6.1 VECTORS IN THE PLANE

Objectives: Apply the arithmetic of vectors; Use vectors to solve real-world problems

Two-Dimension Vectors

- A ______________________________ is an ordered pair of real numbers, denoted in ____________________ as $\langle a, b \rangle$.
  - The numbers a and b are the ____________________ of the vector v
  - The _________________________ of the vector $\langle a, b \rangle$ is the arrow from the origin to the point (a, b)
  - The _________________ of v is the length of the vector and the ___________ of v is the direction in which the arrow is pointing

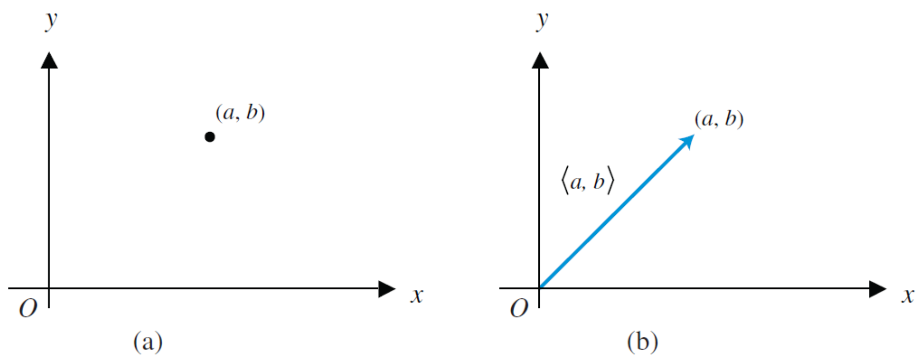
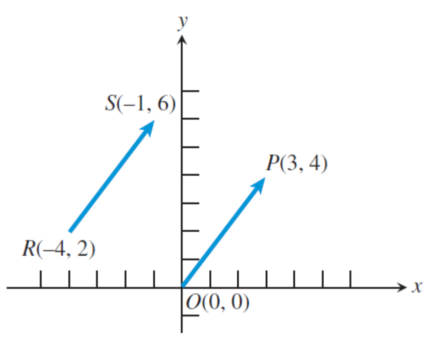


FIGURE 6.1 The point represents the ordered pair (a, b). The arrow (directed line segment) represents the vector $\langle a, b \rangle$.

- The vector $0 = \langle 0, 0 \rangle$, called the _________________________, has zero length and no direction
- Any two arrows with the same length and pointing in the same direction represent the same vector
  - The arrows $\overrightarrow{RS}$ & $\overrightarrow{OP}$ both represent the vector $\langle 3, 4 \rangle$, as would any arrow with the same length pointing in the same direction.
    - Such arrows are called __________________



- Head Minus Tail (HMT) Rule
  - If an arrow has initial point (x_1, y_1) and terminal point (x_2, y_2), it represents the vector _________________________

- Magnitude
  - The magnitude of a vector v is also called the _______________________ of v, |v| or ||v||
  - If v is represented by the arrow from (x_1, y_1) to (x_2, y_2), then

    ____

    - If $\mathrm{v} = \langle a, b \rangle$, then $|\mathrm{v}| =$ _____________________

Example 1: Showing Arrows are Equivalent

Prove that $\overrightarrow{RS}$ and $\overrightarrow{PQ}$ are equivalent by showing that they represent the same vector: $R = (-4, 2), S = (-1, 6), P = (2, -1), \& \ Q = (5, 3)$

Example 2: Finding Magnitude of a Vector

Find the magnitude of the vector v represented by $\overrightarrow{PQ}$, where $P = (-3, 4) \ \& \ Q = (-5, 2)$.

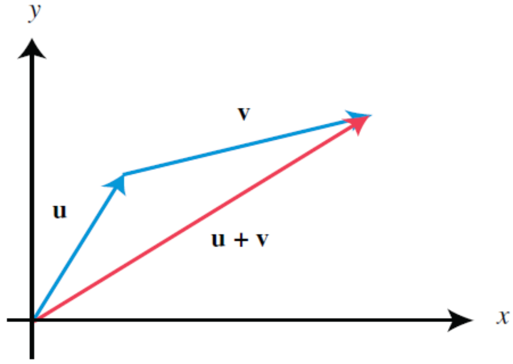
- Vector Operations
  - Vector Addition
    - Let $\mathrm{u} = \langle u_1, u_2 \rangle$ and $\mathrm{v} = \langle v_1, v_2 \rangle$ be vectors and let k be a real number (scalar). The sum (or resultant) of the vectors u & v is the vector…

      ____

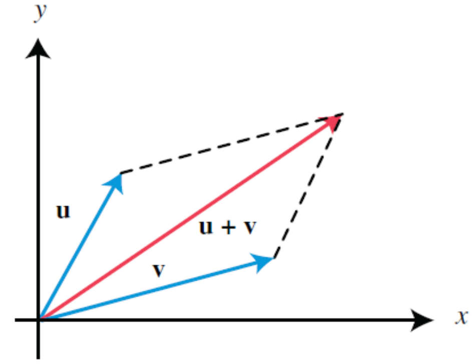
    - The product of the scalar k and the vector u is…

      ____

  - The sum of the vectors u & v can be represented geometrically by arrows in two ways.
    - Tail-to-head representation (a)
    - Parallelogram representation (b)



(a)



(b)

Example 3: Performing Vector Operations

Let $\mathrm{u} = \langle 2, -1 \rangle$, $\mathrm{v} = \langle 1, 3 \rangle$, & $\mathrm{w} = \langle -1, -1 \rangle$. Find the component form of the following vectors:

(a) $-\mathrm{u} + \mathrm{v}$

(b) $\mathrm{u} + (-1)\mathrm{w}$

(c) $-\mathrm{u} + \mathrm{w}$

(d) $3\mathrm{u} + 2\mathrm{v}$

(e) $-\mathrm{v} - 2\mathrm{w}$

Unit Vectors

- A vector u with length $|\mathrm{u}| = 1$ is a unit vector.
  - If v is not the zero vector $\langle 0, 0 \rangle$, then the vector

    ...is a unit vector in the direction of v.

Standard Unit Vectors

- Any vector v can be written as an expression in terms of the standard unit vector:

  ____

  - $\mathrm{v} = \langle a, b \rangle$ is expressed as the ____________ ____________________ $a\mathrm{i} + b\mathrm{j}$ of the vectors i & j
  - The scalars a & b are the ___________________ and ________________________ of the vector v

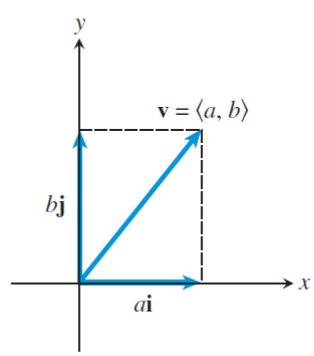


FIGURE 6.9 The vector **v** is equal to $a\mathbf{i} + b\mathbf{j}$.

Example 4: Finding a Unit Vector

Find the unit vector in the direction of the given vector.

(a) $\mathrm{u} = \langle 2, -1 \rangle$

(b) $\mathrm{v} = 3\mathrm{i} + 2\mathrm{j}$

- Direction Angles
  - A simple but precise way to specify the direction of a vector v is to state its direction angle, the angle θ that ______________________________
  - Resolving the Vector
    - If v has direction angle θ, the components of v can be expressed using the formula...

      ____

Example 5: Finding the Components of a Vector

Find the components of a given vector.

(a) u with direction angle $105°$ and magnitude 8

(b) v with direction angle $75°$ and magnitude 7.5

Example 6: Finding the Direction Angle of a Vector

Find the magnitude and direction of each vector.

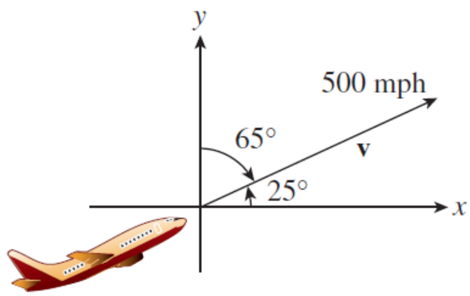
(a) $u = \langle 1, 4 \rangle$

(b) $v = \langle -5, 4 \rangle$

(c) $w = \langle 3, -2 \rangle$

- Applications of Vectors
  - The ________________ of a moving object is a vector because velocity has both magnitude and direction.
    - The magnitude of velocity is speed.

Example 9: Writing Velocity as a Vector



A DC-10 jet aircraft is flying on a bearing of 65° at 500 mph. Find the component form of the velocity of the airplane. Recall that the bearing is the angle that the line of travel makes with due north, measured clockwise.

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6.2 DOT PRODUCT OF VECTORS

Objectives: Calculate the dot product of vectors

The Dot Product

- The dot product or inner product of $\mathrm{u} = \langle u_1, u_2 \rangle$ & $\mathrm{v} = \langle v_1, v_2 \rangle$ is...

  ____

Examples: Finding Dot Products

Find the dot product of the given vectors.

1. $\mathrm{u} = \langle 3, 1 \rangle$ & $\mathrm{v} = \langle 2, -5 \rangle$

2. $\mathrm{u} = \langle 5, 4 \rangle$ & $\mathrm{v} = \langle -4, 5 \rangle$

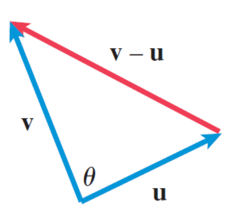
3. $\mathrm{u} = (-2\mathrm{i} + 2\mathrm{j})$ & $\mathrm{v} = (3\mathrm{i} - 5\mathrm{j})$

Example 4: Using Dot Product to Find Length

Use the dot product to find the length of the vector $\mathrm{u} = \langle 4, -3 \rangle$

Angle Between Vectors

- If θ is the angle between the nonzero vectors u & v, then...



$\cos\theta =$ ______________________________

FIGURE 6.16 The angle θ between nonzero vectors **u** and **v**.

Examples: Finding the Angle between Vectors

Find the angle between the two given vectors.

5. $u = \langle 4, 3 \rangle$ & $v = \langle -3, -1 \rangle$

6. $u = \langle 5, 3 \rangle$ & $v = \langle -1, -3 \rangle$

7. $u = i + \sqrt{2}j$ & $v = -\sqrt{2}i - 4j$

Orthogonal Vectors

- The vectors u and v are orthogonal if and only if ______________________
  - The terms "perpendicular" and "orthogonal" almost mean the same thing

Example 8: Proving Vectors are Orthogonal

Prove that the vectors $u = \langle 2, 3 \rangle$ & $v = \langle -6, 4 \rangle$ are orthogonal by showing that their dot product is zero.

Example 8: Parallel, Orthogonal or Neither?

Determine whether the vectors $u = \langle 2, -7 \rangle$ & $v = \langle -4, 14 \rangle$ are parallel, orthogonal, or neither.

6.3 PARAMETRIC EQUATIONS

Objectives: Define parametric equations, eliminate the parameter, and find parametric equations for a line

Parametric Equations & Curves

- The graph of the ordered pairs (x, y) where $x = f(t)$ & $y = g(t)$ are functions defined on an interval I of t-values is a ________________________. The equations are called _____________________________ for the curve, the variable t is a ______________.

Eliminating the Parameter

- When a curve is defined parametrically it is sometimes possible to __________ ________________________ and obtain a rectangular equation in x & y that represents the curve.

Examples: Eliminating the Parameter

Eliminate the parameter and identify the graph of the parametric curve – a line, parabola, or a circle.

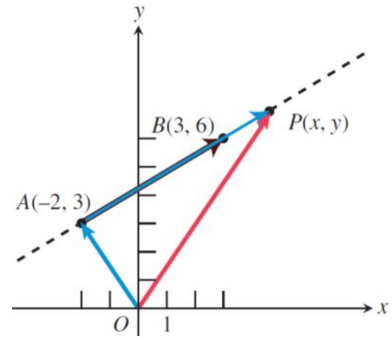
1. $x = 2 - t, y = 2t + 1$

2. $x = t^2, y = 2 - t$

3. $x = t - 2, y = 1 + 2t^2$

4. $x = 6\cos t\,, y = 6\sin t\,, 0 \leq t \leq 2\pi$

Lines and Line Segments



- We can use vectors to help us find parametric equations for a line.
  - Example: Find a parametrization of the line through the points $A(-2,3)$ & $B(3,6)$
    - Let $P(x,y)$ be an arbitrary point on the line through A & B.
    - The vector $\overrightarrow{OP}$ is the tail-to-head vector sum of $\overrightarrow{OA}$ & $\overrightarrow{AP}$ and $\overrightarrow{AP}$ is a scalar multiple of $\overrightarrow{AB}$.

      $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$

      $\overrightarrow{OP} = \overrightarrow{OA} + t \cdot \overrightarrow{AB}$

      $\langle x, y\rangle = \langle -2, 3\rangle + t\langle 3-(-2), 6-3\rangle$

      $\langle x, y\rangle = \langle -2, 3\rangle + t\langle 5, 3\rangle$

      $\langle x, y\rangle = \langle -2+5t, 3+3t\rangle$

Examples: Finding Parametric Equations for a Line Segment

Find the parametrization of the line segment with endpoints A & B.

5. $A = (1,-1), B = (-2,7)$

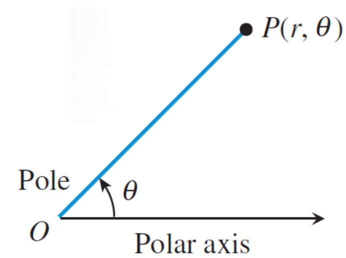
6. $A = (12,-2), B = (4,3)$

6.4 POLAR COORDINATES

Objectives: Convert points and equations from polar to rectangular coordinates and vice versa

Polar Coordinate System

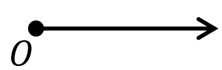
- A ____ is a plane with a point O, the __________, and a ray from O to the _______________
- Each point P in the plane is assigned as _________ _______________ follows:
  - r is the _________________________ from O to P
  - θ is the _________________________ whose initial side is on the polar axis and whose terminal side is on the line OP
    - As in trigonometry, we measure θ as positive when moving counterclockwise; negative when moving clockwise
    - If $r > 0$, then P is on the terminal side of θ
    - If $r < 0$, then P is on the terminal side of $\theta + \pi$

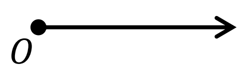


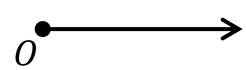
Examples: Plotting Points in the Polar Coordinate System

Plot the points with the given polar coordinates.

1. $P\left(2, \frac{\pi}{3}\right)$
2. $Q\left(-1, \frac{3\pi}{4}\right)$
3. $R(3, -45°)$







Finding all Polar Coordinates of a Point

- Let P have polar coordinates (r, θ). Any other polar coordinate of P must be of the form...

  ____

  ...where n is any integer.
  - In particular, the pole has polar coordinates $(0, \theta)$, where θ is any angle.

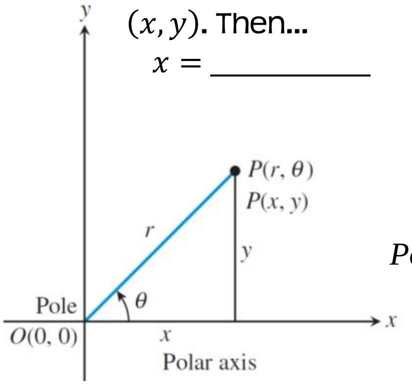
Example 4: Finding all Polar Coordinates for a Point

If point P has polar coordinates $(3, \pi/3)$, find all polar coordinates for P.

Coordinate Conversion

- Let the point P have polar coordinates (r, θ) and rectangular coordinates (x, y). Then...

  $x =$ ___________ $y =$ ___________ $r^2 =$ ___________ $\tan\theta =$ ______



Polar and rectangular coordinates for P.

<u>Examples: Converting from Polar to Rectangular Coordinates</u>

Find the rectangular coordinates of a point with the given polar coordinates.

5. $P\left(3, \frac{3\pi}{4}\right)$

6. $R\left(3, -\frac{5\pi}{6}\right)$

7. $T(2, -215°)$

8. $V(6, 80°)$

When converting rectangular coordinates to polar coordinates, we must remember there are infinitely many possible polar coordinate pairs.

<u>Examples: Converting from Rectangular to Polar Coordinates</u>

For the point with the given rectangular coordinates, find all polar coordinates that satisfy $-\pi \leq \theta \leq \pi$.

9. $P(-3, 3)$

10. $R(3, -5)$

11. $T(-2, -2)$

12. $Q(2, 5)$

Examples: Converting from Polar Form to Rectangular Form

Convert the polar equation to rectangular form.

13. $r = 2\cos\theta + \sin\theta$

14. $r = -6\sin\theta$

15. $r\sec\theta = -12$

16. $r = 4\csc\theta$

Examples: Converting from Rectangular Form to Polar Form

Convert the rectangular equation to polar form.

17. $x^2 + (y-4)^2 = 16$

18. $(x-3)^2 + (y-4)^2 = 25$

19. $y = 3$

20. $3y - 2x = 3$