance from the Sun. (Make a scatter plot; find a power regression model.) Then use the model to predict the orbital period of Neptune, which is 4497 Gm from the Sun on average.

Table 2.10 Average Distances and Orbit Periods for the Six Innermost Planets

Planet	Average Distance from Sun (Gm)	Period of Orbit (days)
Mercury	57.9	88
Venus	108.2	225
Earth	149.6	365.2
Mars	227.9	687
Jupiter	778.3	4332
Saturn	1427	10,760

Source: Shupe, Dorr, Payne, Hunsiker, et al., National Geographic Atlas of the World (rev. 6th ed.). Washington, DC: National Geographic Society, 1992, plate 116.

2.3 Polynomial Functions of Higher Degree with Modeling

- Graphs of Polynomial Functions
  - Recall that a polynomial function of degree n can be written in the form:

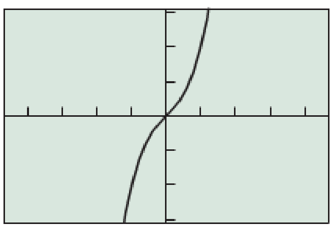
$$\boldsymbol{f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0}$$

  - The Vocabulary of Polynomials
    - Term & Coefficients – SEE YOUR LESSON GUIDE FOR THEIR DEFINITIONS
    - Leading term: a_n
    - Constant term: a_0
      - The constant term a_0 of a polynomial function p is both the ________________ of the function $p(0)$ & the ________________

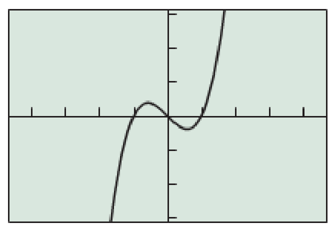
Skill 1: Relating Graphs of Polynomials

Given the polynomial function graphed, locate its extrema & zeros and explain how it is related to the monomials from which it is built: $f(x) = x$ & $f(x) = x^3$

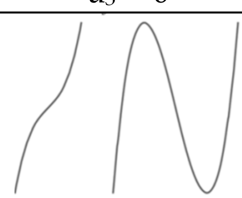
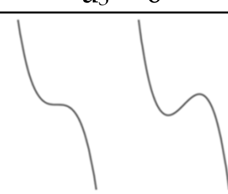
1. $f(x) = x^3 + x$

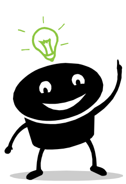


2. $f(x) = x^3 - x$



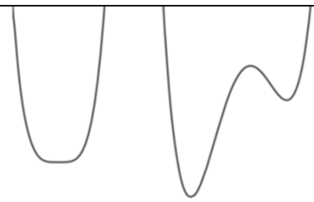
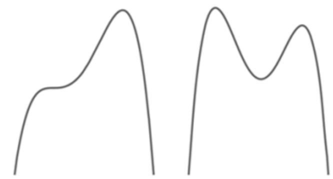
- What do graphs of polynomial functions look like in general?
  - Unbroken without jumps or holes; smooth, unbroken lines or curves with no sharp corners or cusps

Cubic Functions	
$a_3 > 0$	$a_3 < 0$
	
Positive leading coefficients	Negative leading coefficients



Exploration: Imagine horizontal lines that pass through the graphs. Each intersection would be an x-intercept that would correspond to a zero of the function. The high and low points indicate local extrema.

- How many zeros do cubic functions have?
- What about the local extrema?

Quartic Functions	
$a_4 > 0$	$a_4 < 0$
	
Positive leading coefficients	Negative leading coefficients

- How many zeros do quartic functions have?
- What about the local extrema?

Local Extrema & Zeros of Polynomial Functions

- A polynomial function of degree n has at most __________ local extrema and at most _____ zeros.

End Behavior of Polynomial Functions

- The end behavior of a polynomial is closely related to the end behavior of its ____________________.

Exploration: Investigating the End Behavior of $f(x) = a_n x^n$

Graph each function in the window [-5, 5] by [-15, 15]. Describe the end behavior using $\lim_{x\to\pm\infty} f(x)$

(a) $f(x) = -0.3x^5$		(b) $f(x) = -2x^2$		(c) $f(x) = 3x^4$		(d) $f(x) = 2.5x^3$	
$x\to\infty$	$x\to-\infty$	$x\to\infty$	$x\to-\infty$	$x\to\infty$	$x\to-\infty$	$x\to\infty$	$x\to-\infty$
____	____	____	____	____	____	____	____

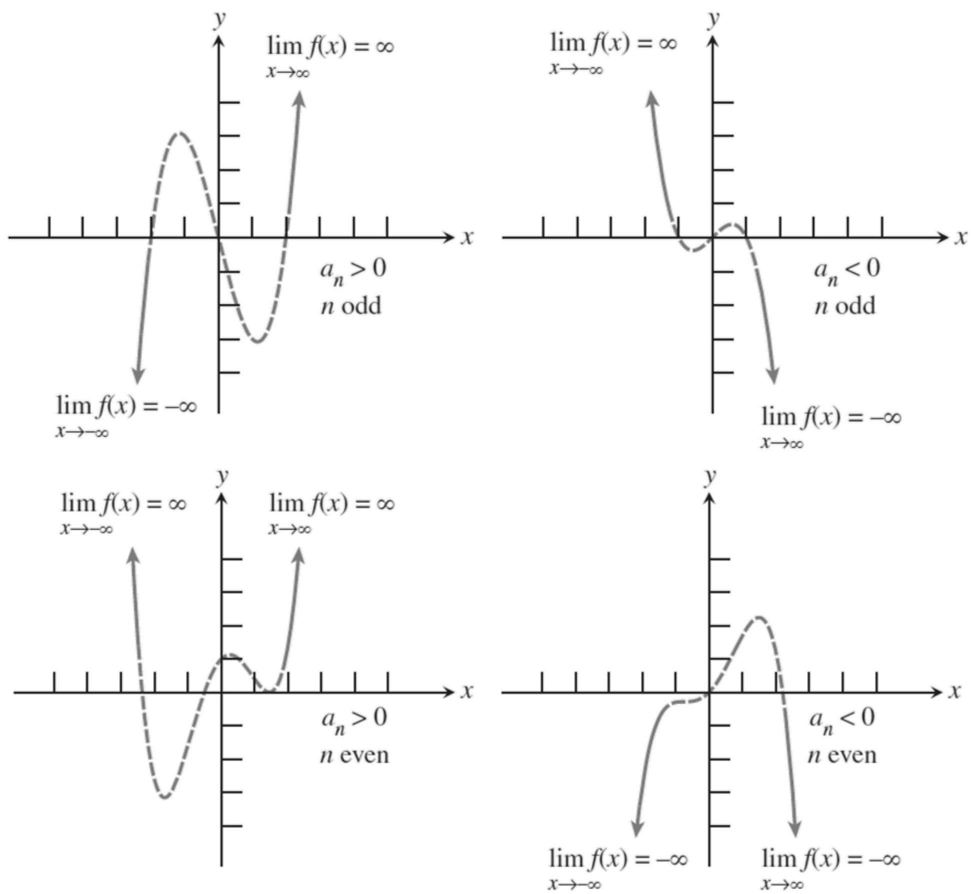
Describe the patterns you observe. In particular, how do the values of the coefficient a_n and the degree n affect the end behavior of $f(x) = a_n x^n$?

End Behavior of Polynomial Functions

- The leading term dominates the behavior of the polynomial as __________
- There are four possible end behavior patterns for a polynomial function.
  - The _______________________________________ tell us which one of the four patterns occurs.

ⵚ Leading Term Test for Polynomial End Behavior

For any polynomial function $f(x) = a_n x^n + \cdots + a_1 x + a_0$, the limits $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ are determined by the degree n of the polynomial and its leading coefficient a_n:



<u>Skill 2: Applying Polynomial Theory</u>

Graph the function in a viewing window that shows all of its extrema and x-intercepts. Describe the end behavior using limits.

	Extrema	Zeros	End Behavior	
3. $f(x) = -6x^3 + 2x^2 + 3x - 8$			$x \to -\infty$	$x \to \infty$
4. $f(x) = 2x^4 - 3x^3 + x - 1$			$x \to -\infty$	$x \to \infty$

ⵚ Zeros of Polynomial Functions

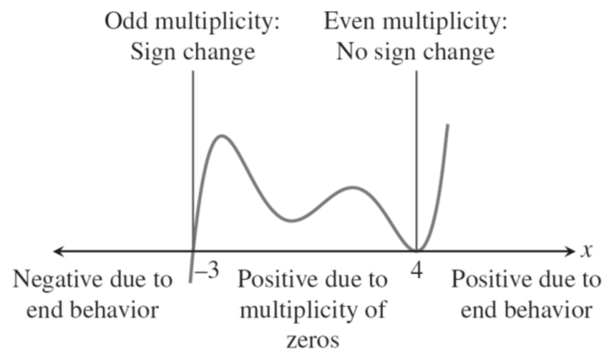
- <u>Zero of a function</u> – SEE YOUR LESSON GUIDE FOR THE DEFINITION
  - When a factor is repeated we say the polynomial function has a repeated zero.

- Multiplicity of a Zero of a Polynomial Function
  -

- Zeros of Odd & Even Multiplicity
  - Odd multiplicity: the graph of f ____________________the x-axis at $(c, 0)$ and the value of f ____________________ sign at $x = c$
  - Even multiplicity: the graph of f ____________________the x-axis at $(c, 0)$ and the value of f ____________________ sign at $x = c$
- Zeros & Their Graphs
  - The __________________ and their __________________ together with the ______ _______________ of a polynomial function give us sufficient information about the polynomial

Knowing where a graph crosses the x-axis and where it doesn't is important in curve-sketching.

The sign chart and graph of $f(x) = (x + 3)(x^2 + 1)(x - 4)^2$ overlaid.



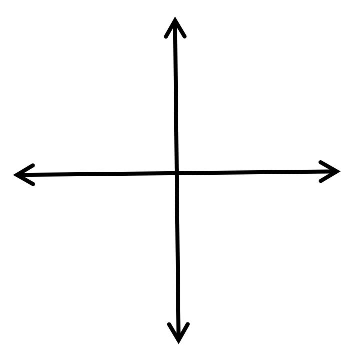
Skill 3: Finding Zeros of Polynomial Functions

5. Find the zeros of $f(x) = x^3 - 8x^2 + 7x$ algebraically.

6. Find the zeros of $f(x) = x^3 + 15x^2 + 56x$ graphically.

Skill 4: Sketching the Graph of a Factored Polynomial

7. $f(x) = (x + 2)(x - 1)^3$



   a) State the degree and sign of the leading coefficient, what does this say about the end behavior of $f(x)$?

   b) Lists the zeros of $f(x)$ and state the multiplicity of each zero and whether the graph crosses the x-axis at the corresponding x-intercept.

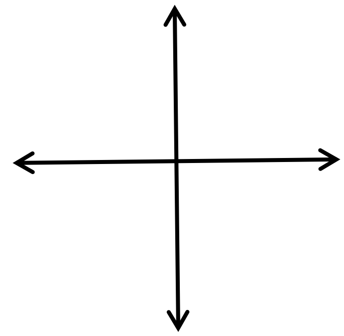
   c) Then sketch the graph of $f(x)$ by hand.

8. $f(x) = -x^4(x-2)$

   a) State the degree and sign of the leading coefficient, what does this say about the end behavior of $f(x)$?

   b) Lists the zeros of $f(x)$ and state the multiplicity of each zero and whether the graph crosses the x-axis at the corresponding x-intercept.

   c) Then sketch the graph of $f(x)$ by hand.



<u>Skill 5: Find Equations of Cubic Functions</u>

9. Using only algebra, find a cubic function with zeros: $1, 1+\sqrt{2}, 1-\sqrt{2}$

<u>Skill 6: Use Quadratic Regression</u>

10. A state highway patrol safety division collected the data on stopping distances in Table 2.14. Draw a scatter plot of the data.

    a) Find the quadratic regression model.

    b) Superimpose the regression curve on the scatter plot. Use the regression model to predict the stopping distance for a vehicle traveling at 25 mph.

    c) Use the regression model to predict the speed of a car if the stopping distance is 300 ft.

Table 2.14 Highway Safety Division

Speed (mph)	Stopping Distance (ft)
10	15.1
20	39.9
30	75.2
40	120.5
50	175.9

2.4 Real Zeros of Polynomial Functions

- The Division Algorithm for Polynomials
  - Given $f(x)$ & $d(x) \neq 0$ there are unique polynomials $q(x)$ (the quotient) and $r(x)$ (the remainder) with ______________________________ with either $r(x) = 0$ or degree of $r(x)$ < degree of $d(x)$
    - the function $f(x)$: __________________
    - $d(x)$: __________________
    - If ______________, we say that $d(x)$ divides evenly into $f(x)$.
  - Write a summary statement in both…
    - polynomial form: $f(x) = d(x) \bullet q(x) + r(x)$
    - and fraction form: $\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$

<u>Skill 1: Using Polynomial Long Division</u>

Divide $f(x)$ by $d(x)$, and write a summary statement in polynomial and fraction form.

1. $f(x) = x^4 - 2x^3 + 3x^2 - 4x + 6; d(x) = x^2 + 2x - 1$

- Synthetic Division
  - <u>Synthetic division</u> – SEE YOUR LESSON GUIDE FOR THE DEFINITION
    - Finds both the quotient and the remainder
  - In order to use synthetic division, the divisor must be a polynomial whose degree is ___ and leading coefficient ___.

Skill 1 (con't): Using Synthetic Division

2. Divide $\frac{5x^3-3x+1}{x+2}$. Write a summary statement in fraction form.

ꝺ Remainder and Factor Theorems

- The Remainder Theorem
  -

Skill 2: Using the Remainder Theorem

Use the Remainder Theorem to find the remainder when $f(x)$ is divided by $x-k$

3. $f(x)=2x^2+4x+3; x-2$
4. $f(x)=3x^2-5x-4; x+3$

When there's no remainder, it isn't necessary to use long division or fancy theorems; traditional factoring methods can do the job.

- The Factor Theorem
  -

Skills 2 & 3: Use the Remainder & Factor Theorems

5. Use the Remainder Theorem to find the remainder when $f(x)=2x^2-4x+2$ is divided by $x-1$. According to the Factor Theorem, what is a factor of $f(x)$?

ꝺ Fundamental Connections for Polynomial Functions

- For a polynomial function f and a real number k, the following statements are equivalent—meaning that any one of the four statements implies the other three.
  1. $x = k$ is a ______________________________ of the equation $f(x) = 0$.
  2. k is a ____________ of the function f.
  3. k is an __________________ of the graph $y = f(x)$.
  4. $x - k$ is a ______________ of $f(x)$.

Skill 4: Finding Polynomial Functions

6. Find the third degree polynomial function with a leading coefficient of 2 and zeros -2, 1 and 4.

7. Using algebraic methods, find the linear factorization of a cubic function with the table of values:

x	-4	0	3	5
$f(x)$	0	180	0	0

The Rational Zeros Theorem

- Real zeros (of polynomial functions): rational or irrational zeros
  - Rational zeros – SEE YOUR LESSON GUIDE FOR THE DEFINITION
- The Rational Zeros Theorem
  - Suppose f is a polynomial function of degree $n \geq 1$ of the form:

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

with every coefficient an integer and $a_0 \neq 0$. If $x = \pm\, {}^{p}/_{q}$ is a rational zero of f, where p and q have no common integer factors other than 1, then:

- p is an integer factor of the ________________ coefficient a_0, and
- q is an integer factor of the ________________ coefficient a_n.

Skill 5: Use the Rational Zeros Theorem

9. Find the rational zeros of $f(x) = 2x^3 - x^2 - 9x + 9$
   a. Identify p & list all of its integer factors:
   b. Identify q & list all of its integer factors:
   c. Potential rational zeros:
   d. Now what? Graph it…of your potential zeros, which one(s) appear to be a rational zero?
   e. Use synthetic division, it tells you whether a number is a zero and, if so, how to factor the polynomial.
   f. Factor & identify the rational zeros

Upper and Lower Bounds

- Upper Bound (for the real zeros) – SEE YOUR LESSON GUIDE FOR THE DEFINITION
- Lower Bound (for the real zeros) – SEE YOUR LESSON GUIDE FOR THE DEFINITION

➤ Upper & Lower Bound Tests for Real Zeros

Let f be a polynomial function of degree $n \geq 1$ with a positive leading coefficient. Suppose $f(x)$ is divided by $x - k$ using synthetic division.

- If $k \geq 0$ & every number in the last line is ____________________, then k is an upper bound
- If $k \leq 0$ & the numbers in the last line are ___________________________________, then k is a lower bound

Skill 6: Establishing Bounds for Real Zeros

10. Determine whether the number $k = 3$ is an upper bound for the real zeros of the function: $f(x) = 2x^3 - 5x^2 - 5x - 1$.

11. Determine whether the number $k = -1$ is a lower bound for the real zeros of the function: $f(x) = 3x^3 - 4x^2 + x + 3$.

Skill 7: Find ALL Real Zeros of a Function

12. Find all of the real zeros of the function:

$$f(x) = 3x^4 - 7x^3 - 3x^2 + 17x + 10,$$

finding exact values whenever possible. (Use the quadratic formula.) Identify each zero as rational or irrational.

~~~~~~

2.5 Complex Zeros & The Fundamental Theorem of Algebra

Existence Theorems

Tell of the existence of zeros and linear factors.

- Fundamental Theorem of Algebra
  -

- Linear Factorization Theorem
  - A polynomial $f(x)$ of degree $n > 0$ has precisely ______________________; the factorization $f(x) = a(x - z_1)(x - z_2) \cdots (x - z_n)$ where _____ is the leading coefficient and the _____ are the complex zeros of f
~~~~~~

- Zeros to Complex Zeros

One connection is lost going from real zeros to complex zeros.

  - If k is a non-real complex zero of a polynomial function $f(x)$, then…

    ____

- Fundamental Polynomial Connections in the Complex Case
  - The following statements about a polynomial function f are equivalent if k is a complex number:
    1. $x = k$ is a solution (or root) of the equation $f(x) = 0$
    2. k is a zero of the function f
    3. $x - k$ is a factor of $f(x)$

Skills 1 & 2: Write a Polynomial Function in Standard Form

1. Write the polynomial $f(x) = (x - 1)(x - 1)(x + 2i)(x - 2i)$ in standard form; identify the zeros of the function and the x-intercepts of its graph.

   Standard form: ________________________________

   Zeros: __________ x-intercepts: ______________

- Complex Conjugate Zeros
  - Suppose that $f(x)$ is a polynomial function with real coefficients. If a & b are real numbers with $b \neq 0$ and $a + b$i is a zero of $f(x)$, then…

    ____

    - Recall that $x - k = 0$ indicates that k is a zero.

Skill 1: Write a Polynomial in Standard Form

2. Write a polynomial function of minimum degree in standard form with real coefficients whose zeros include -2, 1, and 3 - i. Write the linear factorization first.

3. Write a polynomial function of minimum degree in standard form with real coefficients whose zeros and their multiplicities include 2 (multiplicity 2) & 3 + i (multiplicity 1). Write the linear factorization first.

Skill 3: Write the Linear Factorization of a Polynomial; Find its zeros

4. Find all zeros of $f(x) = x^5 - 3x^3 + 6x^2 - 28x + 24$ and write the linear factorization of $f(x)$
   a) Graph $f(x)$
   b) Identify the real zeros of f
   c) Use synthetic division to verify zero(s).
      (This step may have to be repeated.)

Skill 4: Finding the Remaining Zeros (GIVEN A ZERO)

5. The complex number 3 – 2i is a zero of $f(x) = x^4 - 6x^3 + 11x^2 + 12x - 26$ Find the remaining zeros of $f(x)$, and write the linear factorization of $f(x)$

Factors of a Polynomial with Real Coefficients

- Every polynomial function with real coefficients can be written as a product of ____________________ and ___________________________________, each with real coefficients.
  - Standard quadratic form: $ax^2 + bx + c$

Skill 5: Factoring Polynomials

6. Write $f(x) = x^5 - x^4 - 2x^3 + 2x^2 - 3x + 3$ as a product of linear and irreducible quadratic factors, each with real coefficients.
   1. Use the Rational Zeros Theorem to find possible zeros.
   2. Graph $f(x)$ and look for the zeros (in step 1).
   3. Pick a zero (from steps 1 & 2) and use synthetic division.
   4. Repeat until what remains is a quadratic factor.

- Polynomial Function of Odd Degree
  - Every polynomial function of odd degree with real coefficients has ____________ real zero.
- Complex Zeros
  - The number of complex zeros is the __________________________________ of the polynomial.
  - The number of real zeros can be determined from a graph.
    - Differs from the complex zeros by an __________ number.

<u>Skill 6: Determing the Number of Complex and Real Zeros</u>
State how many complex and real zeros the function has.

	Complex Zeros	Real Zeros
7. $f(x) = x^3 - x + 3$		
8. $f(x) = x^4 - 5x^3 + x^2 - 3x + 6$		

2.6 Graphs of Rational Functions

- Rational Functions
  - Let f & g be polynomial functions with $g(x) \neq 0$. Then the function of the following form is a <u>rational function</u>:

$$\boldsymbol{r(x) = \frac{f(x)}{g(x)}}$$

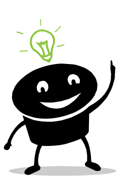
  - Domain: ____
    - Continuous on its domain

- Horizontal & Vertical Asymptotes
  - Horizontal Asymptotes
    - The line __________ is a horizontal asymptote of the graph of f if…

$$\lim_{x \to -\infty} f(x) = b \text{ or } \lim_{x \to \infty} f(x) = b$$

  - Vertical Asymptotes
    - The line __________ is a vertical asymptote of the graph of f if…

$$\lim_{x \to a^-} f(x) = \pm\infty \text{ or } \lim_{x \to a^+} f(x) = \pm\infty$$



<u>Exploration:</u> Use your grapher to graph the following functions and discuss the results in the shift of the graph of $f(x) = \frac{1}{x}$

$$f(x) = \frac{1}{x} \qquad f(x) = \frac{1}{x} + 1 \qquad f(x) = \frac{1}{x} + 2 \qquad f(x) = \frac{1}{x+1} \qquad f(x) = \frac{1}{x+2}$$

If you're seeing the asymptotes of the graph of the rational function, check to see if you're in "dot" and "connected mode." (Get out of connected mode.)

- Transformations to the Reciprocal Function: $\boldsymbol{f(x) = \frac{1}{x}}$
  - The graph of any nonzero rational function of the form:

$$\boldsymbol{g(x) = \frac{ax+b}{cx+d}, c \neq 0}$$

    can be obtained through transformations of the graph of the reciprocal function.

    - If N° ≥ D° → ____

<u>Skill 1: Transforming the Reciprocal Function</u>

Describe how the graph of the given function can be obtained by transforming the graph of the reciprocal function $f(x) = \frac{1}{x}$. Find the domain. Identify the horizontal & vertical asymptotes and use limits to describe the corresponding behavior.

	$1. f(x) = \frac{5}{x+4}$		$2. f(x) = \frac{2x-1}{x+3}$	
Graphing transformations				
Domain				
Horizontal Asymptote(s)				
Vertical Asymptote(s)				
End Behavior	$\lim\limits_{x\to-\infty} =$	$\lim\limits_{x\to\infty} =$	$\lim\limits_{x\to-\infty} =$	$\lim\limits_{x\to\infty} =$

- Characteristics of the Graph of a Rational Function

$$\boldsymbol{y = \frac{f(x)}{g(x)} = \frac{a_n x^n + \cdots}{b_n x^m + \cdots}}$$

  - End Behavior (Horizontal) Asymptote:
    - If $n < m$, then __________
    - If $n = m$, then __________
    - If $n > m$, the end behavior is the ______________________________ where $f(x) = g(x)q(x) + r(x)$
      - There is <u>no</u> horizontal asymptote…*The quotient polynomial represents the equation of a* ____________________
  - Vertical Asymptotes:
    - Occur at the zeros of the ________________ that are not also zeros of the numerator of equal or greater multiplicity

See your Section 1.2 notes for more on finding the equations of asymptotes algebraically.

- Intercepts
  - x-intercepts:
  - y-intercept:

Skill 2: Finding Asymptotes & Intercepts

3. $f(x) = \dfrac{2x+1}{x^2-x}$

4. $f(x) = \dfrac{x^2+2x-3}{x+2}$

Asymptotes: Intercepts: Asymptotes: Intercepts:

Skill 3: Analyzing the Graph of a Rational Function

	5. $f(x) = \dfrac{3x^2-2x+4}{x^2-4x+5}$	6. $f(x) = \dfrac{2x^3-3x+2}{x^3-1}$
Domain		
Range		
Horizontal asymptote(s)		
Vertical asymptote(s)		
x-intercept(s)		
y-intercept		
Increasing		
Decreasing		
Relative extrema: Identify the y-value & the location "at x = …"		
Local maximum(s)		
Local minimum(s)		
Symmetry		
Even/Odd/Neither		
Boundedness		
End behavior:		
$\lim_{x\to\infty} f(x)$		
$\lim_{x\to-\infty} f(x)$		

2.7 Solving Equations in One Variable

$$\frac{f(x)}{g(x)} = 0$$

- Solving Rational Equations
  - If $f(x)$ and $g(x)$ are polynomial functions with no common factors, then the zeros of ________ are the solutions of the equation.
    - How do you rid an equation of fractions?
  - Beware of Extraneous Solutions!
    - The result of multiplying (or dividing) by an expression containing variables

MAKE SURE YOU CHECK YOUR ANSWERS!

Skill 1: Solving Rational Equations

Solve the equation algebraically. Identify any extraneous solutions

1. $\frac{1}{x} - 4x = 3$

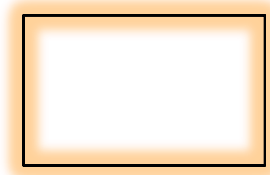
2. $\frac{1}{x^2 + x} = 2 - \frac{1}{x + 1}$

3. $\frac{x - 3}{x} + \frac{3}{x + 2} = \frac{-6}{x^2 + 2x}$

4. Solve the equation graphically; round to the nearest thousandth.
   $x^2 + \frac{5}{x} = 8$

Skill 2: Applications w/Rational Equations

5. Find the dimensions of the rectangle with minimum perimeter if its area is 182 ft^2 and then find this least perimeter.

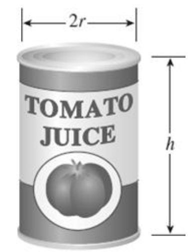


6. Stewart Cannery will package tomato juice 0.5-liter cylindrical cans. Find the radius and height of the cans if the cans have a surface area of 900 cm^2.

   Surface area: $S = 2\pi r^2 + 2\pi rh$

   Volume: $V = \pi r^2 h$

   ∗ 1L = 1000 cm^3



7. The number of wineries for several years is given in Table 2.21. Let $x = 0$ represent 1970, $x = 1$, represent 1971, and so forth. A model for this data is given by:

$$y = 3000 - \frac{39500}{x + 9}$$

   a. Graph the model together with a scatter plot of the data.
   b. Use the model to estimate the number of wineries in 2005.

Table 2.21 Number of Wineries

Year	Number
1975	579
1980	912
1985	1375
1990	1625
1995	1813
2000	2188

Source: American Vintners Association as reported in USA TODAY on June 28, 2002.

2.8 Solving Inequalities in One Variable

- Polynomial Inequalities
  - A polynomial function $p(x)$ is positive, negative, or zero for all real numbers
    - If $f(x) > 0$: find the values that make $f(x)$ positive
    - If $f(x) < 0$: find the values that make $f(x)$ negative
  - If the expression is a product…determine its sign according to its factors.
    - Make a "sign chart"
      - Shows the x-axis as a number line with the ____________________ displayed – as the location of potential ___________________________ – and the factors displayed with their sign value in the corresponding interval

<u>Skill 1: Positive, Negative & Zeros of Polynomials</u>

1. Determine the x values that cause the polynomial function to be

   $f(x) = (2x+5)(x-8)^2(x+1)^3$

   (a) zero:

   (b) positive:

   (c) negative:

<u>Skill 2: Solving Polynomial Inequalities</u>

2. Solve the following inequalities for the given function $f(x) = (x^2+4)(2x^2+3)$

   (a) $f(x) > 0$ (b) $f(x) \geq 0$

   (c) $f(x) < 0$ (d) $f(x) \leq 0$

3. Solve algebraically: $x^3 + 2x^2 - x - 2 < 0$

4. Solve graphically: $3x^3 - 2x^2 - x + 6 \geq 0$

See your Section P.7 notes for more on solving inequalities graphically.

Rational Inequalities

- A rational function $r(x)$ can be positive, negative, zero or undefined
  - A rational function is undefined at the ___________________

    ____

    - A modified sign chart includes the ____

      and ___________________________ to determine the location of potential sign changes

$$\boldsymbol{r(x) = \frac{f(x)}{g(x)}}$$

Skill 3: Zero, Undefined, Positive & Negative Values of Functions

5. Determine the x values that cause the function to be: (a) zero; (b) undefined; (c) positive; and (d) negative.

   $$f(x) = \frac{\sqrt{x+5}}{(2x+1)(x-1)}$$

$$\longleftrightarrow$$

Caution!

- When solving a rational inequality, it's tempting to multiply both sides by the LCD. However, the method of eliminating fractions will not work because the sign of the LCD may depend on x.

Skill 4: Solving Rational (& OTHER) Inequalities

Solve the inequality using a sign chart.

6. $\dfrac{x^3 - x}{x^2 + 1} > 0$

$$\longleftrightarrow$$

7. $(x+3)|x-1| \geq 0$

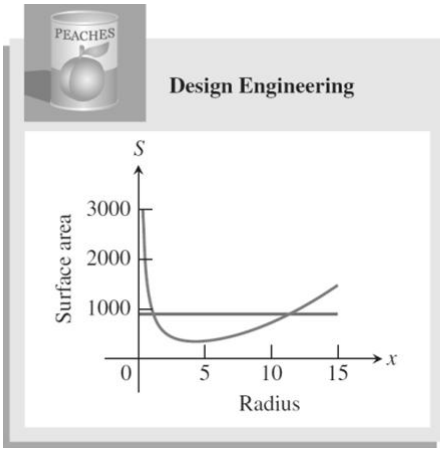
$$\longleftrightarrow$$

Solve the inequality graphically.

8. $\dfrac{(x-5)|x-2|}{\sqrt{2x-3}} \leq 0$

Skill 5: Applications Involving Inequalities

9. Design a Juice Can – Flannery Cannery packs peaches in 0.5-L cylindrical cans. * 1 liter = 1000 cm^3



(a) Express the volume V of the can as a function of the height h (in cm).

$$V = \pi r^2 h$$

(b) Express the surface area S of the can as a function of the radius x (in cm).

$$S = 2\pi r h + 2\pi r^2$$

(c) Find the dimensions of the can if the surface is less than 900 cm^2.

(d) Find the least possible surface area of the can.