

Chapter 6: Transformations of Functions & Their Graphs

6.1 – VERTICAL & HORIZONTAL SHIFTS

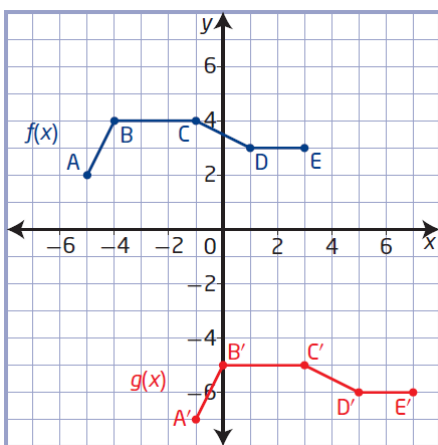
OBJECTIVES:

- Identify the effect on the graph of a function replacing $f(x)$ by $f(x) + k$ and $f(x + k)$ for specific values of k (both positive or negative)
- Describe, write a formula, graph and interpret a function that has been shifted vertically and/or horizontally

❖ **EXPLORING TRANSLATIONS – What do you notice? What do you wonder?**

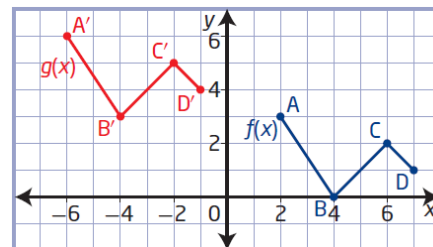
Also identify the domain and range of $f(x)$, as well as the domain and range of the transformed graphs.

$g(x) = f(x - 4) - 9$



	DOMAIN	RANGE
$f(x)$		
$g(x)$		

$g(x) = f(x + 8) + 3$



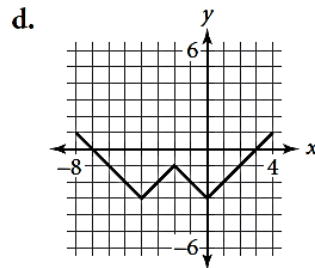
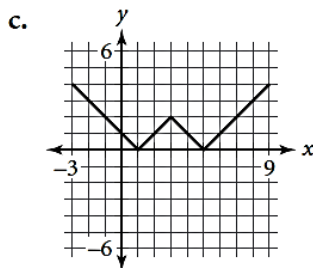
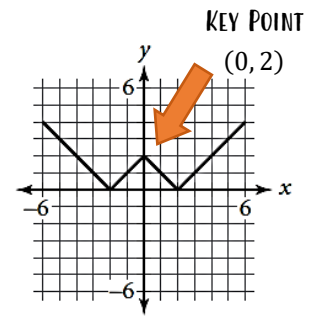
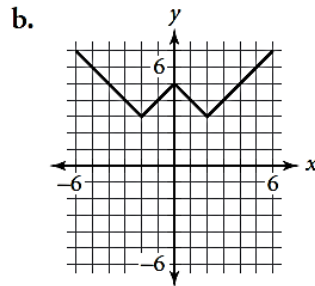
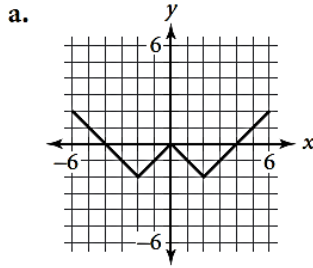
	DOMAIN	RANGE
$f(x)$		
$g(x)$		

❖ **Translations (shifts)**

FUNCTION NOTATION	DESCRIPTION	COORDINATE RULE	DOMAIN OR RANGE CHANGE?
$y = f(x - h)$			
$y = f(x + h)$			
$y = f(x) + k$			
$y = f(x) - k$			

Examples:

1. The graph of $y = f(x)$ is shown at right. Write an equation for each related graph showing how the function has been translated.



2. The graph of $g(x)$ contains the point $(-3, 0)$. Describe the translation and then write a formula for a translation of g that has a graph containing the point $(5, 9)$.
3. Suppose that the x -intercepts of the graph of $y = f(x)$ are -5 & 3 . What are the x -intercepts of the graph of $y = f(x + 2)$?
4. Suppose that the function $y = f(x)$ is increasing on the interval $(-1, 5)$. On what interval is the graph of $y = f(x - 2)$ increasing?
5. The domain of a function $h(x)$ is $[0, 12]$ and its range is $[-4, 2]$. What is the domain and range of $h(x + 5) - 12$?

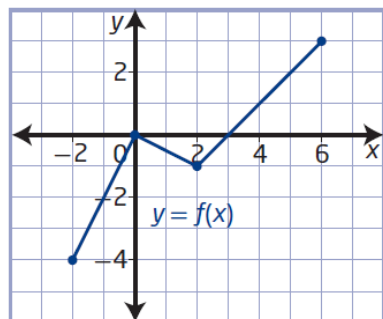
6.2 – REFLECTIONS

OBJECTIVES:

- Identify the effect on the graph of a function replacing $f(x)$ by $-f(x)$ and $f(-x)$
- Describe, write a formula, graph and interpret a function that has been reflected

❖ EXPLORING REFLECTIONS – *What do you notice? What do you wonder?*

Also identify the domain and range of $f(x)$, as well as the domain and range of the transformed graphs.

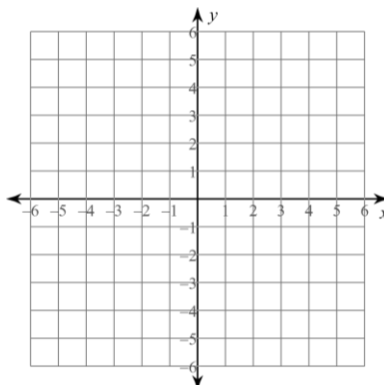


DOMAIN:

RANGE:

Graph $y = -f(x)$

REFLECT OVER THE x -AXIS

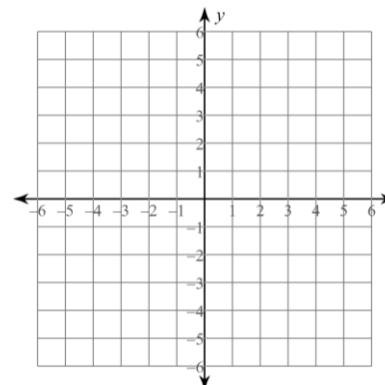


DOMAIN:

RANGE:

Graph $y = f(-x)$

REFLECT OVER THE y -AXIS



DOMAIN:

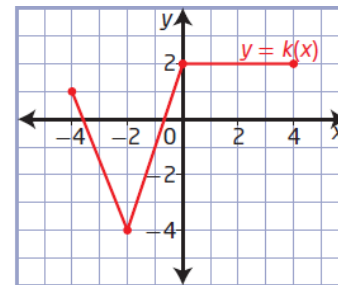
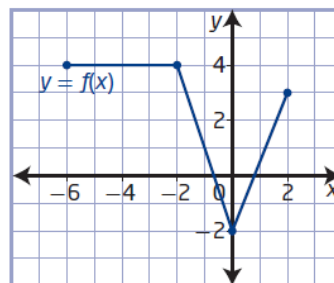
RANGE:

❖ Reflections Across Axes (flips)

FUNCTION NOTATION	DESCRIPTION	COORDINATE RULE	DOMAIN OR RANGE CHANGE?
$y = -f(x)$	Reflect over x -axis		
$y = f(-x)$	Reflect over y -axis		

- The graph of $f(x)$ contains the point $(2, -3)$. What point must lie on the reflected graph if the graph is...
 - reflected about the x -axis?
 - reflected about the y -axis?

- The graph of $y = f(x)$ is shown at left. Describe the transformation and then write the equation of $k(x)$ in terms of $f(x)$.



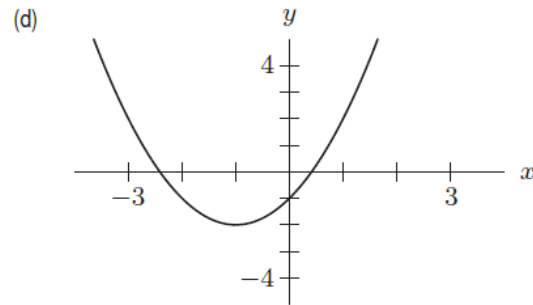
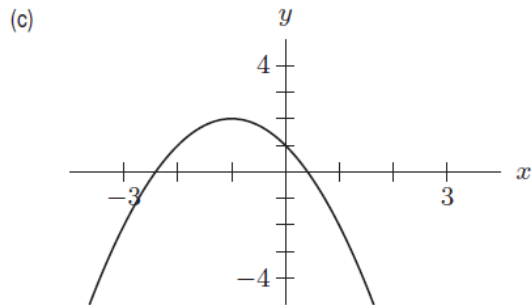
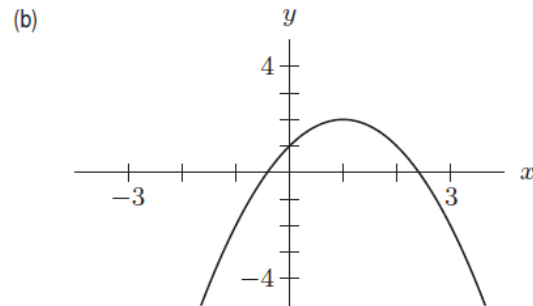
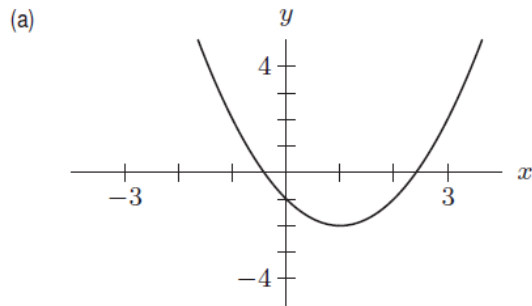
If $f(x) = (x + 1)^2 - 2$, match the following functions to the graphs.

3. $f(x)$

4. $u(x) = -f(x)$

5. $v(x) = f(-x)$

6. $w(x) = -f(-x)$



7. The domain of a function $h(x)$ is $[0, 12]$ and its range is $[-4, 2]$.

a. What is the domain and range of $-h(x - 4)$?

b. What is the domain and range of $h(-x) + 4$?

6.3 – VERTICAL STRETCHES & COMPRESSIONS

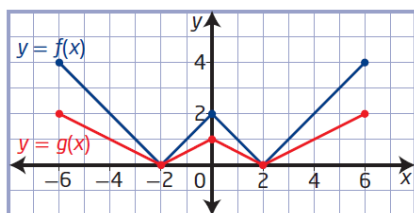
OBJECTIVES:

- Identify the effect on the graph of a function replacing $f(x)$ by $kf(x)$ for specific values of k
- Describe, write a formula, graph and interpret a function that has been vertically stretched or compressed

❖ EXPLORING VERTICAL SIZE CHANGES – *What do you notice? What do you wonder?*

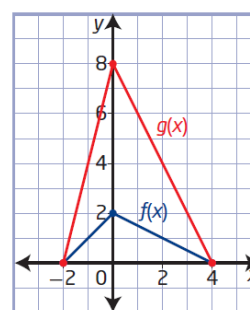
Also identify the domain and range of $f(x)$, as well as the domain and range of the transformed graphs.

$$g(x) = \frac{1}{2}f(x)$$



	DOMAIN	RANGE
$f(x)$		
$g(x)$		

$$g(x) = 4f(x)$$



	DOMAIN	RANGE
$f(x)$		
$g(x)$		

❖ Vertical Stretches & Compressions

FUNCTION NOTATION	DESCRIPTION	COORDINATE RULE	DOMAIN OR RANGE CHANGE?
$y = Af(x), A > 1$			
$y = Af(x), 0 < A < 1$			

Examples:

- The function $g(x)$ is obtained from $f(x)$ by a single transformation. Use the tables below to find a formula for $g(x)$ in terms of $f(x)$.

x	-4	-2	0	2	4
$f(x)$	12	-4	-2	4	6

x	-4	-2	0	2	4
$g(x)$	36	-12	-6	12	18

- The graph of $h(x)$ is found by vertically stretching the graph of $f(x)$ by a factor of 7, reflecting it about the x -axis, and then vertically shifting it down 3 units. Find a formula for $h(x)$ in terms of $f(x)$.
- The graph of $f(x)$ contains the point $(3, -2)$. What corresponding point is on the graph of $g(x) = 3f(x - 8)$?

❖ **ORDER IS IMPORTANT!**

1 ⇒

REFLECTION ABOUT
Y-AXIS

2 ⇒

HORIZONTAL
TRANSLATION

3 ⇒

VERTICAL
STRETCH/ compression

4 ⇒

REFLECTION ABOUT
X-AXIS

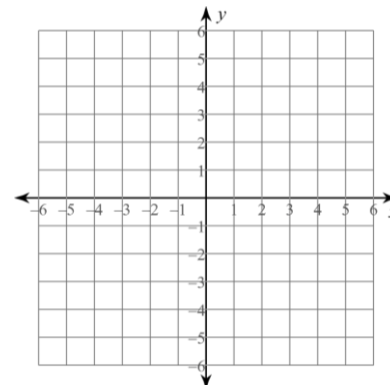
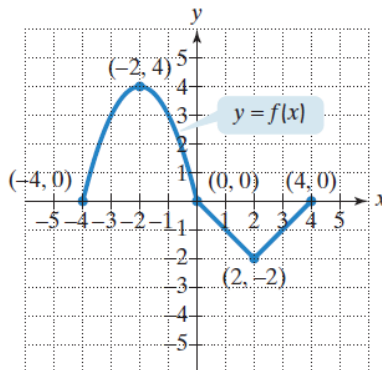
5

VERTICAL
TRANSLATIONS

4. Let $y = f(x)$ be the function whose graph is given. Describe the transformations and then sketch the graphs of the transformations.

$$y = -\frac{1}{2}f(x + 2) - 3$$

Transformations:



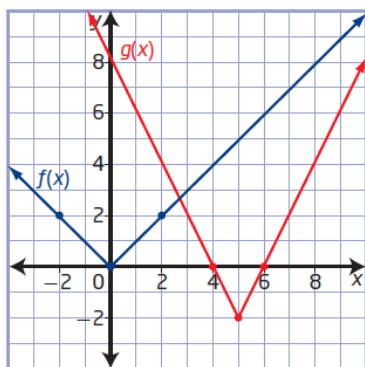
5. The domain of a function $h(x)$ is $[0, 12]$ and its range is $[-4, 2]$. What is the domain and range of $-2h(x + 1) - 3$?

6. The graph of $g(x)$ is the graph of $f(x)$ after it has been vertically stretched or compressed and then translated. The point $(5, 12)$ lies on the graph of $f(x)$; $(2, 4)$ is the corresponding point on $g(x)$.

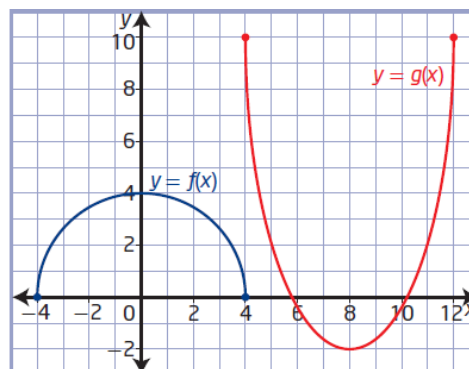
- What is a possible transformation(s) have been performed on $f(x)$?
- What is a possible formula for $g(x)$ in terms of $f(x)$?

Write an equation for $g(x)$ as a transformation of the function $f(x)$.

7.



8.



COMBINING A REFLECTION OVER THE y -AXIS WITH A HORIZONTAL TRANSLATION

Graph the function $f(x) = \sqrt{1 - x} + 2$. Find the domain and the range of f .

Solution Because horizontal shifts require the form $x - h$, we begin by rewriting $f(x)$ as $f(x) = \sqrt{1 - x} + 2 = \sqrt{-(x - 1)} + 2$. Now use the following steps:

STEP 1: $y = \sqrt{x}$ Square root function

STEP 2: $y = \sqrt{-x}$ Replace x by $-x$; reflect about the y -axis.

STEP 3: $y = \sqrt{-(x - 1)} = \sqrt{1 - x}$ Replace x by $x - 1$; horizontal shift to the right 1 unit.

STEP 4: $y = \sqrt{1 - x} + 2$ Add 2; vertical shift up 2 units.

See Figure 59.

Figure 59

