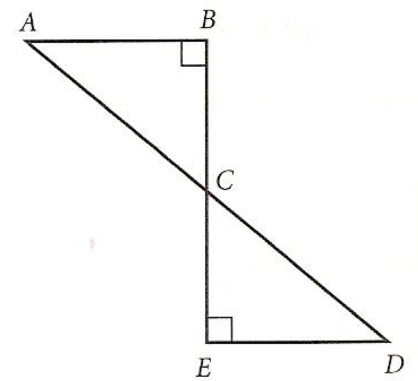
7.7.D2 ~ Using Congruent Triangles

Accessing Prior Knowledge

a. If C is the midpoint of $\overline{BE}$, then what two segments are congruent?

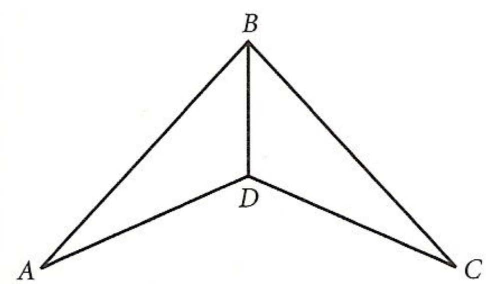
b. If $\overline{BE}$ & $\overline{AD}$ intersect at C, what two angles must be congruent and why?

c. Name two other congruent angles and explain why they are congruent.



d. If $\overrightarrow{BD}$ bisects $\angle ABC$, then what two angles are congruent

e. Why is $\overline{BD} \cong \overline{BD}$?



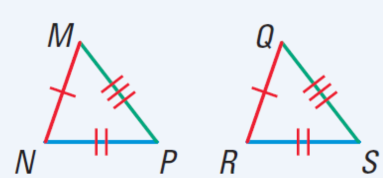
PROVING TRIANGLES CONGRUENT COULD BE A VERY TEDIOUS TASK IF WE HAD TO VERIFY THE CONGRUENCE OF EVERY ONE OF THE SIX PAIRS OF CORRESPONDING PARTS.

TRIANGLES HAVE SOME SPECIAL PROPERTIES THAT WILL ENABLE US TO PROVE TWO TRIANGLES ARE CONGRUENT BY COMPARING <u>ONLY THREE</u> SPECIALLY CHOSEN PAIRS OF CORRESPONDING PARTS.

Side-Side-Side Congruence Postulate (SSS)

Words If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

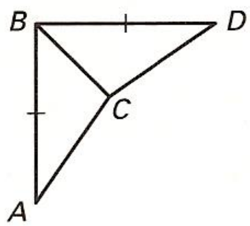
Symbols If **S**ide $\overline{MN} \cong \overline{QR}$, and
Side $\overline{NP} \cong \overline{RS}$, and
Side $\overline{PM} \cong \overline{SQ}$,
then $\triangle MNP \cong \triangle QRS$.



Examples: Using the SSS Congruence Postulate

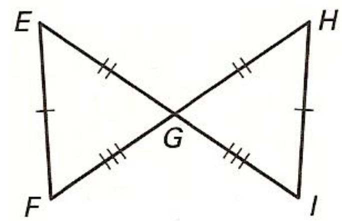
Does the diagram give enough information to use the SSS Congruence Postulate? Explain your reasoning.

1.



List those angles &/or sides you know to be congruent:

2.



List those angles &/or sides you know to be congruent:

Side-Angle-Side Congruence Postulate (SAS)

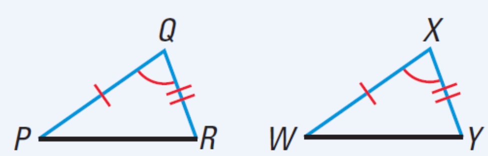
Words If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

Symbols If **S**ide $\overline{PQ} \cong \overline{WX}$, and

Angle $\angle Q \cong \angle X$, and

Side $\overline{QR} \cong \overline{XY}$,

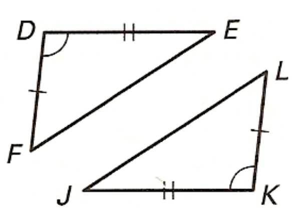
then $\triangle PQR \cong \triangle WXY$.



Examples: Using the SAS Congruence Postulate

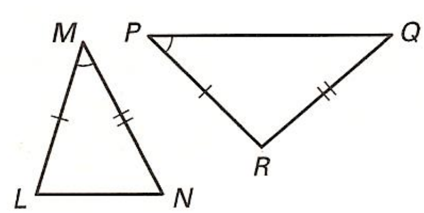
Does the diagram give enough information to use the SAS Congruence Postulate? Explain your reasoning.

3.



List those angles &/or sides you know to be congruent:

4.



List those angles &/or sides you know to be congruent:

Angle-Side-Angle Congruence Postulate (ASA)

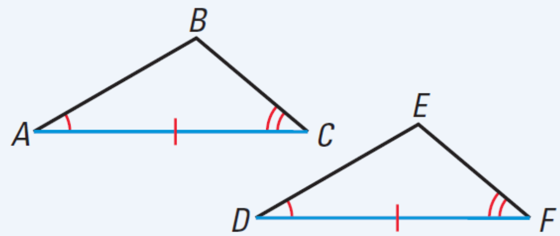
Words If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

Symbols If **A**ngle $\angle A \cong \angle D$, and

Side $\overline{AC} \cong \overline{DF}$, and

Angle $\angle C \cong \angle F$,

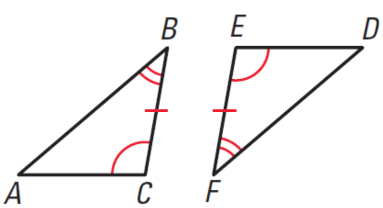
then $\triangle ABC \cong \triangle DEF$.



Examples: Using the ASA Congruence Postulate

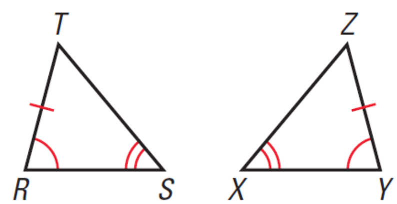
Does the diagram give enough information to use the ASA Congruence Postulate? Explain your reasoning.

5.



List those angles &/or sides you know to be congruent:

6.



List those angles &/or sides you know to be congruent:

Angle-Angle-Side Congruence Theorem (AAS)

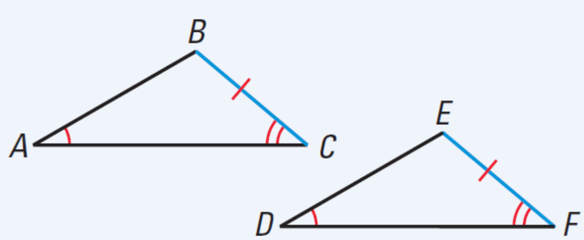
Words If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.

Symbols If **A**ngle $\angle A \cong \angle D$, and

Angle $\angle C \cong \angle F$, and

Side $\overline{BC} \cong \overline{EF}$,

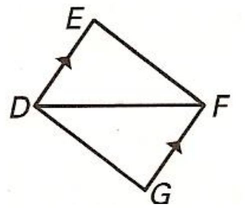
then $\triangle ABC \cong \triangle DEF$.



Examples: Using the AAS Congruence Theorem

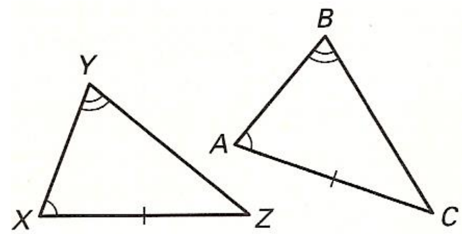
Based on the diagram, can you use the AAS Congruence Theorem to show that the triangles are congruent? If not, what additional congruence is needed?

7.



List those angles &/or sides you know to be congruent:

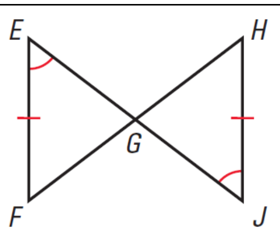
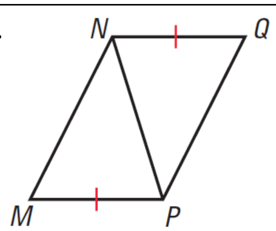
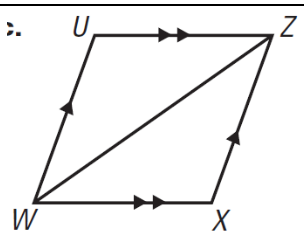
8.



List those angles &/or sides you know to be congruent:

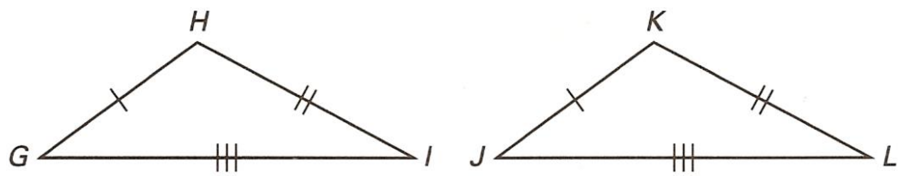
Examples: Deciding Whether Triangles are Congruent

Does the diagram given enough information to show that the triangles are congruent? If so, state the method – SSS, SAS, ASA or AAS – you would use.

Diagram	Congruences	Method
9. 		
10. 		
11. 		

7.8 ~ Congruent Triangle Proofs

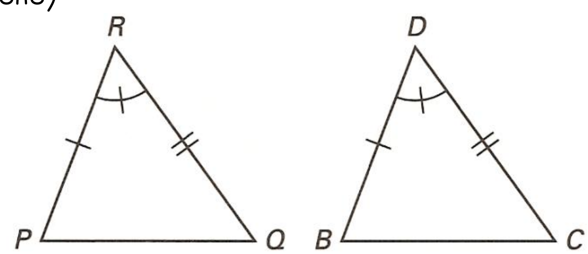
- Methods of Proving Triangles Congruent
  - The Side-Side-Side Congruence Theorem
    - If three sides of one triangle are congruent to the corresponding sides of another triangle, then the triangles are congruent. (SSS)



Side | Side | Side

$\overline{GH} \cong$ _______ _______ $\cong \overline{LJ}$ $\overline{HI} \cong$ _______

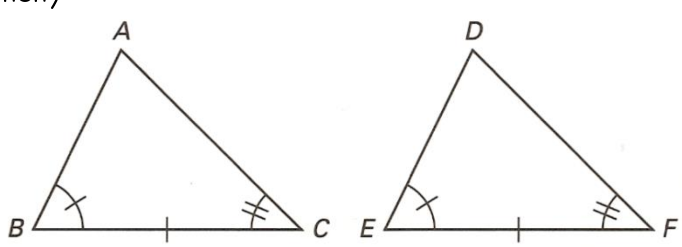
  - The Side-Angle-Side Congruence Theorem
    - If two sides and the included angle of one triangle are congruent to the corresponding two sides and the included angle of a second triangle, then the triangles are congruent. (SAS)



Side | Angle | Side

$\overline{RP} \cong$ _______ $\angle QRP \cong$ _______ _______ $\cong \overline{DC}$

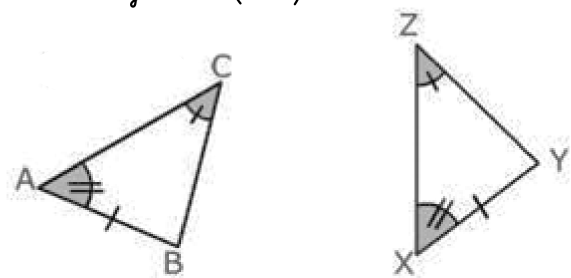
  - The Angle-Side-Angle Congruence Theorem
    - If two angles and the included side of one triangle are congruent to the corresponding two angles and included side of another triangle, then the triangles are congruent. (ASA)



Angle | Side | Angle

$\angle CBA \cong$ _______ _______ $\cong \overline{EF}$ _______ $\cong \angle DFE$

- <u>The Angle-Angle-Side Congruence Theorem</u>
  - If two angles and a non-included side of one triangle are congruent to the corresponding two angles and the corresponding non-included side of a second triangle, then the triangles are congruent. (AAS)

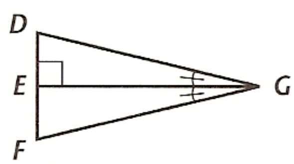


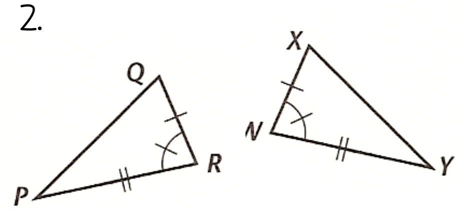
Angle	Angle	Side
$\angle CAB \cong$ ______	______ $\cong \angle XZY$	______ $\cong \overline{XY}$

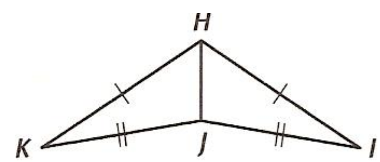
BE ON THE LOOKOUT FOR: (1) VERTICAL ANGLES & (2) SHARED SIDES – REFLEXIVE PROPERTY

<u>Examples: Congruent? SSS, SAS, ASA or AAS?</u>

Determine whether you could prove that the triangles are congruent. If so, write a congruence statement & identify the postulate you could use.

1.


2.


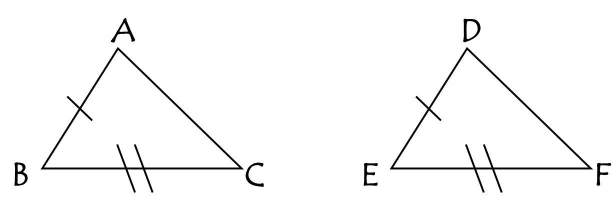
3.


<u>Examples:</u>

In 4 – 7, you are given the congruent angles and sides shown by the tick marks. Name the additional congruent sides or angles needed to prove that the triangles are congruent by each specified method.

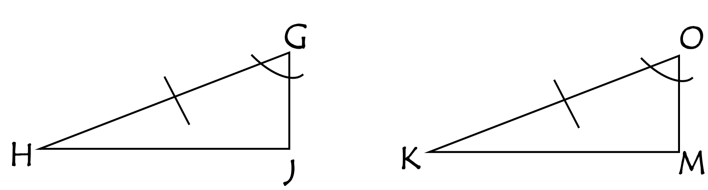
4. a. SSS

   b. SAS



5. a. ASA

   b. AAS

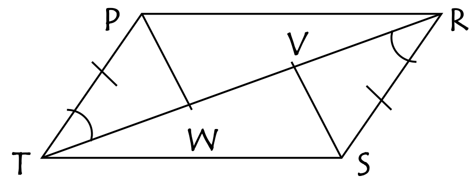


6. Prove: $\triangle PWT \cong \triangle SVR$

a. SAS

b. ASA

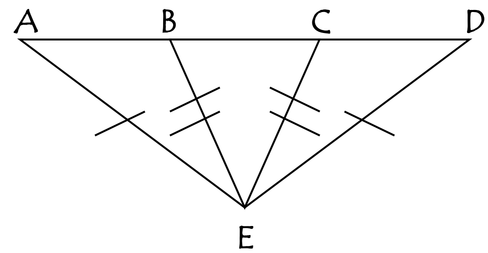
c. AAS



7. Prove: $\triangle AEC \cong \triangle DEB$

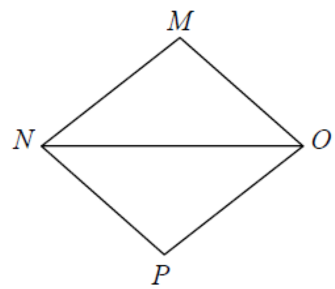
a. SSS

b. SAS



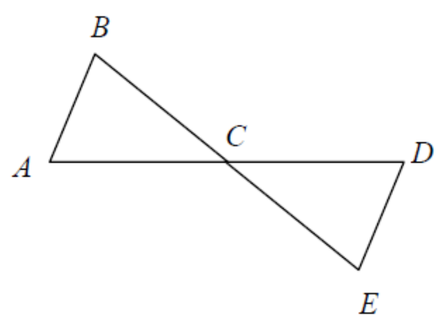
PROOFS:

8. Given: $\overline{MN} \cong \overline{PO}$
$\overline{MO} \cong \overline{PN}$
Prove: $\triangle MNO \cong \triangle PON$



Statements	Reasons

9. Given: $\overline{AC} \cong \overline{DC}$
$\overline{BC} \cong \overline{CE}$
Prove: $\triangle ABC \cong \triangle DEC$



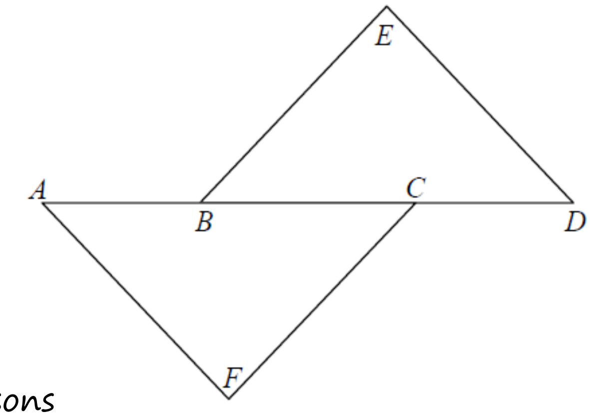
Statements	Reasons

10. Given: $\overline{AB} \cong \overline{CD}$

$\overline{AF} \cong \overline{DE}$

$\angle A \cong \angle D$

Prove: $\triangle FAC \cong \triangle EDB$



Statements	Reasons

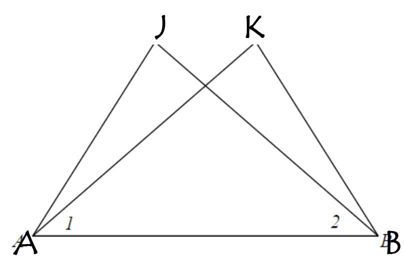
Proofs 11 & 12: Save for 7.8D3

11. Overlapping Triangles

Given: $\angle JAB \cong \angle KBA$

$\angle 1 \cong \angle 2$

Prove: $\triangle JAB \cong \triangle KBA$



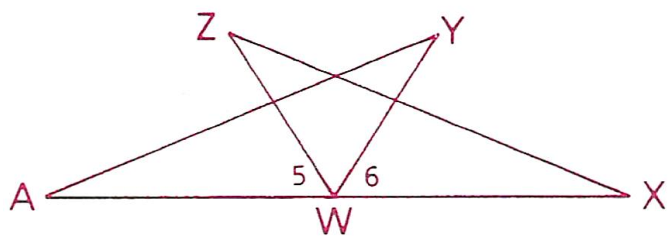
Statements	Reasons

Helpful Hints w/Overlapping Triangles

- Draw the triangles separately.
- Outline the two triangles in different colors.
- ALSO…there will be a reflexive step—that shared side or angle.

12. Given: $\overline{YW}$ bisects $\overline{AX}$
$\angle A \cong \angle X$
$\angle 5 \cong \angle 6$

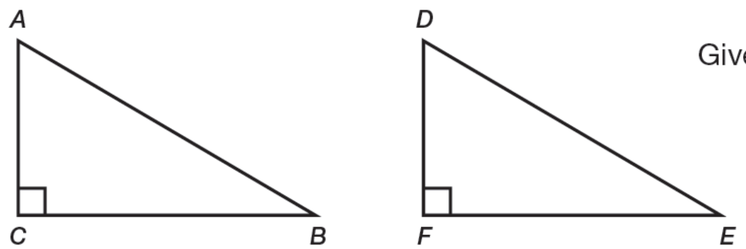
Prove: $\triangle AWY \cong \triangle XWZ$



Statements	Reasons

8.1 ~ HL Congruence Theorem

- The Hypotenuse-Leg Congruence Theorem
  - If the hypotenuse and leg of one right triangle are congruent to the hypotenuse and leg of another right triangle, then the triangles are congruent. (HL)



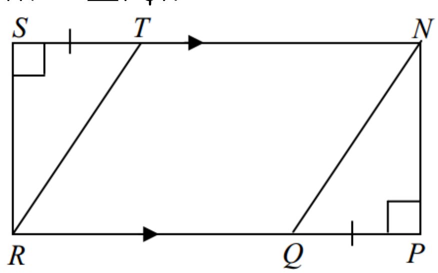
Given: $\angle C$ and $\angle F$ are right angles

$\overline{AC} \cong \overline{DF}$

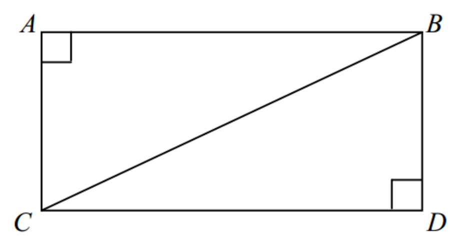
$\overline{AB} \cong \overline{DE}$

<u>Examples:</u> What additional information would you need to prove the triangles congruent by the HL Congruence Theorem?

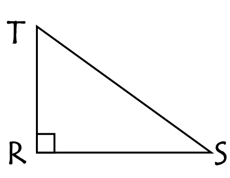
1. $\triangle STR \cong \triangle PQN$

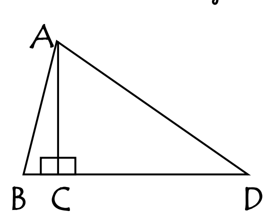


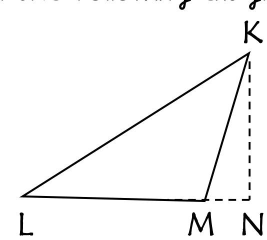
2. $\triangle ABC \cong \triangle DCB$



- Altitudes of Triangles
  - If a segment is an <u>altitude</u> of a triangle, then it forms right angles with the side to which it is drawn.
    - Every triangle has three altitudes.
    - An altitude of a triangle forms right angles with one of the sides.
      - Identify the altitude shown & the right angles formed in the following diagrams:







$\triangle TRS$ ~ altitude: ______

right angle(s): ____________

$\triangle ABD$ ~ altitude: ______

right angle(s): ____________

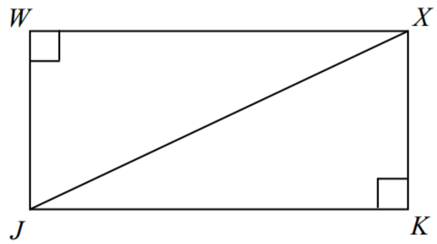
$\triangle KLM$ ~ altitude: ______

right angle(s): ____________

PROOFS:

3. Given: $\overline{WJ} \cong \overline{KX}$
$\angle JWX$ is a right angle
$\angle XKJ$ is a right angle

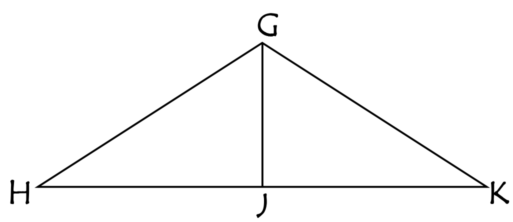
Prove: $\triangle WJX \cong \triangle KJX$



Statements	Reasons

4. Given: $\overline{GH} \cong \overline{GK}$
$\overline{GJ}$ is an altitude

Prove: $\triangle GHJ \cong \triangle GKJ$



Statements	Reasons

8.2 ~ CPCTC & CIRCLES

If two triangles are congruent, then each part of one triangle is congruent to the corresponding part of the other triangle. "**Corresponding parts of congruent triangles are congruent**," is abbreviated as **CPCTC**, is often used as reasons in proofs. CPCTC states that corresponding angles or sides in two congruent triangles are congruent. This reason can only be used after you have proven that the triangles are congruent.

To use CPCTC in a proof, follow these steps:

Step 1: Identify two triangles in which segments or angles are corresponding parts.

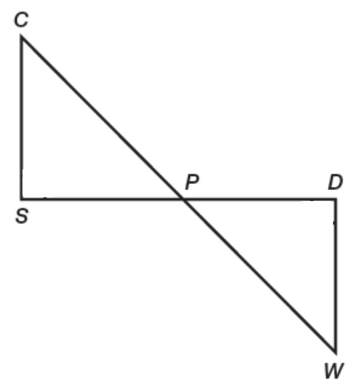
Step 2: Prove the triangles congruent.

Step 3: State the two parts are congruent using CPCTC as the reason.

A PROOF:

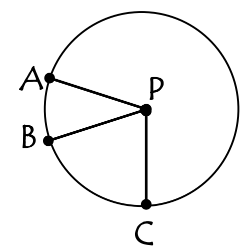
1. Given: $\overline{CW}$ & $\overline{SD}$ bisect each other

   Prove: $\overline{CS} \cong \overline{WD}$



Statements	Reasons

- Circles
  - A circle is named by its center; this circle is called circle P (or ⊙P)

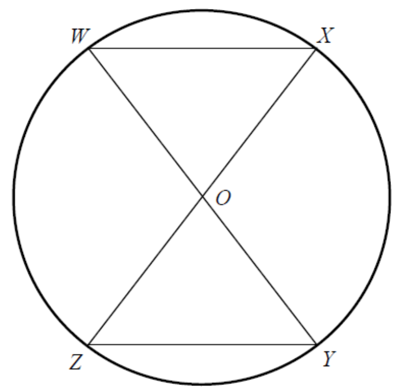


- Radii
  - Points A, B, and C lie on circle P (⊙P)
    - $\overline{PA}$ is called a radius
    - $\overline{PA}, \overline{PB}, \& \overline{PC}$ are called radii
  - Theorem: All radii of a circle are congruent.

A PROOF:

2. Given: ⊙O

   Prove: $\overline{XW} \cong \overline{ZY}$



Statements	Reasons

- Auxiliary Lines
  - Need there to be line connecting two points? No problem!
    - Auxiliary lines connect two points already in the diagram.

Whenever we use an auxiliary line in a proof, we must be able to show that such a line can be drawn & then justify it with the following postulate:

Two points determine a line.

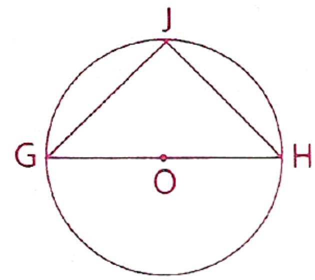
Statements	Reasons
⋮	⋮
Draw $\overline{AL}$	Two points determine a line.

A PROOF:

3. Given: $\odot O$

$\overline{GJ} \cong \overline{HJ}$

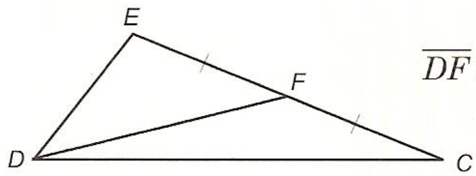
Prove: $\angle G \cong \angle H$



Statements	Reasons

8.3 ~ TRIANGLES IN PROOFS

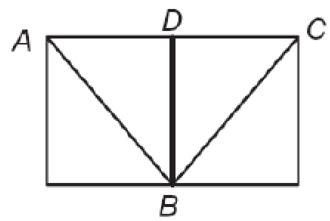
- Medians of Triangles
  - If a segment is a <u>median</u> of a triangle, then it divides the opposite side into two congruent segments.



$\overline{DF}$ is a median of $\triangle DEC$.

A PROOF:

1. Given: $\overline{AB} \cong \overline{CB}$
   $\overline{BD}$ is a median of $\triangle ABC$
   Prove: $\triangle ABD \cong \triangle CBD$



Statements	Reasons

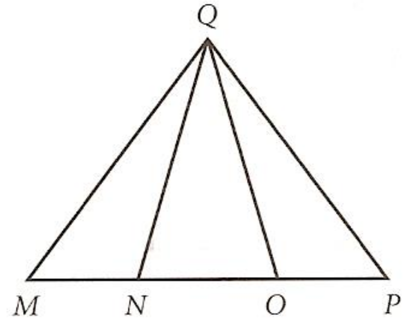
- Triangles in Proofs
  - Isosceles Triangles
    - If at least two sides of a triangle are congruent, then the triangle is an isosceles triangle.
  - Equilateral Triangles
    - If all sides of a triangle are congruent, then the triangle is an equilateral triangle.
  - Right Triangles
    - If a triangle has a right angle, then it is a right triangle.

- Isosceles Triangle Theorems
  - Isosceles Triangle Base Angle Theorem
    - If two sides of a triangle are congruent, then the angles opposite these sides are congruent.
  - Isosceles Triangle Base Angle Converse Theorem
    - If two angles of a triangle are congruent, then the sides opposite these angles are congruent.

PROOFS:

2. Given: $\overline{QM} \cong \overline{QP}$
   $\overline{MN} \cong \overline{PO}$

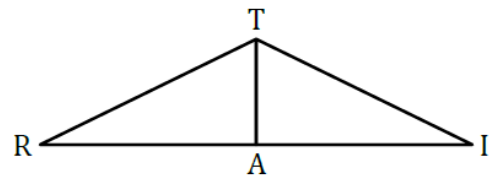
   Prove: $\angle QNP \cong \angle QOM$



Statements	Reasons

3. Given: $\overline{TA}$ is a median of $\triangle RIT$
   $\triangle RIT$ is isosceles with base $\overline{RI}$

   Prove: $\triangle TRA \cong \triangle TIA$



Statements	Reasons

- More Isosceles Triangle Theorems
  - Isosceles Triangle Base Theorem
    - The altitude to the base of an isosceles triangle bisects the base.
  - Isosceles Triangle Vertex Angle Theorem
    - The altitude to the base of an isosceles triangle bisects the vertex angle.
  - Isosceles Triangle Perpendicular Bisector Theorem
    - The altitude from the vertex of an isosceles triangle is the perpendicular bisector of the base.
  - Isosceles Triangle Altitude to Congruent Sides Theorem
    - In an isosceles triangle, the altitudes to the congruent sides are congruent.
  - Isosceles Triangle Bisector to Congruent Sides Theorem
    - In an isosceles triangle, the angle bisectors to the congruent sides are congruent.