

# Chapter 9: Trigonometry

## 9.2 – 9.4: SINE, COSINE, & TANGENT RATIOS

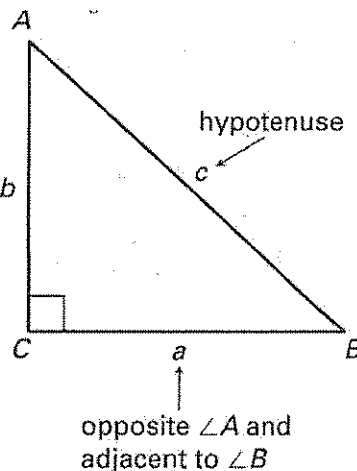
### Trigonometric Ratios

➤ A ratio of the lengths of two sides of a right triangle.

▪ For any acute angle, there is a leg...

- Opposite the angle
- Adjacent to the angle

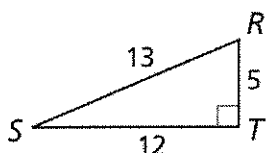
opposite  $\angle B$  and  
adjacent to  $\angle A$  →  $b$



### Trigonometric Ratios

DEFINITION	SYMBOLS	DIAGRAM
The <b>sine</b> of an angle is the ratio of the length of the leg opposite the angle to the length of the hypotenuse.	$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{a}{c}$ $\sin B = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{b}{c}$	
The <b>cosine</b> of an angle is the ratio of the length of the leg adjacent to the angle to the length of the hypotenuse.	$\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{b}{c}$ $\cos B = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{a}{c}$	
The <b>tangent</b> of an angle is the ratio of the length of the leg opposite the angle to the length of the leg adjacent to the angle.	$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{a}{b}$ $\tan B = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{b}{a}$	

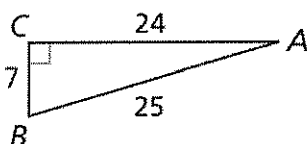
Write each trigonometric ratio as a fraction in simplest form.



1.  $\sin R = \frac{12}{13}$

2.  $\cos R = \frac{5}{13}$

3.  $\tan S = \frac{5}{12}$

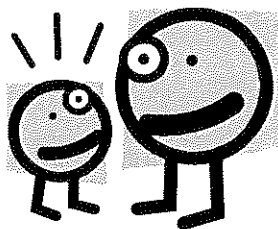


4.  $\sin B = \frac{24}{25}$

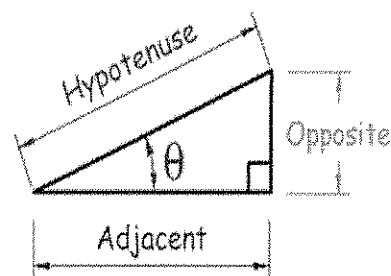
5.  $\cos A = \frac{24}{25}$

6.  $\tan B = \frac{24}{7}$

So how will I remember all these trig ratios?



I use SOH-CAH-TOA.



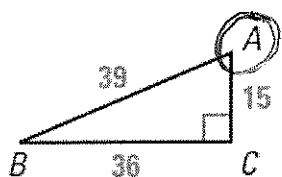
**SOH - CAH - TOA**

$$\text{sine} = \frac{\text{opp}}{\text{hyp}}$$

$$\text{tangent} = \frac{\text{opp}}{\text{adj}}$$

$$\text{cosine} = \frac{\text{adj}}{\text{hyp}}$$

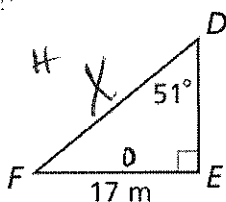
7. Use the above-mentioned mnemonic device to find the sine, cosine and tangent ratios of  $\angle A$ . Express as a fraction in simplest form.



$$\sin A = \frac{36}{39} = \frac{12}{13} \quad \cos A = \frac{15}{39} = \frac{5}{13} \quad \tan A = \frac{36}{15} = \frac{12}{5}$$

Set up and solve a trigonometric equation to find the indicated length rounded to the nearest hundredth.

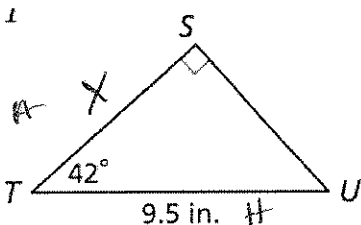
8. DF



$$\sin 51 = \frac{17}{X}$$

$$X = \frac{17}{\sin 51} \approx 21.87 \text{ m}$$

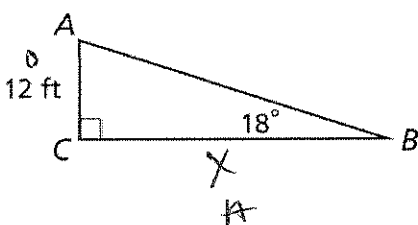
9. ST



$$\cos 42 = \frac{X}{9.5}$$

$$X = 9.5 \cos 42 \approx 7.06 \text{ in}$$

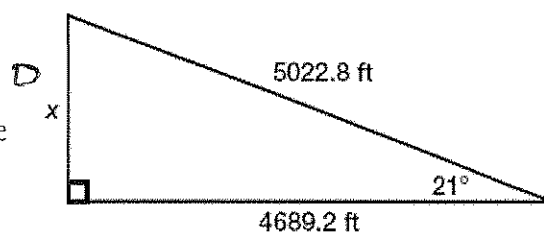
10. BC



$$\tan 18 = \frac{12}{X}$$

$$X = \frac{12}{\tan 18} \approx 36.93 \text{ ft}$$

11. A ski slope at Snowy Valley has an average angle of elevation of  $21^\circ$  (as shown).



- a. Which trigonometric ratio would you use to calculate the vertical height of the ski slope,  $x$ ? Explain your reasoning.

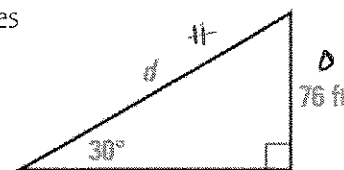
Either Sine OR tangent b/c you want to find the side opposite the angle  $\therefore$  you know the adjacent side  $\therefore$  the hypotenuse.

- b. Set up and solve a trigonometric equation to find the vertical height of the ski slope to the nearest tenth of a foot.

$$\begin{aligned}\sin 21^\circ &= \frac{x}{5022.8} \\ x &= 5022.8 \cdot \sin 21^\circ \\ x &\approx 1800.64\end{aligned}$$

$$\begin{aligned}\tan 21^\circ &= \frac{x}{4689.2} \\ x &= 4689.2 \cdot \tan 21^\circ \\ x &\approx 1800.64\end{aligned}$$

12. The escalator at the Wilshire/Vermont Metro Rail Station in Los Angeles rises 76 feet at a  $30^\circ$  angle (as shown).



- a. Which trigonometric ratio would you use to calculate the distance,  $d$ , a person travels on the escalator stairs? Explain your reasoning.

Sine b/c you know the side opposite the angle  $\therefore$  you want to know the hypotenuse.

- b. Set up and solve a trigonometric equation to find the distance,  $d$ , a person travels to the nearest tenth of a foot.

$$\begin{aligned}\sin 30^\circ &= \frac{76}{d} \\ d &= \frac{76}{\sin 30^\circ} = 152.0\end{aligned}$$

## 9.2 – 9.5 (D1): INVERSE SINE, INVERSE COSINE, INVERSE TANGENT, AND COMPLEMENTARY ANGLE RELATIONSHIPS

### Calculating Angle Measures

➤ Is your calculator in DEGREE mode?

- MODE: If your calculator is in RADIAN mode, cursor to DEGREE and press ENTER

➤ 2<sup>nd</sup> [trig]

- Find  $x$  if  $\sin x = 0.5431$ .

$$x = \sin^{-1}(0.5431) \approx 32.9^\circ$$

Use the inverse trigonometric functions to calculate the measure of the indicated angle to the nearest tenth of a degree.

1.  $\cos A = 0.5431$   $57.1^\circ$

2.  $\tan B = 0.5431$   $28.5^\circ$

3.  $\sin M = 0.8426$   $57.4^\circ$

4.  $\tan P = 3.5$   $74.1^\circ$

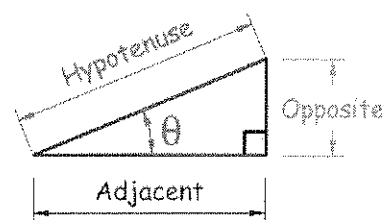
5.  $\sin W = \frac{3}{7}$   $25.4^\circ$

6.  $\cos Y = \frac{4}{13}$   $72.1^\circ$

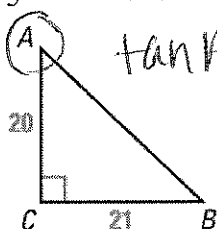
### Inverse Sine, Inverse Cosine, Inverse Tangent

➤ Inverse trig functions are used to calculate the measure of an acute angle in a right triangle if the length of two sides are known

- Inverse Sine – used when you know the length of the **LEG OPPOSITE** the angle and the length of the **HYPOTENUSE**
- Inverse Cosine – used when you know the length of the **LEG ADJACENT** the angle and the length of the **HYPOTENUSE**
- Inverse Tangent – used when you know the length of **BOTH LEGS**

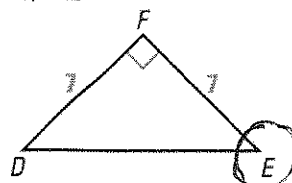


Write a trigonometric ratio and use inverse trigonometric functions to find the measure of the indicated angle rounded to the nearest tenth of a degree.

7.  $m\angle A$ 

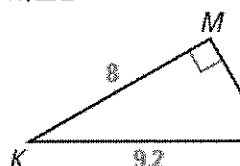
$$\tan A = \frac{21}{20}$$

$$m\angle A = \tan^{-1}\left(\frac{21}{20}\right) \approx 46.4^\circ$$

8.  $m\angle E$ 

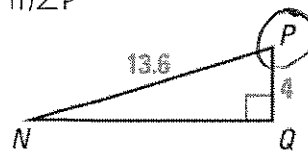
$$\tan E = \frac{7}{7}$$

$$m\angle E = \tan^{-1}\left(\frac{7}{7}\right) = 45^\circ$$

9.  $m\angle L$ 

$$\sin L = \frac{8}{9.2}$$

$$m\angle L = \sin^{-1}\left(\frac{8}{9.2}\right) \approx 60.4^\circ$$

10.  $m\angle P$ 

$$\cos P = \frac{4}{13.6}$$

$$m\angle P = \cos^{-1}\left(\frac{4}{13.6}\right) \approx 72.9^\circ$$

## d Trigonometric Ratios & Complementary Angles

➤ The two acute angles of a right triangle are **complementary** angles.

- If the measure of one of the two acute angles is given, the measure of the second acute angle can be found by subtracting the given measure from  $90^\circ$ .

➤ The trigonometric function of the complement of an angle is called a **cofunction**

- $\sin x = \cos(90 - x)$

- $\cos x = \sin(90 - x)$

11. Find the sine and cosine of the acute angles in the right triangle shown.

a.  $\sin A = \frac{6}{10} = \frac{3}{5}$

b.  $\cos A = \frac{8}{10} = \frac{4}{5}$

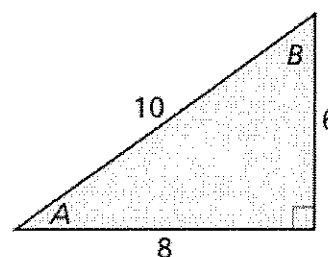
c.  $\sin B = \frac{8}{10} = \frac{4}{5}$

d.  $\cos B = \frac{6}{10} = \frac{3}{5}$

e. What do you observe?

$$\sin A = \cos B$$

$$\cos A = \sin B$$



12. Write each trigonometric function in terms of its cofunction.

a. Write  $\sin 42^\circ$  in terms of cosine.

$$\cos 48^\circ$$

b. Write  $\cos 36^\circ$  in terms of sine.

$$\sin 54^\circ$$

c. Write  $\sin 28^\circ$  in terms of cosine.

$$\cos 62^\circ$$

d. Write  $\cos 51^\circ$  in terms of sine.

$$\sin 39^\circ$$

## 9.2 – 9.5 (D2): SOLVING RIGHT TRIANGLES

Every right triangle has one right angle, two acute angles, one hypotenuse, and two legs.

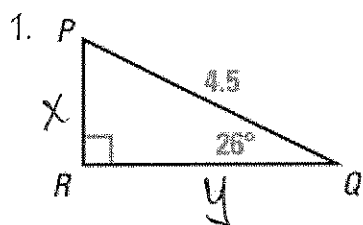
### ☺ Solving a Right Triangle

- To **solve a right triangle** means to find the measures of both acute angles and the lengths of all three sides.
- You can solve a right triangle if you know either of the following:
  - Two side lengths
  - One side length and one acute angle measure

### ☺ How to Solve a Right Triangle

- Use the **Pythagorean Theorem** to find a missing side (if two sides are known)
- Use the **relationship between the sine and cosine of complementary angles** to find the measure of a missing angle (if two angles are known)
- Write a **tangent, sine or cosine ratio** to find a missing acute angle measure and/or a missing side length.

Solve each right triangle. Round angle measures to the nearest tenth of a degree; round side lengths to the nearest hundredth.



$$\begin{aligned}\angle P &= 24^\circ \\ PR &= 1.97 \\ QR &= 4.04\end{aligned}$$

PR

$$\sin 26^\circ = \frac{x}{4.5}$$

$$x = 4.5 \cdot \sin 26^\circ$$

$$x \approx 1.97$$

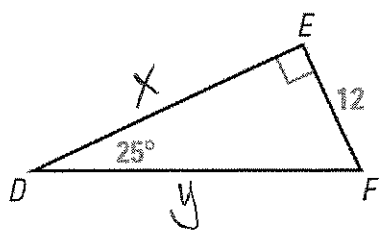
QR

$$\cos 26^\circ = \frac{y}{4.5}$$

$$y = 4.5 \cos 26^\circ$$

$$y \approx 4.04$$

2.



$$\angle F = \underline{65^\circ}$$

$$DE = \underline{25.73}$$

$$DF = \underline{28.39}$$

$$\frac{DE}{\tan 25} = \frac{12}{x}$$

$$\tan 25 = \frac{12}{x}$$

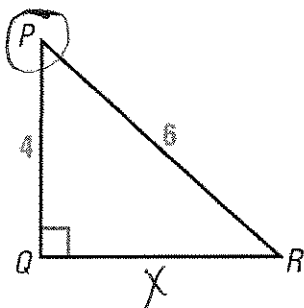
$$x = \frac{12}{\tan 25} \approx 25.73$$

$$\frac{DF}{\sin 25} = \frac{12}{y}$$

$$\sin 25 = \frac{12}{y}$$

$$y = \frac{12}{\sin 25} \approx 28.39$$

3.



$$\frac{QR}{x^2 + 4^2 = 6^2}$$

$$x^2 + 16 = 36$$

$$x^2 = 20$$

$$x \approx 4.47$$

$$\cos P = \frac{4}{6}$$

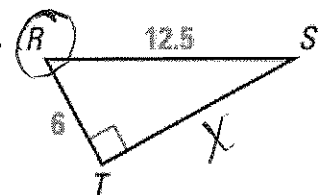
$$m\angle P = \cos^{-1}\left(\frac{4}{6}\right) \approx 48.2^\circ$$

$$\angle P = \underline{48.2^\circ}$$

$$\angle R = \underline{41.8^\circ}$$

$$QR = \underline{4.47}$$

4.



$$\frac{ST}{x^2 + 6^2 = 12.5^2}$$

$$x^2 + 36 = 156.25$$

$$x^2 = 120.25$$

$$x \approx 10.97$$

$$\cos R = \frac{6}{12.5}$$

$$m\angle R = \cos^{-1}\left(\frac{6}{12.5}\right) \approx 101.3^\circ$$

$$ST = \underline{10.97}$$

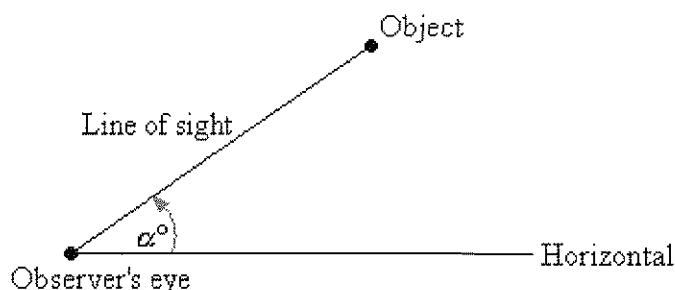
$$\angle R = \underline{101.3^\circ}$$

$$\angle S = \underline{28.7^\circ}$$

## 9.2 – 9.5 (D3): ANGLES OF ELEVATION & DEPRESSION

### ☺ Angle of Elevation

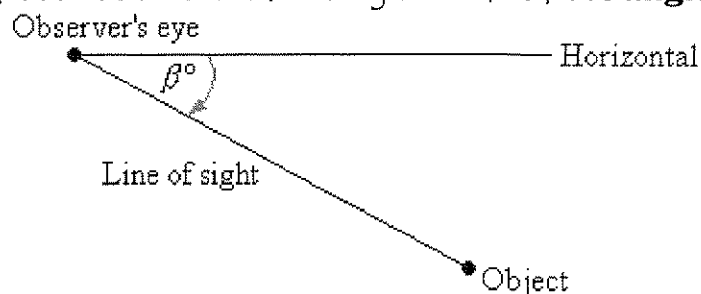
- The **angle of elevation** of an object as seen by an observer is the angle between the horizontal and the line from the object to the observer's eye (line of sight).



- The angle of elevation of the object from the observer is  $\alpha^\circ$

### ☺ Angle of Depression

- If the object is below the level of the observer, then the angle between the horizontal and the observer's line of sight is called the **angle of depression**.

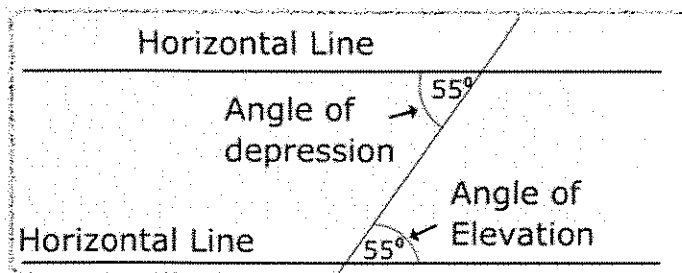


- The angle of depression of the object from the observer is  $\beta^\circ$

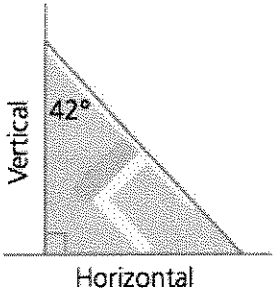
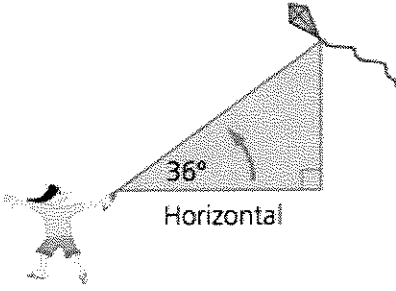
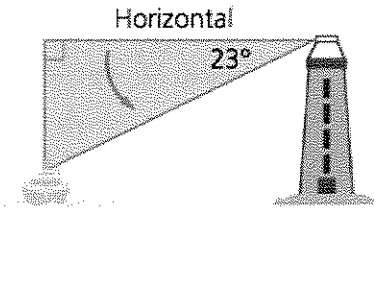
### Angle of Elevation $\cong$ Angle of Depression

Why?

Horizontal lines are parallel. The angle of elevation  $\cong$  the angle of depression b/c they are alternate interior  $\angle$ s.





Angle of inclination	Angle of elevation	Angle of depression
Formed between the vertical or horizontal as defined by the situation. The angle of inclination with the vertical is $42^\circ$ .	Formed by looking up from the horizontal The angle of elevation is $36^\circ$ .	Formed by looking down from the horizontal. The angle of depression is $23^\circ$ .
		

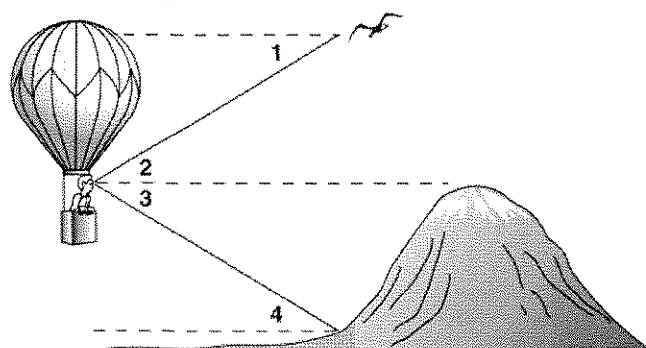
Describe each angle as it relates to the situation in the diagram.

1.  $\angle 1$   $\angle$  of depression

2.  $\angle 2$   $\angle$  of elevation

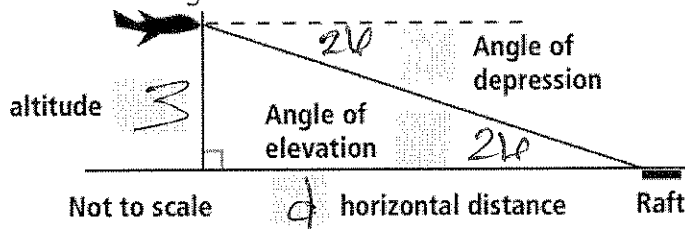
3.  $\angle 3$   $\angle$  of depression

4.  $\angle 4$   $\angle$  of elevation



5. An airplane pilot sights a life raft at a  $26^\circ$  angle of depression. The airplane's altitude is 3 km. What is the airplane's horizontal distance  $d$  from the raft?

a. Label the diagram below:



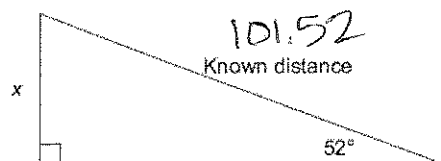
b. Set up and solve a trigonometric equation to find  $d$ .

$$\tan 26^\circ = \frac{3}{d}$$

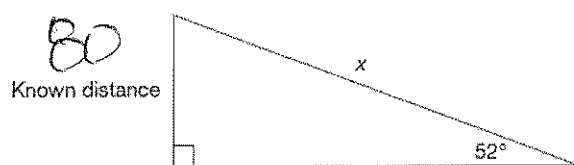
$$d = \frac{3}{\tan 26^\circ} \approx 10.2 \text{ km}$$

Match each diagram with the appropriate situation and identify the trigonometric function that would be most helpful in answering each question. Then calculate the unknown measurement.

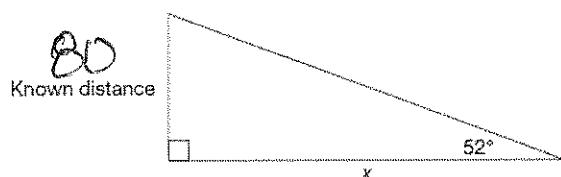
#8 Diagram A



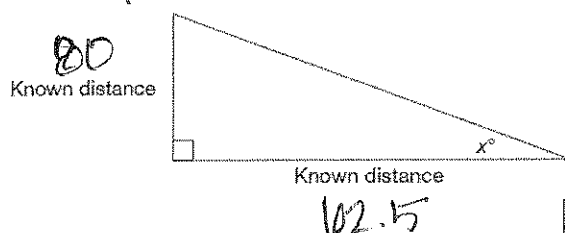
#10 Diagram B



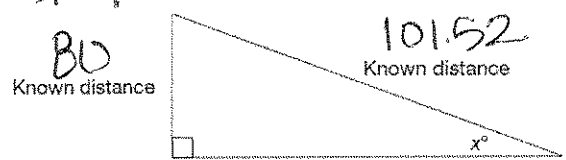
#11 Diagram C



#7 Diagram D



#9 Diagram E



6. A building is 80 feet high. An observer stands an unknown distance away from the building and, looking up to the top of the building, notes that the angle of elevation is  $52^\circ$ . Determine the distance from the base of the building to the observer.

$$\tan 52 = \frac{80}{x} \quad x = \frac{80}{\tan 52} \quad x \approx 62.5 \text{ ft}$$

7. A building is 80 feet high. An observer stands 62.5 feet away from the building and looking up to the top of the building he ponders the measure of the angle of elevation. Determine the measure of the angle of elevation.

$$\tan x = \frac{80}{62.5} \quad \tan^{-1}\left(\frac{80}{62.5}\right) = 52^\circ$$

8. An observer stands an unknown distance away from a building and looking up to the top of the building notes that the angle of elevation is  $52^\circ$ . He also knows the distance from where he is standing to the top of the building is 101.52 feet. Determine the height of the building.

$$\sin 52 = \frac{x}{101.52} \quad x = 101.52 \sin 52 \approx 80 \text{ ft}$$

9. A building is 80 feet high. An observer stands an unknown distance away from the building and looking up to the top of the building he ponders the measure of the angle of elevation. He also knows the distance from where he is standing to the top of the building is 101.52 feet. Determine the measure of the angle of elevation.

$$\sin x = \frac{80}{101.52} \quad \sin^{-1}\left(\frac{80}{101.52}\right) = 52^\circ$$

10. A building is 80 feet high. An observer stands an unknown distance away from the building and looking up to the top of the building notes that the angle of elevation is  $52^\circ$ . Determine the distance from the observer to the top of the building.

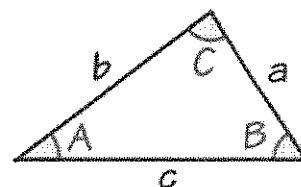
$$\sin 52 = \frac{80}{x}$$

$$x = \frac{80}{\sin 52} \approx 101.52 \text{ ft}$$

## 9.6 (D1): THE LAW OF SINES

### The Law of Sines

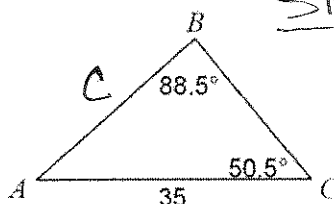
- The ratio of the sine of an angle to the length of its opposite side is the same for all 3 angles of any triangle.
- You can use the Law of Sines when...
  - You know the lengths of two sides of a triangle and the measure of an angle opposite one of the sides
  - You know the measures of two angles of a triangle and the length of a side opposite one of those angles
- For any oblique (non-right) triangle ABC...



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Use the Law of Sines to solve for the indicated side or angle. Round angle measures to the nearest tenth of a degree; round side lengths to the nearest hundredth.

1. Find AB.

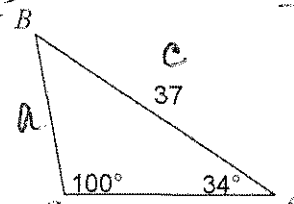


$$\frac{\sin 88.5}{35} = \frac{\sin 50.5}{c}$$

$$\frac{c \sin 88.5}{\sin 88.5} = \frac{35 \sin 50.5}{\sin 88.5}$$

$$AB \approx 27.01$$

2. Find BC.

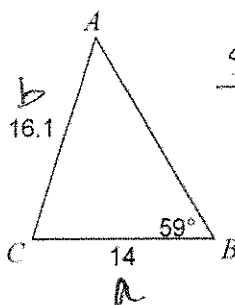


$$\frac{\sin 34}{a} = \frac{\sin 100}{37}$$

$$\frac{a \sin 100}{\sin 100} = \frac{37 \sin 34}{\sin 100}$$

$$BC \approx 21$$

3. Find  $m\angle A$ .



$$\frac{\sin A}{14} = \frac{\sin 59}{16.1}$$

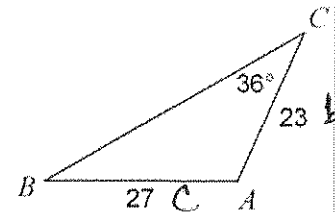
$$\frac{16.1 \sin A}{16.1} = \frac{14 \sin 59}{16.1}$$

$$\sin A \approx 0.7454$$

$$m\angle A = \sin^{-1}(0.7454)$$

$$\angle A \approx 48.2^\circ$$

4. Find  $m\angle B$ .



$$\frac{\sin B}{23} = \frac{\sin 36}{27}$$

$$\frac{27 \sin B}{27} = \frac{23 \sin 36}{27}$$

$$\sin B \approx 0.5007$$

$$m\angle B = \sin^{-1}(0.5007)$$

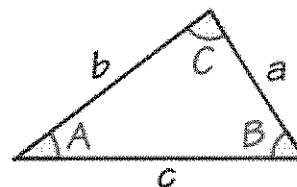
$$\angle B \approx 30^\circ$$

## 9.6 (D2): THE LAW OF COSINES

### The Law of Cosines

➤ You can use the Law of Cosines when...

- You know the lengths of all three sides of a triangle
  - Continue with the Law of Cosines to find the 2<sup>nd</sup> angle
- You know the lengths of two sides of a triangle and the measure of the included angle



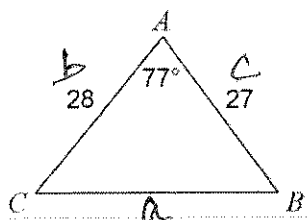
➤ For any oblique (non-right) triangle ABC...

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\frac{a^2 - b^2 - c^2}{-2bc} = \cos A$$

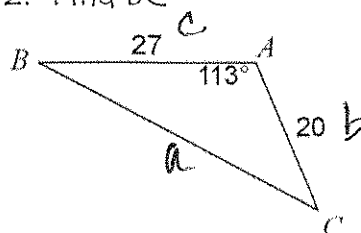
Use the Law of Cosines to solve for the indicated side or angle. Round angle measures to the nearest tenth of a degree; round side lengths to the nearest hundredth.

1. Find BC



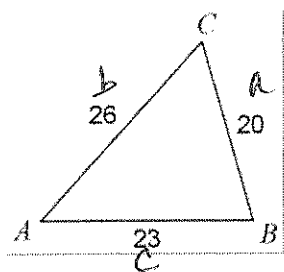
$$\begin{aligned} a^2 &= 28^2 + 27^2 - 2(28)(27) \cos 77 \\ a^2 &= 1513 - 1512 \cos 77 \\ a^2 &\approx 1172.8740 \\ BC &\approx 34.25 \end{aligned}$$

2. Find BC



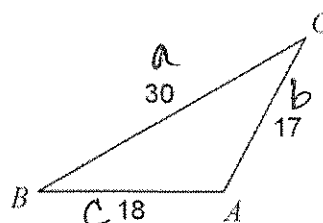
$$\begin{aligned} a^2 &= 20^2 + 27^2 - 2(20)(27) \cos 113 \\ a^2 &= 1129 - 1080 \cos 113 \\ a^2 &\approx 1550.9896 \\ BC &\approx 39.38 \end{aligned}$$

3. Find  $m\angle A$ .



$$\begin{aligned} \cos A &= \frac{20^2 - 26^2 - 23^2}{-2(26)(23)} = \frac{-805}{-1196} \\ \cos A &\approx 0.6731 \\ \angle A &\approx 47.7^\circ \end{aligned}$$

4. Find  $m\angle A$ .



$$\begin{aligned} \cos A &= \frac{30^2 - 17^2 - 18^2}{-2(17)(18)} = \frac{287}{-612} \\ \cos A &\approx -0.46895 \\ \angle A &\approx 110^\circ \end{aligned}$$