Chapter 7: Right Triangles & Trigonometry

C7.APK ~ Simplifying Radicals

OBJECTIVES:

- Use properties of radicals to simplify expressions
- Simplify expressions by rationalizing the denominator
- Perform operations with radicals

Radical expressions like $2\sqrt{3}$ and $\sqrt{x+3}$ contain a radical. You can simplify a radical expression by removing perfect square factors from the radicand. Recall that a radicand is the quantity or expression UNDER the radical sign.

❖ **Multiplication Property of Square Roots**

➢ For every number $a \geq 0$ & $b \geq 0$, $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

- $\sqrt{54} = \sqrt{9} \cdot \sqrt{6} = 3\sqrt{6}$
- Ex: $-2\sqrt{5} \cdot 3\sqrt{2} = (-2 \cdot 3) \cdot \sqrt{5} \cdot \sqrt{2} = -6\sqrt{10}$

Multiply the insides with the insides & the outsides with the outsides.

❖ **Simplifying Square Roots**

➢ Use the Multiplication Property of Square Roots

- Rewrite the radicand as a product of the perfect-square factors times the remaining factors

➢ Making a Tree Diagram

▪ Multiplication Property	▪ Tree Diagram
1. Find the <u>largest</u> perfect square which will divide evenly into the radicand—the number under the radical sign	1. Find the prime factorization. $\sqrt{72} = \sqrt{2 \times 2 \times 2 \times 3 \times 3}$ 2. When working w/ squares look for pairs. (Circle these.) $\sqrt{72} = \sqrt{2 \times 2 \times 2 \times 3 \times 3}$
2. Write the radicand as a product containing the perfect square; each factor should be its own square root $\sqrt{48} = \sqrt{16} \times \sqrt{3}$	3. "The Buddy System" - Only a "factor pair" can be removed from the radicand.
3. Reduce the "perfect" radical to obtain your answer. $\sqrt{48} = 4\sqrt{3}$	4. The square root of a "factor pair is the factor: $\sqrt{72} = (2 \times 3)\sqrt{2} = 6\sqrt{2}$

EXAMPLES:

Simplify the following square roots using one of the methods from above.

1. $\sqrt{50}$

2. $\sqrt{98}$

EXAMPLES:

Use the Multiplication Property of Square Roots and find the product of the two radical expressions. Write the solution in simplest radical form.

3. $\sqrt{8} \cdot 2\sqrt{5}$

4. $3\sqrt{2} \cdot 4\sqrt{3}$

- ***Division Property of Square Roots***
  - For every number $a \geq 0$ and $b \geq 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

$$\sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}$$

- Tricks When Simplifying Quotients Involving Square Roots
  - *Try simplifying the fraction in the radicand first and then simplify the result.*

$$\sqrt{\frac{32}{50}} = \sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}$$

  - *Look to see if the denominator of the radicand is a perfect square; it is easier to simplify the numerator and denominator separately.*

$$\sqrt{\frac{10}{81}} = \frac{\sqrt{10}}{\sqrt{81}} = \frac{\sqrt{10}}{9}$$

EXAMPLES:

Simplify the following radical expressions.

5. $\sqrt{\frac{9}{4}}$

6. $\sqrt{\frac{11}{49}}$

7. $\sqrt{\frac{90}{5}}$

- ***Rationalizing the Denominator***
  - The process of simplifying a fraction with a radical in the denominator is called <u>rationalizing the denominator</u>.
    - *Multiply both the numerator AND the denominator by the radical in the denominator*
    - *When finished, see if the outside numbers reduce*

$$\frac{5}{\sqrt{10}} = \frac{5}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{5\sqrt{10}}{\sqrt{100}} = \frac{5\sqrt{10}}{10} = \frac{\sqrt{10}}{2}$$

Reduce: $\frac{5}{10} = \frac{1}{2}$

EXAMPLES:

Simplify the following radical expressions by rationalizing the denominator.

8. $\frac{5}{\sqrt{7}}$

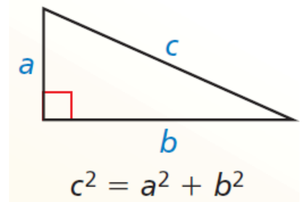
9. $\frac{6}{\sqrt{18}}$

7.1 ~ THE PYTHAGOREAN THEOREM

OBJECTIVES:

- Use the Pythagorean Theorem to find missing side lengths in a right triangle
- Use the Converse of the Pythagorean Theorem and Pythagorean Inequalities to classify triangles by their angles given their side lengths

❖ The Pythagorean Theorem



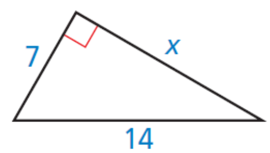
- ➢ In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs
  - $c^2 = a^2 + b^2$
  - $\text{hyp}^2 = \text{leg}^2 + \text{leg}^2$
- ➢ A <u>Pythagorean triple</u> is a set of three positive integers *a, b,* and *c*, that satisfy the equation $c^2 = a^2 + b^2$.

Common Pythagorean Triples and Some of Their Multiples

3, 4, 5	**5, 12, 13**	**8, 15, 17**	**7, 24, 25**
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39	24, 45, 51	21, 72, 75
$3x, 4x, 5x$	$5x, 12x, 13x$	$8x, 15x, 17x$	$7x, 24x, 25x$

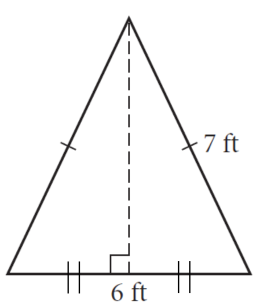
EXAMPLES:

1. Use the Pythagorean Theorem to find the value of x. If necessary, express as a radical in simplest form.

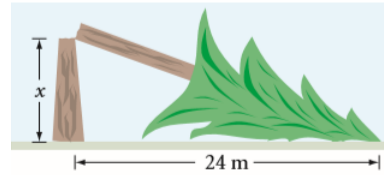


2. A soccer field is a rectangle 90 meters wide and 120 meters long. The coach ask players to run from one corner to the corner diagonally across. How many meters do the players run? *(Approximate your answer to the nearest tenth.)*

3. Find the area of the isosceles triangle.



4. A giant California redwood tree 36 meters tall cracked in a violent storm and fell as if hinged. The tip of the once beautiful tree hit the ground 24 meters from the base. Researcher Red Woods wishes to investigate the crack. How many meters up from the base of the tree does he have to climb?



❖ Pythagorean Inequalities

➢ For $\triangle ABC$, with c as the length of the longest side…

ACUTE: $c^2 < a^2 + b^2$ RIGHT: $c^2 = a^2 + b^2$ OBTUSE: $c^2 > a^2 + b^2$

EXAMPLES

5. Consider a triangle with side lengths 5.3, 6.7, and 7.8. Is the triangle acute, right, or obtuse?

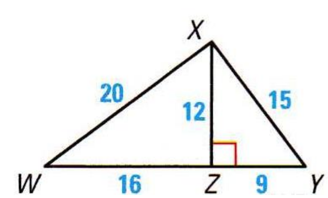
4.5 – SIMILARITY IN RIGHT TRIANGLES

OBJECTIVE:

- Use the Right Triangle/Altitude Similarity Theorem and the geometric mean to find missing side and segments lengths of similar right triangles and solve application problems

❖ INVESTIGATION

➢ Explain and show <u>how</u> $\triangle XZW \sim \triangle YZX \sim \triangle YXW$.



❖ Geometric Mean

➢ For any two positive numbers a and b, the geometric mean is the positive number x: $\dfrac{a}{x} = \dfrac{x}{b}$

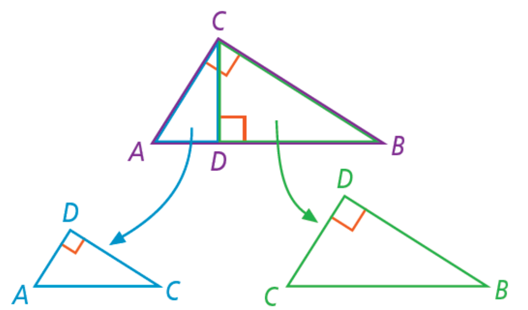
EXAMPLES – Find the geometric mean between the following numbers. If necessary, express as a radical in simplest form.

1. 5 and 8
2. 4 and 18

❖ Right Triangle/Altitude Similarity Theorem

➢ *If an altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.*

If . . .
$\triangle ABC$ is a right triangle with right $\angle ACB$, and $\overline{CD}$ is the altitude to the hypotenuse



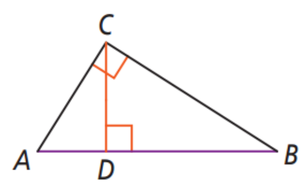
Then . . .
$\triangle ABC \sim \triangle ACD$
$\triangle ABC \sim \triangle CBD$
$\triangle ACD \sim \triangle CBD$

❖ Right Triangle Altitude/Hypotenuse Theorem

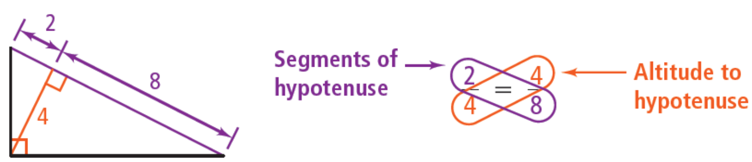
➢ *The measure of the altitude (drawn from the vertex of the right angle of a right triangle to its hypotenuse) is the geometric mean between the measures of the two segments of the hypotenuse.*

- Given: altitude $\overline{CD}$ & the two segments of the hypotenuse: $\overline{AD}$ & $\overline{BD}$

  GEOMETRIC MEAN: $\dfrac{CD}{AD} = \dfrac{BD}{CD}$



Example



❖ Right Triangle Altitude/Leg Theorem

➢ *If the altitude is drawn to the hypotenuse of a right triangle, each leg of the right triangle is the geometric mean of the hypotenuse and the segment of the hypotenuse adjacent to the leg.*

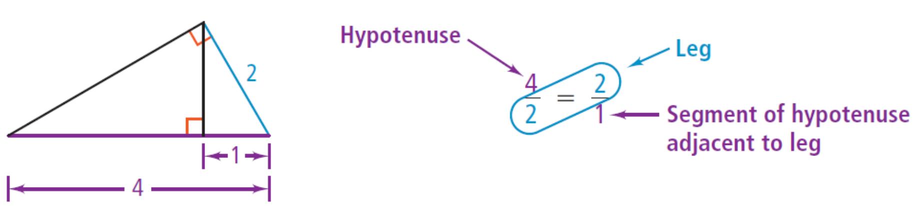
- Given: leg $\overline{AC}$, segment $\overline{AD}$ & hypotenuse $\overline{AB}$

  GEOMETRIC MEAN: $\dfrac{AC}{AB} = \dfrac{AD}{AC}$

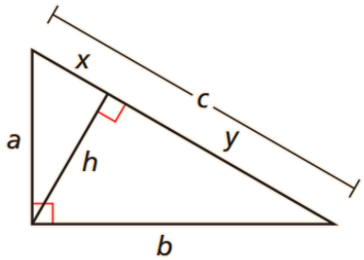
- Given: leg $\overline{BC}$, segment $\overline{BD}$ & hypotenuse $\overline{AB}$

  GEOMETRIC MEAN: $\dfrac{BC}{AB} = \dfrac{BD}{BC}$

Example



- You can label a diagram as shown below.
  - **The legs are a & b, the hypotenuse is c, and the altitude is h.**
    - Notice that segment x is adjacent to leg a, and that segment y is adjacent to leg b.



ALTITUDE/HYPOTENUSE

$$\frac{h}{x} = \frac{y}{h} \Rightarrow h^2 = xy$$

ALTITUDE/LEG

$$\frac{a}{c} = \frac{x}{a} \Rightarrow a^2 = cx$$

$$\frac{b}{c} = \frac{y}{b} \Rightarrow b^2 = cy$$

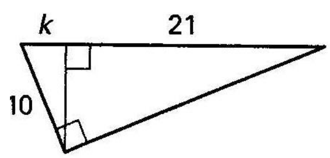
ADDITIONAL FORMULAS

$$x + y = c$$

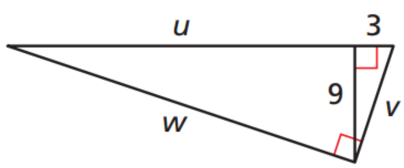
You can also use the Pythagorean Theorem for any of the right triangles.

EXAMPLES – Find the value of the variable(s). If necessary, express as a radical in simplest form.

3.



4.

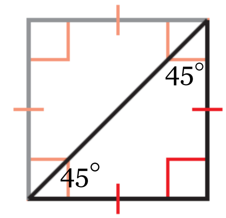


Investigating Special Right Triangles

Objective:

- Use the Pythagorean Theorem to explore the relationship between the side lengths of a triangle and the measures of its interior angles

The table below contains information about squares of various side lengths. Use the Pythagorean Theorem to find the length of the diagonal. Express your answer as a radical in simplest form.

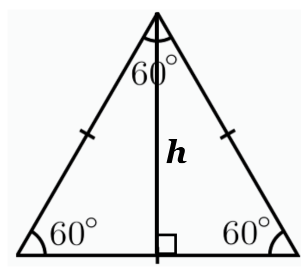


Side Length	Diagonal Length
2	
3	
4	
5	
x	

What do you notice?

How long is the diagonal of a square with sides measuring 100 units?

The table below contains information about equilateral triangles of various side lengths. Use the Pythagorean Theorem to find the height of the triangle. Express your answer as a radical in simplest form.



Side Length (hypotenuse)	½ side length (short leg)	Height (long leg)
2		
4		
6		
8		
x		

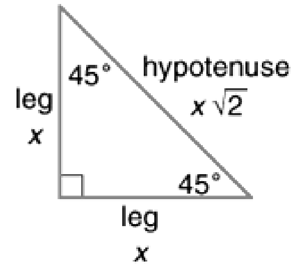
What do you notice?

What are the lengths of the short and long legs if the hypotenuse measures 100 units?

7.2 ~ Special Right Triangles

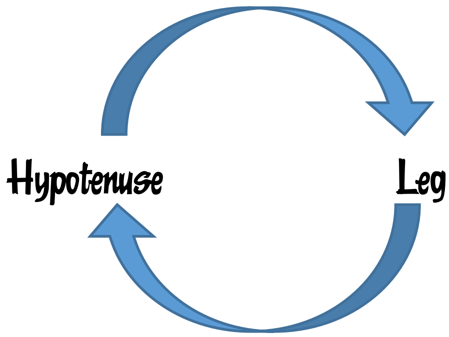
Objective:

- Apply the 45° – 45° – 90° Triangle Theorem and the 30° – 60° – 90° Triangle Theorem



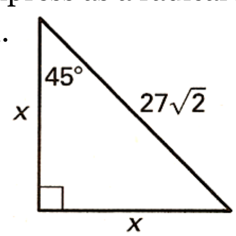
❖ The 45° – 45° – 90° Triangle Theorem

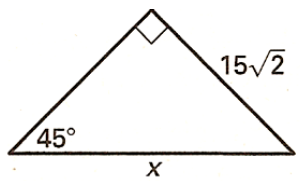
➢ In a triangle whose angles have the measures 45°, 45°, & 90°, the lengths opposite theses angles can be represented by $x, x, \& \ x\sqrt{2}$ respectively.

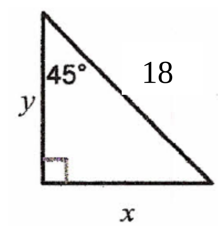


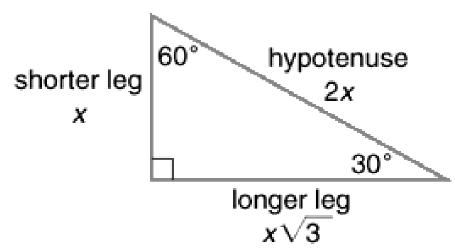
Examples:

The triangles below are isosceles right triangles. Find the missing side length indicated. If necessary, express as a radical in simplest form.

1.


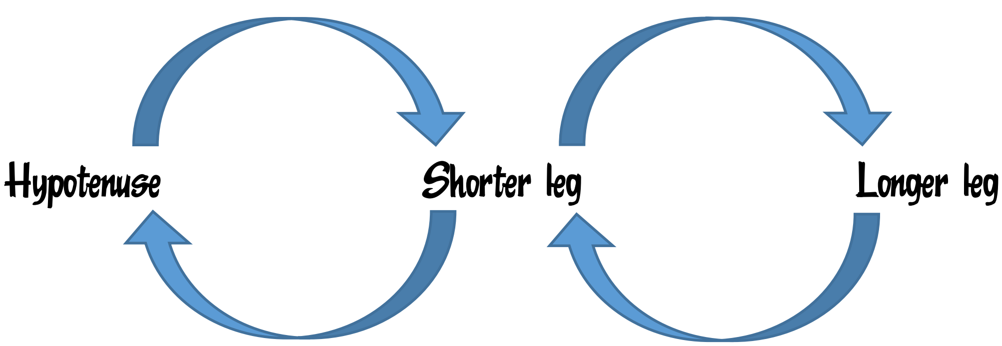
2.


3.




❖ The 30° – 60° – 90° Triangle Theorem

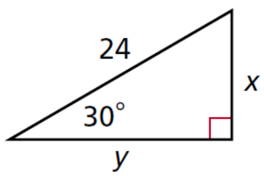
➢ In a triangle whose angles have the measures 30°, 60°, & 90°, the lengths opposite theses angles can be represented by $x, x\sqrt{3}, \& \ 2x$ respectively.



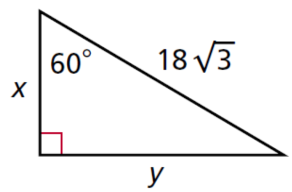
EXAMPLES:

Find the missing side lengths in each 30°-60°-90° triangle. If necessary, express as radicals in simplest form.

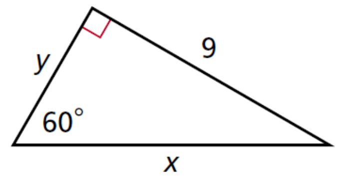
4.



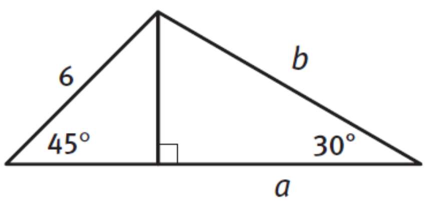
5.



6.



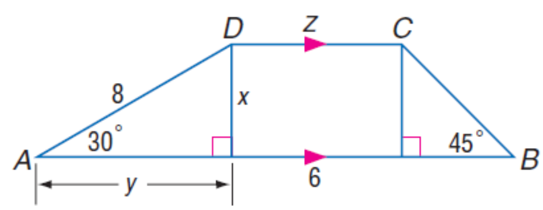
7. Find *a* and *b*. If necessary, express radicals in simplest form.



8. Given: *ABCD* is a trapezoid.
   a. Find *x*, *y*, and *z*. If necessary, express radicals in simplest form.
   b. What is the perimeter of *ABCD*?
   c. What is the area of *ABCD*?

   $A_{\text{trapezoid}} = \frac{1}{2}h(b_1 + b_2)$

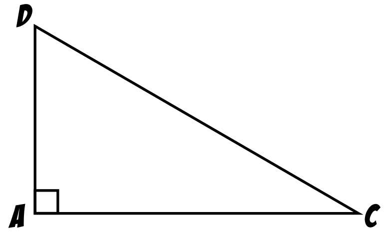
   Round the perimeter and the area to the nearest tenth.



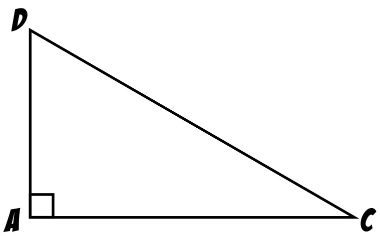
TRIG INVESTIGATION

A trigonometric ratio is a ratio of the lengths of two sides in a right triangle.

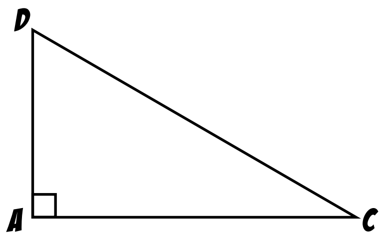
- Go to www.geogebra.org
  - Search for: **Trig Ratios on Right Triangle**
    Author: GeoGebra Forum
- Instructions:
  - Adjust angles C and D to a triangle of your choosing and record the angle measures in the diagram provided:



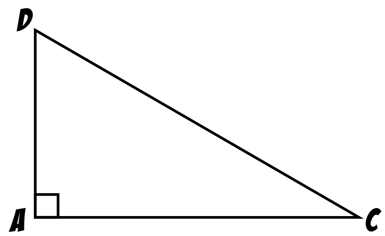
  - Now drag point A to change the size of the triangle. Record the side lengths in the diagram provided. *Do this three times; creating three triangles.*



△1



△2



△3

  - Using $\angle C$, as the reference angle, write the side ratio and calculate its value, rounded to two decimal places.

	$\frac{\text{opposite}}{\text{adjacent}} = \frac{AD}{AC}$	$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{AD}{DC}$	$\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{AC}{DC}$
△1			
△2			
△3			

- Observations:
  - In right triangles, when the angle measures are unchanged, what can be concluded about…
    - …the ratio of the side opposite the $\angle C$ and the side adjacent to the angle?
    - …the ratio of the side opposite the $\angle C$ and the hypotenuse?
    - …the ratio of the side adjacent to $\angle C$ and the hypotenuse?

- Calculate and round to two decimal places. Then compare to the side ratios (above).

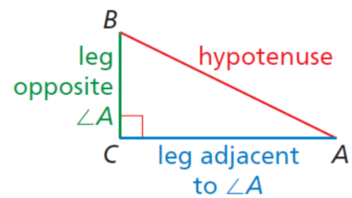
	SINE	COSINE	TANGENT
CALCULATE	$\sin C \approx$	$\cos C \approx$	$\tan C \approx$
SIDE RATIO			

7.3 ~ The Tangent Ratio

Objectives:

- Calculate the tangent of the acute angles in right triangles
- Use the tangent ratio to solve for unknown side lengths in right triangles
- Use the inverse tangent to solve for unknown angle measures in right triangles

❖ The Tangent Ratio

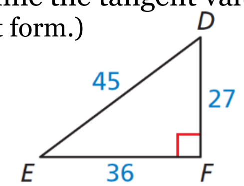


➢ Let $\triangle ABC$ be a right triangle with acute $\angle A$. The tangent of $\angle A$ (written $\tan A$) is defined as:

$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}$$

Examples:

Determine the tangent values of all the acute angles in the right triangles shown. (Express as a ratio in simplest form.)

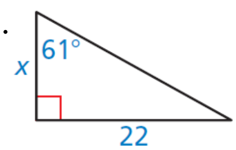


1. $\tan D$

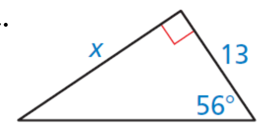
2. $\tan E$

Write a trigonometric equation using tangent to find the indicated side length, x. Given an exact answer, solve the equation for x, and an approximate answer rounded to the nearest hundredth.

3.



4.



What about unknown acute angle measures in right triangles?

If you know the length of any two sides of a right triangle, it IS possible to compute the measure of either acute angle by using an inverse trigonometric function.

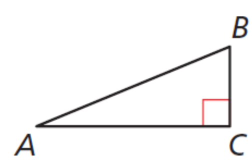
❖ Inverse Tangent

➢ The inverse tangent – or arc tangent – of x is the measure of an acute angle whose tangent is x.

➢ Let $\angle A$ be an acute angle:

- If $\tan A = x$, then $\tan^{-1} x = m\angle A$.

➢ Calculating Angle Measures: 2nd [trig]

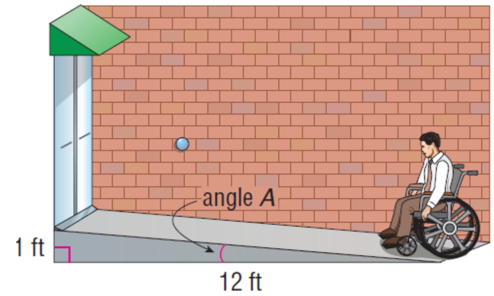


$$\tan^{-1}\frac{BC}{AC} = m\angle A$$

Examples:

3. Look back at Examples 1 & 2, and calculate $m\angle D$ and $m\angle E$ (to the nearest tenth of a degree).

4. The Americans with Disabilities Act (ADA) provides regulations designed to make public buildings accessible to all. Under this act, the slope of an entrance ramp designed for those with mobility disabilities must not exceed a ratio of 1:12. This means that every 1 unit of vertical rise requires 12 units of horizontal run.



   a. Write a tangent ratio that represents $\angle A$ and then calculate its measure to the nearest tenth of a degree.

   b. Use a tangent ratio to calculate the run of a wheel chair ramp with a vertical rise of 24 inches.

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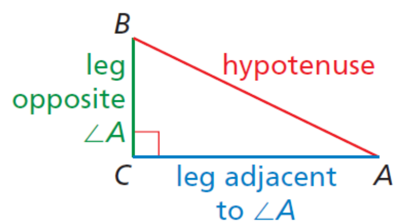
7.4 ~ The Sine & Cosine Ratios

OBJECTIVES:

- Calculate the sine or cosine of acute angles in right triangles
- Use the sine or cosine ratio to solve for unknown side lengths in right triangles
- Use the inverse sine or cosine to solve for unknown angle measures in right triangles

❖ The Sine & Cosine Ratios



- Let $\triangle ABC$ be a right triangle with acute $\angle A$.
- The sine of $\angle A$ (written $\sin A$) is defined as:

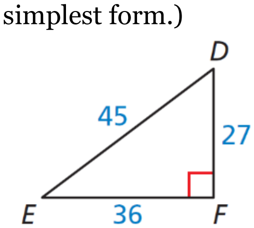
$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}$$

- The cosine of $\angle A$ (written $\cos A$) is defined as:

$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AC}{AB}$$

EXAMPLES:

Determine the sine and cosine values of all the acute angles in the right triangles shown. (Express as a ratio in simplest form.)



1. $\sin D$
2. $\sin E$
3. $\cos D$
4. $\cos E$

SOH – CAH – TOA

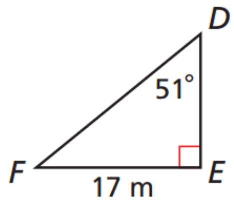
$\sin\theta =$ ________ $\tan\theta =$ ________

$\cos\theta =$ ________
~~~~~~

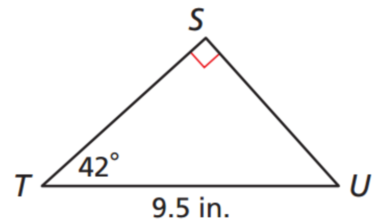
EXAMPLES:

Set up and solve a trigonometric equation to find the indicated side length. Give an exact answer, solve the equation for the variable, and an approximate answer rounded to the nearest hundredth.

5. *DF*

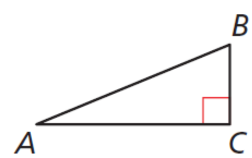


6. *ST*



What about unknown acute angle measures in right triangles?

If you know the length of any two sides of a right triangle, it IS possible to compute the measure of either acute angle by using an inverse trigonometric function.



$$\sin^{-1}\frac{BC}{AB} = m\angle A$$

$$\cos^{-1}\frac{AC}{AB} = m\angle A$$

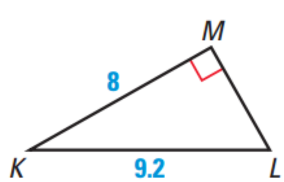
- Inverse Sine
  - The inverse sine – or arc sine – of x is the measure of an acute angle whose sine is x.
    - If $\sin A = x$, then $\sin^{-1} x = m\angle A$.
- Inverse Cosine
  - The inverse cosine – or arc cosine – of x is the measure of an acute angle whose cosine is x.
    - If $\cos A = x$, then $\cos^{-1} x = m\angle A$.
- Calculating Angle Measures
  - Is your calculator in DEGREE mode?
    - MODE: If your calculator is in RADIAN mode, cursor to DEGREE and press ENTER
  - 2^{nd} [trig]
    - Find x if $\sin x = 0.5431$.

$$x = \sin^{-1}(0.5431) \approx 32.9°$$

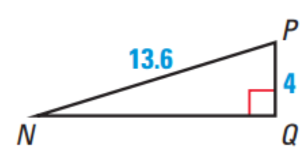
EXAMPLES:

Write a trigonometric ratio – using sine or cosine – then use inverse trigonometric functions to find the measure of the indicated angle rounded to the nearest tenth of a degree.

7. $m\angle L$



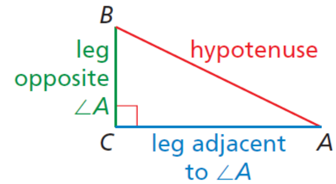
8. $m\angle P$



7.5 ~ Other Trigonometric Relationships

OBJECTIVES:

- Calculate the tangent, sine, cosine, cotangent, cosecant, or secant of indicated angles in right triangles
- Explore complement angle relationships in a right triangle
- Use one trigonometric ratio to find the others



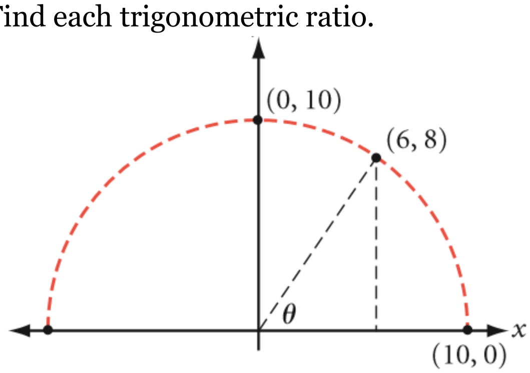
❖ Reciprocal Trigonometric Ratios

- The Cotangent Ratio
  - *The cotangent of an acute angle in a right triangle is the ratio of the length of the side that is adjacent to the angle to the length of the side that is opposite the angle.*
- The Cosecant Ratio
  - *The cosecant of an acute angle in a right triangle is the ratio of the length of the hypotenuse to the length of the side that is opposite the angle.*
- The Secant Ratio
  - *The secant of an acute angle in a right triangle is the ratio of the length of the hypotenuse to the length of the side that is adjacent to the angle.*

TANGENT & COTANGENT		SINE & COSECANT		COSINE & SECANT	
$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$	$\cot\theta = \dfrac{\text{adj}}{\text{opp}}$	$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$	$\csc\theta = \dfrac{\text{hyp}}{\text{opp}}$	$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$	$\sec\theta = \dfrac{\text{hyp}}{\text{adj}}$

EXAMPLES:

1. Find each trigonometric ratio.



$\sin\theta =$ $\qquad$ $\csc\theta =$

$\cos\theta =$ $\qquad$ $\sec\theta =$

$\tan\theta =$ $\qquad$ $\cot\theta =$

What is the measure of θ?

Sketch the right triangle described, with right angle C. Use the Pythagorean Theorem to find the third side. Find the value of the other five trigonometric ratios expressed as fractions in simplest radical form.

2. $\sec A = \dfrac{16}{9}$

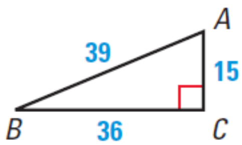
$\sin A =$ $\qquad$ $\csc A =$

$\cos A =$ $\qquad$ $\sec A = \dfrac{16}{9}$

$\tan A =$ $\qquad$ $\cot A =$

What is the measure of $\angle A$?

3. Find the sine and cosine ratios of each acute angle. Express as a fraction in simplest form.

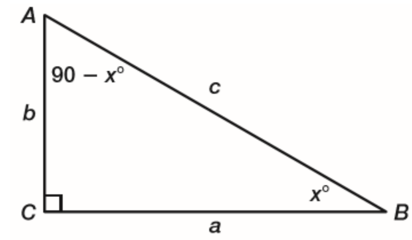


$\sin A =$ $\cos A =$

$\sin B =$ $\cos B =$

- *Compare the ratios in Example 3. What do you notice?*

❖ Complementary Angle Relationships in a Right Triangle



- Angles A and B are complementary angles because the sum of their measures is equal to 90°.
- The trigonometric ratios also have complementary relationships.

❖ The Relationship Between the Trigonometric Ratios of Complementary Angles

$\sin x =$ $\cos x =$

EXAMPLES:

4. The expression $\sin 57°$ is equal to
   a. $\tan 33°$
   b. $\cos 33°$
   c. $\tan 57°$
   d. $\cos 57°$

5. Find the value of R that will make the equation $\sin 73° = \cos R$ true.

6. In a right triangle $\sin(2x + 4)° = \cos 46°$. What is the value of x?

7. In a right triangle $\sin(40 - x)° = \cos(3x)°$. What is the value of x?

~~~~~~

7.6 ~ Solving Right Triangles

OBJECTIVE: Solve right triangles

Every right triangle has one right angle, two acute angles, one hypotenuse, and two legs.

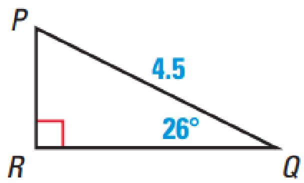
❖ Solving a Right Triangle
- To solve a right triangle means to find the measures of both acute angles and the lengths of all three sides.
- You can solve a right triangle if you know either of the following:
  - Two side lengths **OR** one side length and one acute angle measure

❖ How to Solve a Right Triangle
- Use the Pythagorean Theorem to find a missing side (*if two sides are known*)
- Use complementary angles to find the measure of a missing angle (*if two angles are known*)
- Write a tangent, sine or cosine ratio to find a missing acute angle measure and/or a missing side length.
~~~~~~

EXAMPLES:

Solve each right triangle. Round angle measures to the nearest tenth of a degree; round side lengths to the nearest hundredth.

1.

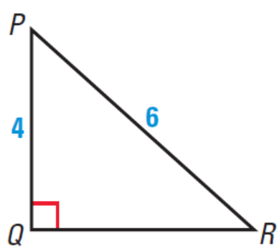


$\angle P =$ ____________

$PR =$ ____________

$QR =$ ____________

2.



$\angle P =$ ____________

$\angle R =$ ____________

$QR =$ ____________

3. Sketch the right triangle described, with right angle *C*. Use the Pythagorean Theorem to find the third side. Use inverse trigonometric ratios to find the missing angle measures. Round angle measures to the nearest tenth of a degree; round side lengths to the nearest hundredth.

$$\sec A = \frac{23}{2}$$

$AB =$ ____________

$BC =$ ____________

$AC =$ ____________

$\angle A =$ ____________

$\angle B =$ ____________

7.7 ~ Applications with Right Triangles

Objective:

- *Use trigonometric relationships to solve application problems involving angles of elevation or angles of depression*

❖ Angle of Elevation

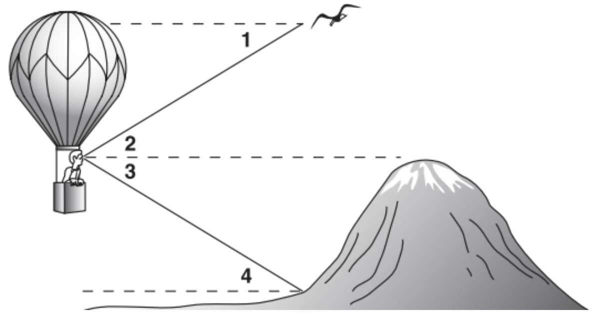
➢ The angle of elevation of an object as seen by an observer is the angle between the horizontal and the line from the object to the observer's eye (line of sight).

❖ Angle of Depression

➢ If the object is below the level of the observer, then the angle between the horizontal and the observer's line of sight is called the angle of depression**.**

Describe each angle as it relates to the situation in the diagram.

1. $\angle 1$
2. $\angle 2$
3. $\angle 3$
4. $\angle 4$

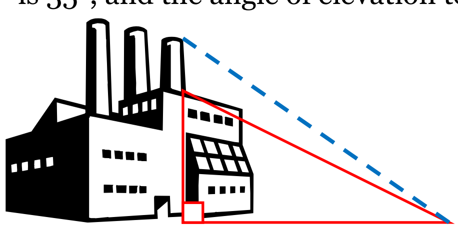


Examples:

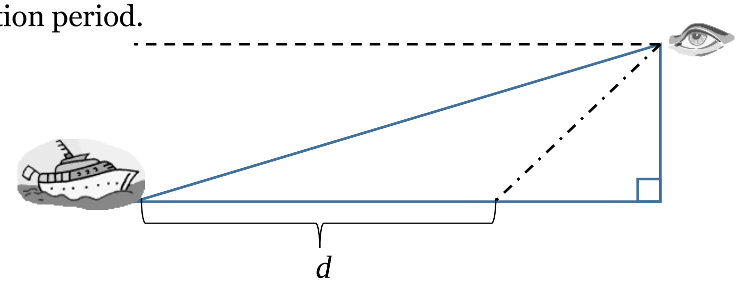
5. An airplane pilot sights a life raft at a 26° angle of depression. The airplane's altitude is 3 km. What is the airplane's horizontal distance d from the raft?

6. A sledding run is 300 yards long with a vertical drop of 27.6 yards. Find the angle of depression, to the nearest tenth of a degree, of the run.

7. At a point 200 feet from the base of a building, the angle of elevation to the bottom of a smokestack is 35°, and the angle of elevation to the top is 53°. Find the height of the smokestack.



8. Standing at the midpoint of the Golden Gate Bridge you observe a speed boat coming directly toward you. Suppose your eyes are 270 feet above the water. The angle of depression, your horizontal line of sight, changes from 20° to 45° while you are watching the boat. Approximate the distance the boat traveled during the observation period.

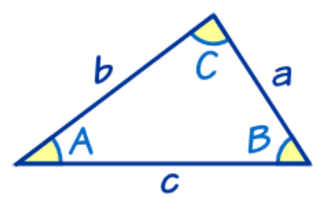


7.8 ~ THE LAW OF SINES

OBJECTIVES:

- Use the Law of Sines to determine unknown side lengths and/or angle measures of oblique triangles
- Derive and apply the formula for the area of a triangle using the sine function

❖ The Law of Sines



➢ The ratio of the sine of an angle to the length of its opposite side is the same for all 3 angles of any triangle.

For any oblique (non-right) triangle *ABC*...

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

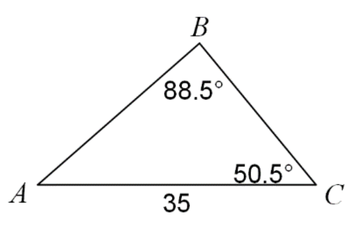
REMEMBER –

To find a missing angle measure, you need to use an inverse trigonometric function.

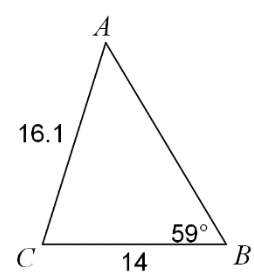
EXAMPLES:

Use the Law of Sines to solve for the indicated side or angle. Round angle measures to the nearest tenth of a degree; round side lengths to the nearest hundredth.

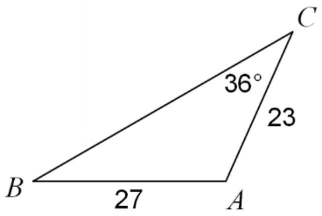
1. Find *BC*.



2. Find $m\angle A$.



3. Use the Law of Sines to solve the oblique triangle; that is, find ALL missing side lengths and angle measures. Round angle measures to the nearest tenth of a degree; round side lengths to the nearest hundredth.



$m\angle A =$ ____________

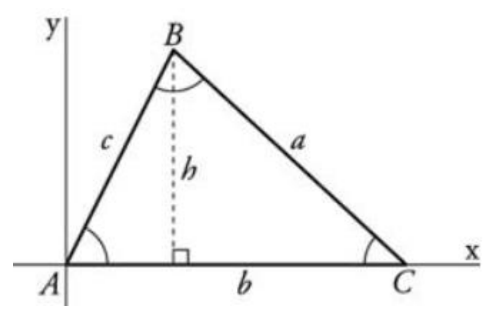
$m\angle B =$ ____________

$BC =$ ____________

❖ Area of a Triangle:

$$A = \frac{1}{2}bh$$

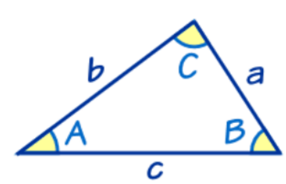
➢ Use trigonometry to come up with an expression to represent the height. Use $\angle A$.



❖ Area of a Triangle in Trigonometry

➢ The area of a triangle is one-half the product of the lengths of two sides and the sine of the included angle.

- The area K of any $\triangle ABC$ is given by any one of these formulas:
  -
  -
  -



Find the area of the triangle to the nearest square unit.

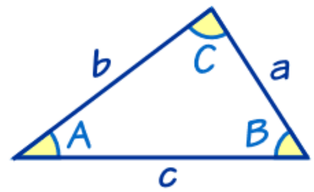
4. $\angle B = 81°$, $a = 6$, and $c = 11$

7.9 ~ THE LAW OF COSINES

OBJECTIVE:

- Use the Law of Cosines to determine unknown side lengths and/or angle measures of oblique triangles

❖ The Law of Cosines



- You can use the Law of Cosines when…
  - You know the lengths of all three sides of a triangle
    - Continue with the Law of Cosines to find the 2nd angle
  - You know the lengths of two sides of a triangle and the measure of the included angle
- For any oblique (non-right) triangle *ABC*…

$$a^2 = b^2 + c^2 - 2bc\cos A \qquad \frac{a^2 - b^2 - c^2}{-2bc} = \cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B \qquad \frac{b^2 - a^2 - c^2}{-2ac} = \cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C \qquad \frac{c^2 - a^2 - b^2}{-2ab} = \cos C$$

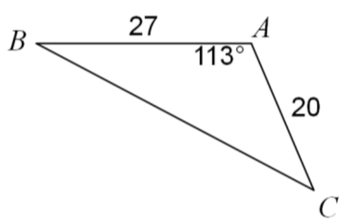
REMEMBER:

To find a missing angle measure, you need to use an inverse trigonometric function.

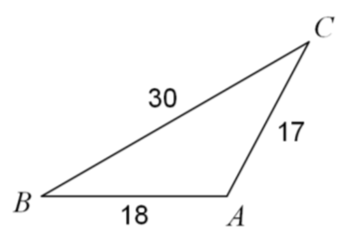
EXAMPLES:

Use the Law of Cosines to solve for the indicated side or angle. Round angle measures to the nearest tenth of a degree; round side lengths to the nearest hundredth.

1. Find *BC*.



2. Find $m\angle A$.



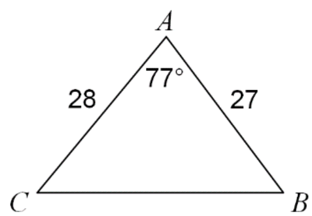
EXAMPLES:

Use the Law of Cosines to solve the oblique triangle; that is, find ALL missing side lengths and angle measures. Round angle measures to the nearest tenth of a degree; round side lengths to the nearest hundredth.

Solving a SAS Triangle

- Use the Law of Cosines to find the side opposite the given angle
- Use the Law of Sines to find the angle opposite the shorter of the two given sides. This angle is always acute.
- Subtract to find the third angle.

3.



$m\angle B =$ ____________

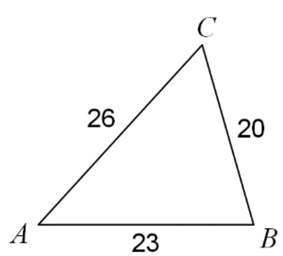
$m\angle C =$ ____________

$BC =$ ____________

Solving a SSS Triangle

- You MUST use the Law of Cosines to find TWO of the unknown angles.
- Subtract to find the third angle.

4.



$m\angle A =$ ____________

$m\angle B =$ ____________

$m\angle C =$ ____________

7.10 ~ Applications with Oblique Triangles

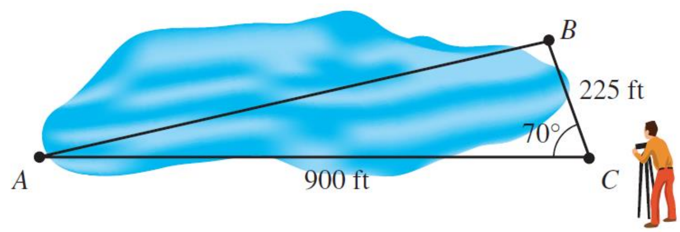
OBJECTIVE:

- Use the Law of Sines or the Law of Cosines to solve application problems involving oblique triangles

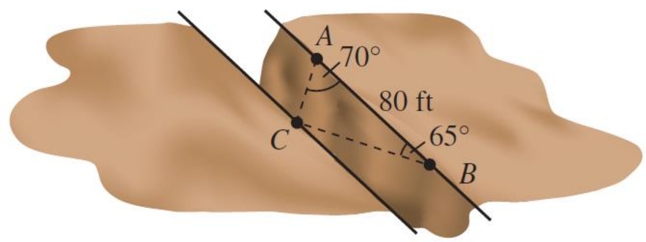
EXAMPLES:

For each problem below, first determine which law to use: the Law of Sines or the Law of Cosines. Then set up and solve a trigonometric equation that represents the situation.

1. In order to determine the distance between two points *A* and *B* on opposite sides of a lake, a surveyor chooses a point *C* that is 900 feet from *A* and 225 feet from *B*, as shown. If the measure of the angle at *C* is 70°, find the distance between *A* and *B* to the nearest foot.



2. Two markers *A* & *B* on the same side of a canyon rim are 80 feet apart, as shown in the figure. A hiker is located across the rim at point *C*. A surveyor determines that $m\angle BAC = 70°$ and $m\angle ABC = 65°$. Find the distance, to the nearest foot, between the hiker and point *A*.



3. A hot-air balloon is seen over Tucson, Arizona, simultaneously by two observers at points *A* and *B* that are 1.75 miles apart on level ground and in line with the balloon. The angles of elevation are as shown here. How high above the ground is the balloon?