

# Chapter 6: Transformations of Functions & Their Graphs

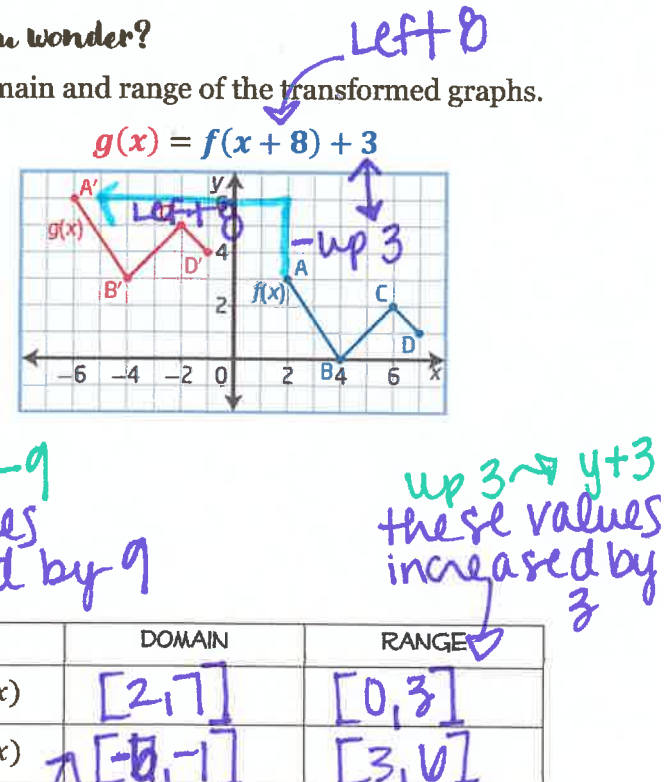
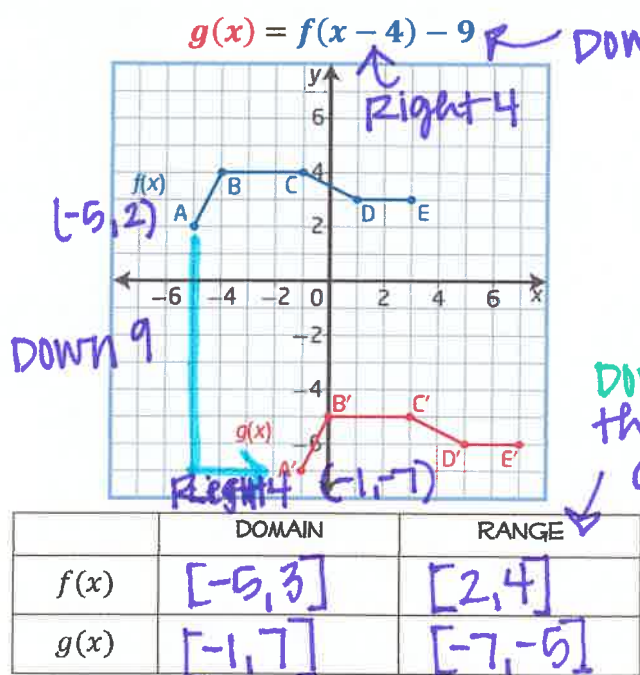
## 6.1 – VERTICAL & HORIZONTAL SHIFTS

**OBJECTIVES:**

- Identify the effect on the graph of a function replacing  $f(x)$  by  $f(x) + k$  and  $f(x + k)$  for specific values of  $k$  (both positive or negative)
- Describe, write a formula, graph and interpret a function that has been shifted vertically and/or horizontally

❖ **EXPLORING TRANSLATIONS – What do you notice? What do you wonder?**

Also identify the domain and range of  $f(x)$ , as well as the domain and range of the transformed graphs.



Down 9 →  $y - 9$   
these values decreased by 9

up 3 →  $y + 3$   
these values increased by 3

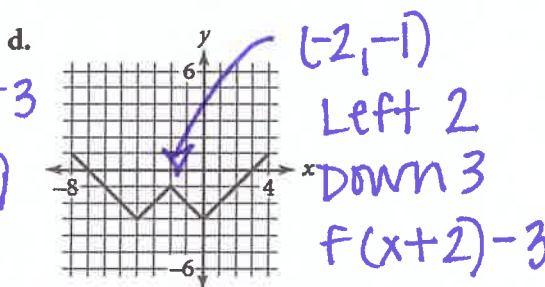
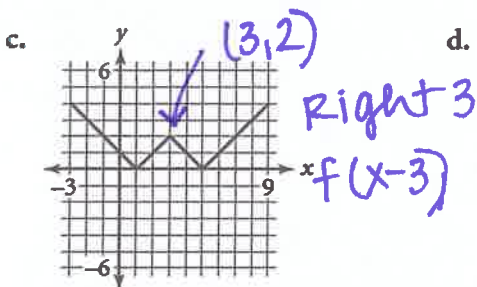
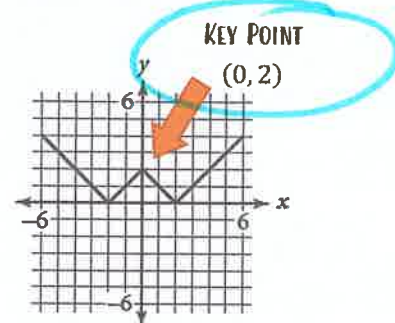
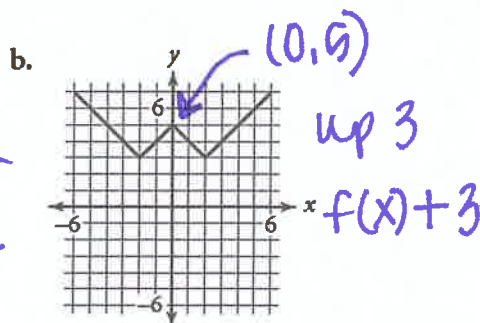
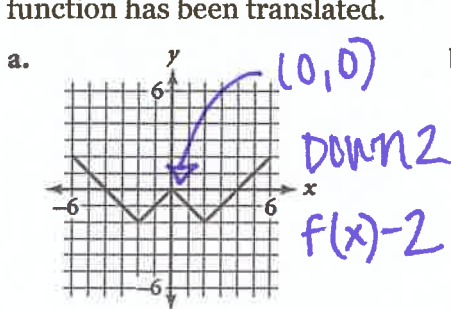
❖ **Translations (shifts)** Right 4 →  $x + 4$

these values decreased by 8 Left 8 →  $x - 8$

FUNCTION NOTATION	DESCRIPTION	COORDINATE RULE	DOMAIN OR RANGE CHANGE?
$y = f(x - h)$	Right $h$	$(x + h, y)$	Domain
$y = f(x + h)$	Left $h$	$(x - h, y)$	Domain
$y = f(x) + k$	Up $k$	$(x, y + k)$	Range
$y = f(x) - k$	Down $k$	$(x, y - k)$	Range

Examples:

1. The graph of  $y = f(x)$  is shown at right. Write an equation for each related graph showing how the function has been translated.



2. The graph of  $g(x)$  contains the point  $(-3, 0)$ . Describe the translation and then write a formula for a translation of  $g$  that has a graph containing the point  $(5, 9)$ .

$x = -3 \rightarrow 5$  Right 8  
 $y = 0 \rightarrow 9$  up 9  
 $g(x - 8) + 9$

3. Suppose that the  $x$ -intercepts of the graph of  $y = f(x)$  are  $-5$  &  $3$ . What are the  $x$ -intercepts of the graph of  $y = f(x + 2)$ ?

Left 2 =  $x - 2$   
 $-5 - 2 = -7$   
 $3 - 2 = 1$   
 $x\text{-int: } -7 \text{ \& } 1$

4. Suppose that the function  $y = f(x)$  is increasing on the interval  $(-1, 5)$ . On what interval is the graph of  $y = f(x - 2)$  increasing?

Right 2 =  $x + 2$   
 $\frac{-1 + 2 \quad 5 + 2}{(1, 7)}$

5. The domain of a function  $h(x)$  is  $[0, 12]$  and its range is  $[-4, 2]$ . What is the domain and range of  $h(x + 5) - 12$ ?

Left 5 =  $x - 5$   
 Down 12 =  $y - 12$   
 Range:  $[-16, -10]$   
 Domain:  $[-5, 7]$

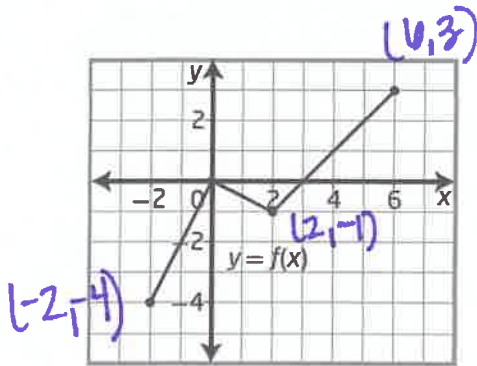
# 6.2 – REFLECTIONS

**OBJECTIVES:**

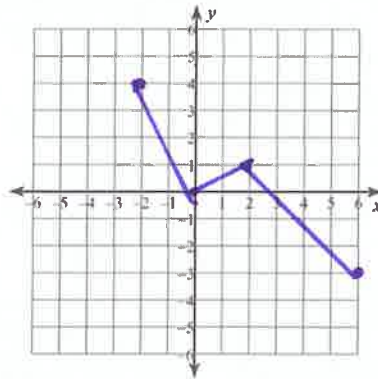
- Identify the effect on the graph of a function replacing  $f(x)$  by  $-f(x)$  and  $f(-x)$
- Describe, write a formula, graph and interpret a function that has been reflected

❖ **EXPLORING REFLECTIONS – What do you notice? What do you wonder?**

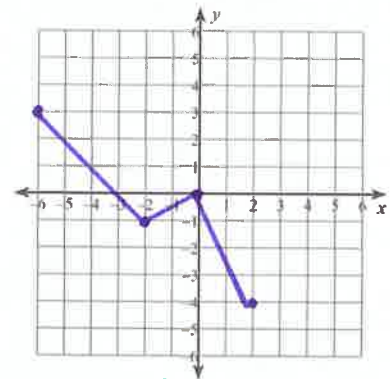
Also identify the domain and range of  $f(x)$ , as well as the domain and range of the transformed graphs.



Graph  $y = -f(x)$   
REFLECT OVER THE X-AXIS



Graph  $y = f(-x)$   
REFLECT OVER THE Y-AXIS



DOMAIN:

$[-2, 6]$

RANGE:

$[-4, 3]$

DOMAIN:

$[-2, 6]$

no change

RANGE:  $= -1 \cdot y$

$[-3, 4]$

opposite & flipped

DOMAIN:  $= -1 \cdot x$

$[-6, 2]$

opp. & flip

RANGE:

$[-4, 3]$

no change

❖ Reflections Across Axes (flips)

FUNCTION NOTATION	DESCRIPTION	COORDINATE RULE	DOMAIN OR RANGE CHANGE?
$y = -f(x)$	Reflect over x-axis	$(x, -1 \cdot y)$	Range
$y = f(-x)$	Reflect over y-axis	$(-1 \cdot x, y)$	Domain

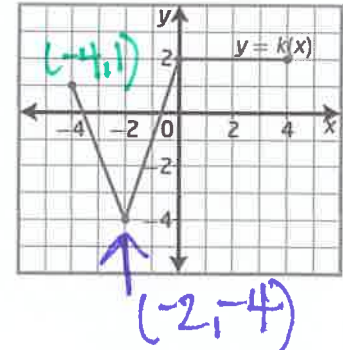
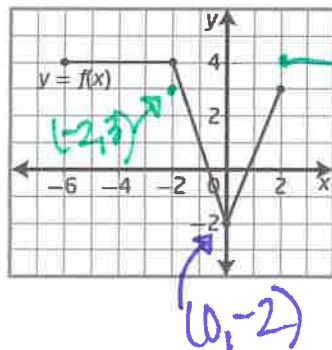
- The graph of  $f(x)$  contains the point  $(2, -3)$ . What point must lie on the reflected graph if the graph is...
  - reflected about the x-axis?
  - reflected about the y-axis?

$(2, 3)$   
 $(x, -1 \cdot y)$

$(-2, -3)$   
 $(-1 \cdot x, y)$

- The graph of  $y = f(x)$  is shown at left. Describe the transformation and then write the equation of  $k(x)$  in terms of  $f(x)$ .

$f(-x)$  Reflect over y-axis  
 $(x+2)$  Left 2  
 $f(x)+2$  up 2  
 $f(- (x+2)) + 2$



$k(x) = f(-x-2) + 2$

vertex  $(-1, -2)$

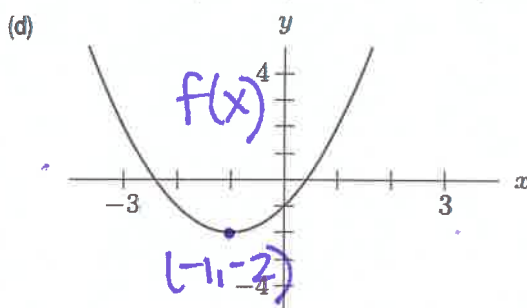
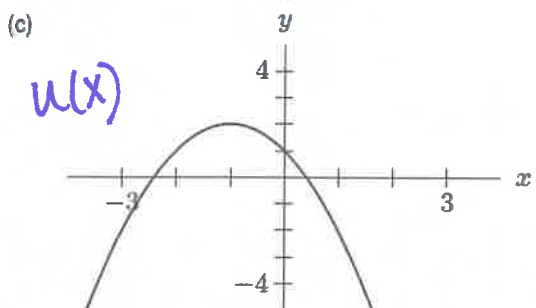
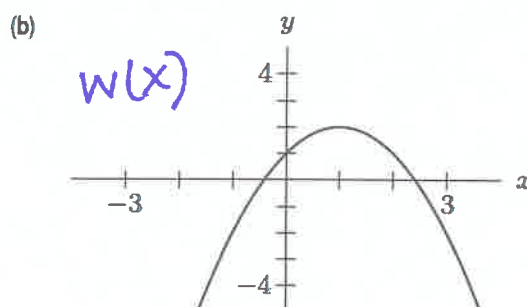
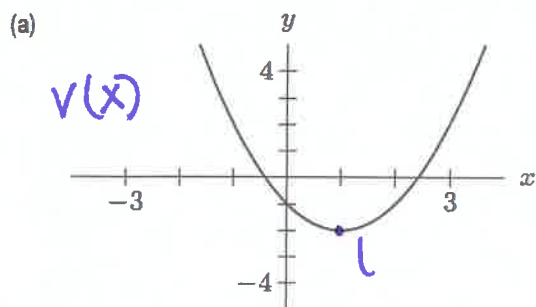
If  $f(x) = (x + 1)^2 - 2$ , match the following functions to the graphs.

3.  $f(x)$  **D**

4.  $u(x) = -f(x)$  **C**

5.  $v(x) = f(-x)$  **A**

6.  $w(x) = -f(-x)$  **B**



7. The domain of a function  $h(x)$  is  $[0, 12]$  and its range is  $[-4, 2]$ .

a. What is the domain and range of  $-h(x - 4)$ ?

Reflect  $x$ -axis  
 $-1 \cdot y$   
 (Range)

Right 4  
 $x + 4$   
 (Domain)

$D: [4, 16]$   
 $R: [-2, 4]$

b. What is the domain and range of  $h(-x) + 4$ ?

Reflect  $y$ -axis  
 $-1 \cdot x$   
 (Domain)

up 4  
 $y + 4$   
 (Range)

$D: [-12, 0]$   
 $R: [0, 6]$

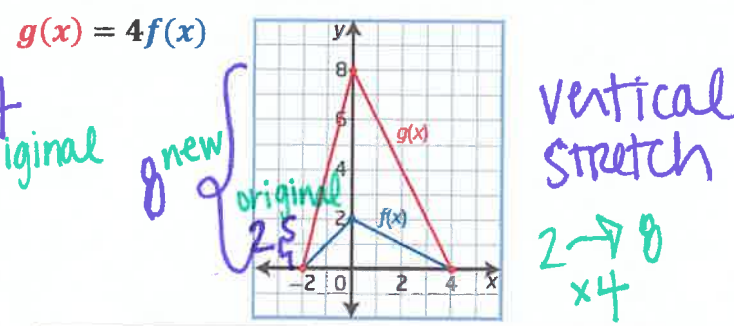
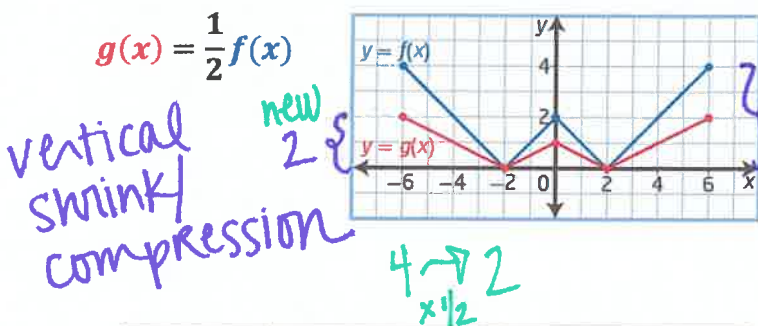
# 6.3 – VERTICAL STRETCHES & COMPRESSIONS

## OBJECTIVES:

- Identify the effect on the graph of a function replacing  $f(x)$  by  $kf(x)$  for specific values of  $k$
- Describe, write a formula, graph and interpret a function that has been vertically stretched or compressed

### ❖ EXPLORING VERTICAL SIZE CHANGES – What do you notice? What do you wonder?

Also identify the domain and range of  $f(x)$ , as well as the domain and range of the transformed graphs.



	DOMAIN	RANGE
$f(x)$	$[-6, 6]$	$[0, 4]$
$g(x)$	$[-6, 6]$	$[0, 2]$

*Handwritten notes: 'no change' under domain, 'x 1/2' next to range change.*

	DOMAIN	RANGE
$f(x)$	$[-2, 4]$	$[0, 2]$
$g(x)$	$[-2, 4]$	$[0, 8]$

*Handwritten notes: 'no change' under domain, 'x 4' next to range change.*

### ❖ Vertical Stretches & Compressions

FUNCTION NOTATION	DESCRIPTION	COORDINATE RULE	DOMAIN OR RANGE CHANGE?
$y = Af(x),  A  > 1$	vert. stretch $\times A$	$(x, A \cdot y)$	Range
$y = Af(x), 0 <  A  < 1$	vert. comp. $\times A$	$(x, A \cdot y)$	

### Examples:

1. The function  $g(x)$  is obtained from  $f(x)$  by a single transformation. Use the tables below to find a formula for  $g(x)$  in terms of  $f(x)$ . *Handwritten note: 'vert. stretch x3'*

$x$	-4	-2	0	2	4
$f(x)$	12	-4	-2	4	6

$x$	-4	-2	0	2	4
$g(x)$	36	-12	-6	12	18

$12 \times 3 = 36$     $-4 \times 3 = -12$     $g(x) = 3f(x)$

2. The graph of  $h(x)$  is found by vertically stretching the graph of  $f(x)$  by a factor of 7, reflecting it about the  $x$ -axis, and then vertically shifting it down 3 units. Find a formula for  $h(x)$  in terms of  $f(x)$ .

$h(x) = -7f(x) - 3$

3. The graph of  $f(x)$  contains the point  $(3, -2)$ . What corresponding point is on the graph of  $g(x) = 3f(x - 8)$ ?

$+8 \times 3$   
 $(11, -6)$

vert. stretch  $\times 3$   
 $3 \cdot y$   
Right?  $x + 8$

❖ **ORDER IS IMPORTANT!**

- |                            |                           |                                  |                            |                          |
|----------------------------|---------------------------|----------------------------------|----------------------------|--------------------------|
| 1 ⇒                        | 2 ⇒                       | 3 ⇒                              | 4 ⇒                        | 5                        |
| REFLECTION ABOUT<br>Y-AXIS | HORIZONTAL<br>TRANSLATION | VERTICAL<br>STRETCH/ compression | REFLECTION ABOUT<br>X-AXIS | VERTICAL<br>TRANSLATIONS |

4. Let  $y = f(x)$  be the function whose graph is given. Describe the transformations and then sketch the graphs of the transformations.

$$y = -\frac{1}{2}f(x+2) - 3$$

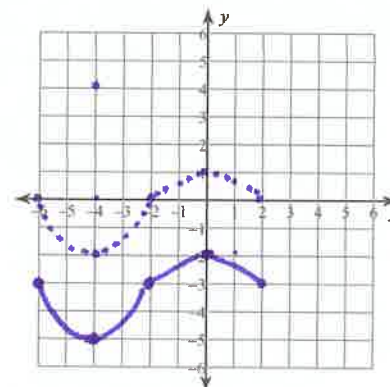
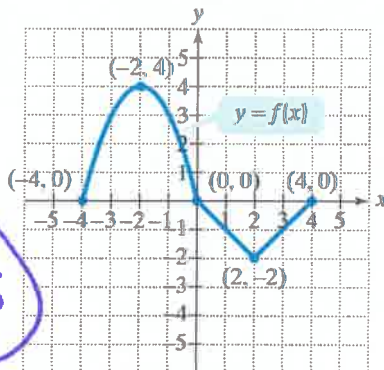
Transformations:

Left 2

vert. comp.  $\times \frac{1}{2}$

Reflect over x-axis

DOWN 3



5. The domain of a function  $h(x)$  is  $[0, 12]$  and its range is  $[-4, 2]$ . What is the domain and range of  $-2h(x+1) - 3$ ?

Left 1 =  $x-1$   
 Reflect over x-axis  
 vert. stretch  $\times 2$   
 $-2 \cdot y$   
 Down 3 =  $y-3$

$$(x, y) \rightarrow (x-1, -2y-3)$$

Domain  $[-1, 11]$

Range  $[5, -7] \rightarrow [-7, 5]$

6. The graph of  $g(x)$  is the graph of  $f(x)$  after it has been vertically stretched or compressed and then translated. The point  $(5, 12)$  lies on the graph of  $f(x)$ ;  $(2, 4)$  is the corresponding point on  $g(x)$ .

a. What is a possible transformation(s) have been performed on  $f(x)$ ?

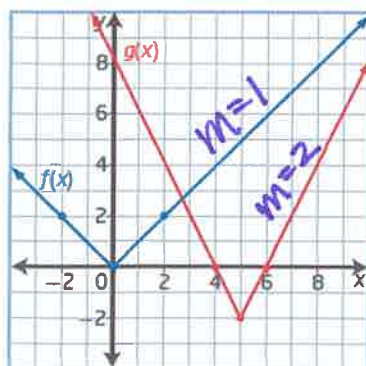
$x=5 \rightarrow 2$  Left 3      $y=12 \rightarrow 4$  Vert. comp.  $\times \frac{1}{3}$

b. What is a possible formula for  $g(x)$  in terms of  $f(x)$ ?

$$g(x) = \frac{1}{3}f(x+3)$$

Write an equation for  $g(x)$  as a transformation of the function  $f(x)$ .

7.

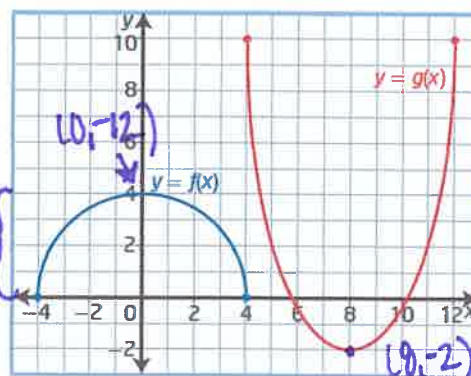


vert. stretch  $\times 2$

DOWN 2  
Right 5

$$g(x) = 2f(x-5) - 2$$

8.



height = 12

$h=4$

4  $\rightarrow$  12 vert. stretch  $\times 3$   
 Reflect x-axis

Right 0  
up 10

$$g(x) = -3f(x-0) + 10$$