College Algebra ~ Notes Packet

Name: ______________________________

CHAPTER 2: LINEAR FUNCTIONS

2.1.D1 ~ Intro to Linear Functions

OBJECTIVES:

- Identify linear functions given a data table
- Determine the slope of the line drawn through two points
- Find & interpret the contextual meaning of the average rate of change

❖ AVERAGE RATE OF CHANGE

➢ The average rate of change of a function f over an interval $[a, b]$ is: $\dfrac{f(b) - f(a)}{b - a}$ where $(a, f(a))$ & $(b, f(b))$ are any two data points in the table.

❖ LINEAR FUNCTIONS

➢ If the average rate of change in output with respect to input remains constant (stays the same) for *any* two points in a data set, then all points will lie on a single straight line.

➢ Any function with a constant rate of change is called a **LINEAR FUNCTION.**

EXAMPLE:

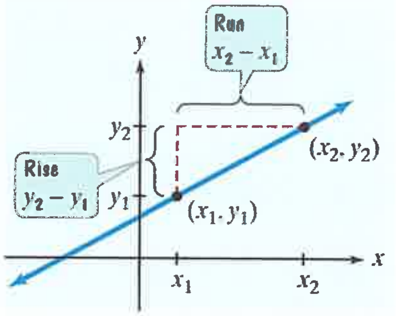
1. Does the data in the table indicate a linear function? Explain your reasoning.

x	200 +300	230	300 +20	320	400
$f(x)$	70 −1.5	68.5	65 −1	64	60

yes! The Rate of change is constant.

$$ARDC = \frac{-1.5}{300} = -0.05 \qquad ARDC = \frac{-1}{20} = -0.05$$

❖ SLOPE OF A LINE



m from the French verb *monter* which means "to rise"

➢ The average rate of change for a linear function determines the steepness of the line and is called the slope of the line.

$$\text{slope} = m = \frac{\text{change in output}}{\text{change in input}} = \frac{y_2 - y_1}{x_2 - x_1}$$

where (x_1, y_1) & (x_2, y_2) are any two points on the line and $x_1 \neq x_2$.

EXAMPLE:

2. Determine the slope of the line containing the points $\overset{x_1\ y_1}{(1, -2)}$ and $\overset{x_2\ y_2}{(-3, 8)}$.

$$m = \frac{8-(-2)}{-3-1} = \frac{10}{-4} = -\frac{5}{2}$$

3. Determine the constant rate of change (slope) of the linear function and explain what it means in terms of the context: At a local community college, a student's tuition bill was $1008 for 9 credit hours. The student adds 4 credit hours and his tuition increases to $1456.

(9,1008) (13,1456)

$$ROC = \frac{1456-1008}{13-9} = \frac{448}{4} = 112$$

The tuition bill increases by $112 for every credit hour.

2.1.D2 ~ Slope-Intercept Form of Linear Equations

OBJECTIVES:
- Use the slope formula and interpret slope as an average rate of change
- Determine vertical and horizontal intercepts and interpret their practical meaning
- Use the slope-intercept form to write the equation of a line
- Graph linear functions

Recall: A function is a relationship between two variables such that any given input has exactly one corresponding output.

EXPLORE – You have decided to buy a new Honda Civic, but you are concerned about the value of the car depreciating over time. You search the Internet and obtain the following information about the value of the car after n years of ownership.

Years, n	0	1	3	5	8
Value, V	20905	19155	15655	12155	6905

1. Select two ordered pairs of the form (n, V) from the table and determine the average rate of change.

$n=1$
$n=3$ $\text{AROC} = \frac{15655-19155}{3-1} = \frac{-3500}{2} = -1750$

2. Select two different ordered pairs, not used in part a, and compute the average rate of change.

$n=0$
$n=8$ $\text{AROC} = \frac{6905-20905}{8-0} = \frac{-14000}{8} = -1750$

3. Is the value, V, of the car a *linear* function of the number of years, n, of ownership? Explain your reasoning.

yes! the AROC is constant!

Any function with a constant rate of change is called a **LINEAR FUNCTION.**

❖ VERTICAL / y-INTERCEPT

➢ The point where the graph of a function crosses, or intercepts, the vertical axis

- (0, #)
- Let x=0 & solve for y

❖ SLOPE-INTERCEPT FORM

➢ The coordinates of all points (x, y) on the line with slope m and vertical intercept $(0, b)$ satisfy the equation $y = mx + b$

❖ HORIZONTAL / x-INTERCEPTS (AKA ZEROS)

➢ A point where the graph meets or crosses the horizontal axis.

- To determine the horizontal intercept, let $y = 0$ and solve for x.

the output

the input

4. Determine the vertical intercept (V-intercept) of the Honda Civic problem. What is the practical meaning of the vertical intercept in this situation? *Include units.*

(0, 20905) the initial value of the Civic is $2090

5. Write a linear equation that represents the value, V, of the Honda Civic for a given number of years, n, of ownership.

$V = 20905 - 1750n$

6. Determine the horizontal intercept (n-intercept) of the graph of the car value equation. What is the practical significance of the horizontal intercept? *Include units.*

$0 = 20905 - 1750n$

$1750n = 20905$

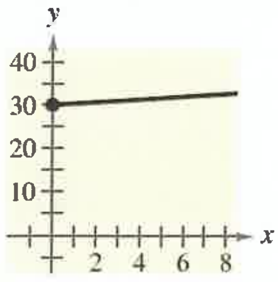
$n = 11.95 \approx 12$

After 12 years, the value of the Civic is $0

EXAMPLES:

Match the description of the situation with its graph AND write its corresponding linear function of the form $y = mx + b$.

A



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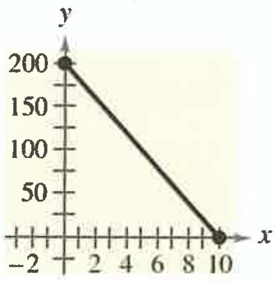
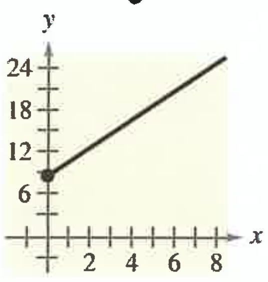
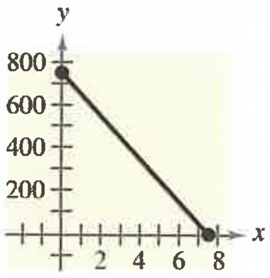


C



D



7. A person is paying \$20 per week to a friend to repay a \$200 loan. $y=200-20x$, B
8. An employee is paid \$8.50 per hour plus \$2 for each unit produced each hour. $y=8.50+2x$, C
9. A sales representative received \$30 per day for food plus \$0.32 for each mile traveled. $y=30+0.32x$, A
10. A computer that was purchased for \$750 depreciates \$100 per year. $y=750-100x$, D

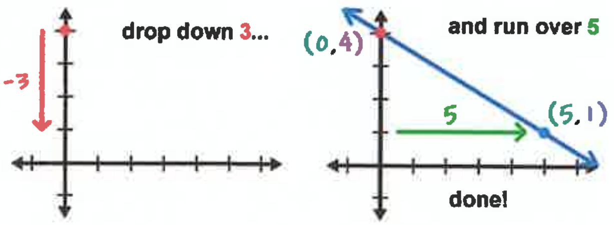
❖ GRAPHING A FUNCTION IN SLOPE-INTERCEPT FORM: $y = mx + b$

- Plot the vertical intercept: $(0, b)$
- From there, use the slope, m, to determine a second point
  - If the slope is positive, go up the number of units in the numerator
  - If the slope is negative, go down the number of units in the numerator
  - Always go right the number of units in the denominator

Graph $y = \frac{-3}{5}x + 4$

$\boxed{1}$ It crosses the y-axis at 4, so we start there:

$\boxed{2}$ the slope is $\frac{-3}{5}$ so we

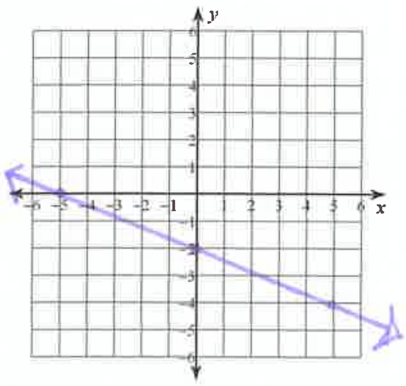


drop down 3... and run over 5

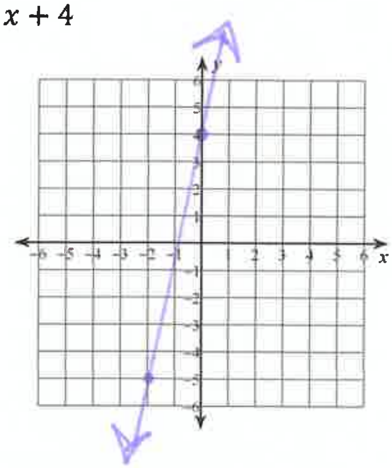
done!

EXAMPLES: Graphing a Function in Slope-Intercept Form

11. $y = -\frac{2}{5}x - 2$



12. $y = \frac{9}{2}x + 4$



2.1.D3 ~ Point-Slope Form of a Linear Equation

OBJECTIVES:

- Use the point-slope form to write a linear equation in slope-intercept form
- Write equations of parallel and perpendicular lines

❖ POINT-SLOPE FORM

➢ Let (x_1, y_1) be a known point on a line, m the slope, and (x, y) is any other point on the line, the point-slope form of a linear equation is: $y - y_1 = m(x - x_1)$.

POINT-SLOPE FORM

$y - y_1 = m(x - x_1)$

$m = -2$ $\qquad (\overset{x_1}{4}, \overset{y_1}{-3})$

$y - (-3) = -2(x - 4)$

$y + 3 = -2x + 8$ ← be careful here!

$-3 \qquad\qquad -3$

$y = -2x + 5$ done

SLOPE-INTERCEPT FORM

You could substitute the point & the slope into the slope-intercept equation of a line and then solve for b:

$$y = mx + b$$
$$-3 = -2(4) + b$$
$$-3 = -8 + b$$
$$5 = b$$

Then only plug in the slope & y-intercept (b):

$$y = -2x + 5$$

EXAMPLES:

1. Find an equation for the line, in slope-intercept form, that passes through the point $(\overset{x_1}{-3}, \overset{y_1}{-4})$ and has slope 2.

$y + 4 = 2(x + 3)$

$y + 4 = 2x + 6$

$y = 2x + 2$

2. Write an equation for the linear function f such that $f(\overset{x_1}{-2}) = \overset{y_1}{3}$ and $f(\overset{x_2}{4}) = \overset{y_2}{1}$.

$m = \frac{3-1}{-2-4} = \frac{2}{-6} = \frac{1}{-3}$

$y = mx + b$

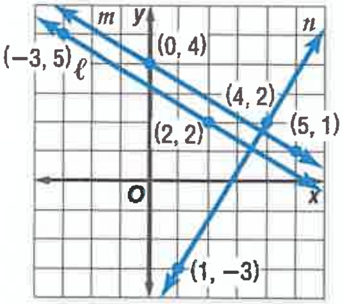
$1 = -\frac{1}{3}(4) + b$

$1 = -\frac{4}{3} + b$

$\frac{7}{3} = b$

PARALLEL AND PERPENDICULAR LINES

Examine the graphs of lines ℓ, m, and n. Lines ℓ and m are parallel, and n is perpendicular to ℓ and m. Let's investigate the slopes of these lines.



slope of ℓ		slope of m		slope of n
$m = \frac{2-5}{2-(-3)}$		$m = \frac{1-4}{5-0}$		$m = \frac{2-(-3)}{4-1}$
$= -\frac{3}{5}$	$\ell \parallel m$	$= -\frac{3}{5}$	$m \perp n$	$= \frac{5}{3}$

➢ If two nonvertical lines are parallel, then their slopes are the same

- Any two vertical or horizontal lines are parallel.

➢ The slopes of perpendicular lines are opposite reciprocals

- Any horizontal line and vertical line are perpendicular.

EXAMPLE:

3. Consider the linear function, *f*, given by the equation: $y = \frac{2}{3}x + 9$. Find the equation of a linear function whose graph is perpendicular to the linear function *f* and the two lines intersect at $x = 6$.

1. Slope of the given line: $m=\frac{2}{3}$

2. perpendicular slope: $\boxed{m_\perp = \frac{-3}{2}}$

3. point: if $x=6$, then $y=\frac{2}{3}(6)+9=13$ $\boxed{(6,13)}$

4. $y=mx+b$

$13=-\frac{3}{2}(6)+b$ $y=\frac{-3}{2}x+22$

$13=-9+b$ $b=22$

2.1.D4 ~ Standard Form of Linear Equations

OBJECTIVES:

- Determine whether two lines are parallel, perpendicular, or neither
- Graph linear functions

❖ Standard Form of Linear Equations

- ➢ $Ax + By = C$
  - ▪ A, B & C represents constants; A & B cannot both be zero
- ➢ Standard Form → Slope-Intercept Form
  - ▪ Solve the equation for y

$$\begin{aligned} 3x + 7y &= 5 \\ -3x \qquad &\;\; -3x \end{aligned}$$

Add $-3x$ to each side of the equation.

$$7y = -3x + 5$$

$$\frac{7y}{7} = \frac{-3x}{7} + \frac{5}{7}$$

Divide each side of the equation by 7, the coefficient of y.

$$y = -\frac{3}{7}x + \frac{5}{7}$$

RECALL: PARALLEL & PERPENDICULAR LINES

❖ Parallel Lines = same slope

❖ Perpendicular Lines = opposite reciprocal slopes

EXAMPLE: $m=2$

1. Consider the line $y = 2x - 5$. Use slopes to determine which of the following lines are parallel, perpendicular (or neither) to the given line.

a. $6x - 3y = 8$

$-3y=-6x+8$

$y=\boxed{2}x-\frac{8}{3}$

parallel

b. $2x + 5y = 3$

$5y=-2x+3$

$y=\boxed{-\frac{2}{5}}x+\frac{3}{5}$

neither

c. $4x + 8y = 5$

$8y=-4x+5$

$y=\boxed{-\frac{1}{2}}x+\frac{5}{8}$

perpendicular

❖ GRAPHING A FUNCTION IN STANDARD FORM

- ➢ Find and plot the vertical and horizontal intercepts of the function.

Graph $x + 2y = 4$

To find the horizontal/x-intercept:
Let y = 0 & solve for x.

$x = 4$

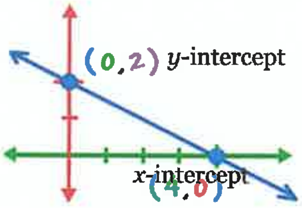
$x = 4$

To find the vertical/y-intercept:
Let x = 0 & solve for y.

$+2y = 4$

$y = 2$

This is also ***b*** *in slope-intercept form.*



EXAMPLE: Graphing a Linear Function in Standard Form

2. Find the vertical and horizontal intercepts of the function. Then graph the function.

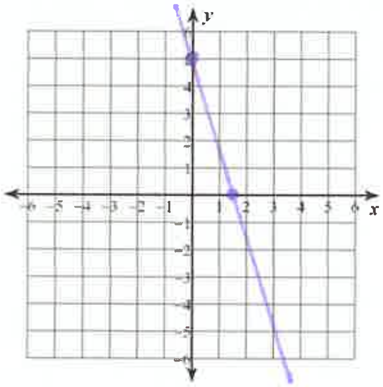
$$10x + 3y = 15$$

$10x = 15$

$x = 1.5$

$3y = 15$

$y = 5$

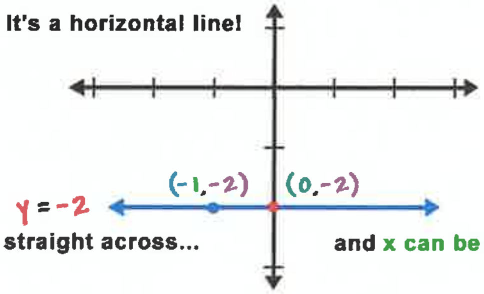


❖ HORIZONTAL & VERTICAL LINES

- ➢ A horizontal line with vertical intercept $(0, b)$ has equation $y = b$.
  - The slope of a horizontal line is 0.
- ➢ A vertical line with horizontal intercept $(a, 0)$ has equation $x = a$.
  - The slope of a vertical line is undefined.

Horizontal Lines: $y = \#$

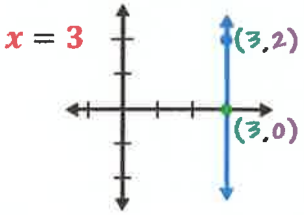
It's a horizontal line!



straight across...

and x can be anything!

Vertical Lines: $x = \#$



It's a vertical line!

What's the slope?

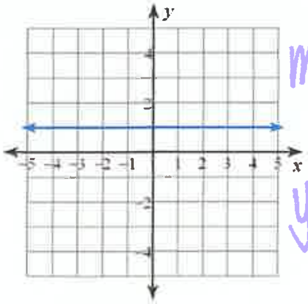
$m = \frac{2-0}{3-3} = \frac{2}{0}$

The slope of a vertical line is undefined.

This is the official word.

EXAMPLES: Find the slope and equation for each line shown.

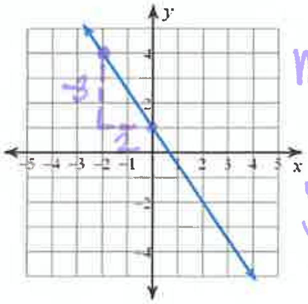
3.



$m = 0$

$y = 1$

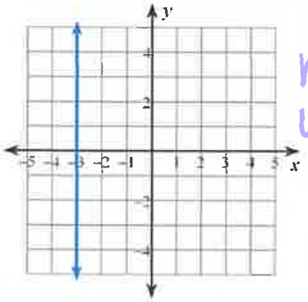
4.



$m = \frac{-3}{2}$

$y = \frac{-3}{2}x + 1$

5.



m is undefined

$x = -3$

2.2 ~ Modeling with Linear Functions

OBJECTIVES:

- Determine a symbolic rule for a linear function from contextual information
- Identify the slope and intercepts of a linear function and interpret their practical meaning

❖ RECOGNIZING WHEN TO USE A LINEAR MODEL

- Several key phrases alert us to the fact that a linear model may be used to model a data set.
  - Increasing at a rate of 20 people per year
  - Tickets cost $37 per person
  - Sales decrease by 100 tickets for every $1 increase in price
  - There are 350 students today. The number of students is increasing by 10 students each month.
  - The price is $12.25 and is decreasing at a constant rate of $0.02 per day

❖ CONSTRUCTING A LINEAR MODEL

- What do I know?
  - Rate of change; the initial value; data points
- Which form of a linear equation should I use?
  - SLOPE-INTERCEPT FORM: $y = mx + b$
    - Use if given the rate of change and the initial value
  - POINT-SLOPE FORM: $y - y_1 = m(x - x_1)$
    - Use if given the rate of change and a data point
    - Use if given two data points
  - STANDARD FORM: $Ax + By = C$
    - Use if combining the two quantities and given a total numerical amount.

SLOPE

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

EXAMPLES:

1. You have purchased a laptop computer so that you can use the software products you have acquired through your job. The initial cost of the computer is $1350. You expect that the computer will depreciate at the rate of $450 per year.

   a. Write a symbolic rule that will determine the value, $v(t)$, of the computer in terms of the number of years, t, that you own it.

   $V(t) = 1350 - 450t$

   b. What is the practical domain for this situation?

   time (years) $[0,3]$

   $0 = 1350 - 450t$
   $450t = 1350$
   $t = 3$

   c. What is the practical range for this situation?

   value ($) $[0,1350]$

2. The Recommended Daily Allowance (RDA) for fat a person on a 2000-calorie-per-day diet is 65 grams. A McDonald's Big N' Tasty sandwich, s, contains 32 grams of fat. A super-size order of French fries, f, contains 29 grams of fat.

   a. Write the equation for the total fat grams consumed.

   $32s + 29f = 65$

   b. If a person consumes only the Big N' Tasty – hold the fries – how many sandwiches can a person eat without exceeding the RDA for fat?

   $32s + 29(0) = 65$
   $32s = 65$
   $s \approx 2$ sandwiches

3. Despite the decrease in housing prices in recent years, there has been a steady increase in housing prices in your neighborhood since 2002. The house across the street sold for $125,000 in 2005 and then sold again in 2009 for $150,000. Let x represent the number of years since 2002 and y represent the sale price of a typical house in your neighborhood. (3, 125000) (7, 150000)

   a. What is the average rate of change? What is the practical meaning of the average rate of change in this situation?

m=6250
(7, 150000)

$$\text{AROC} = \frac{150000-125000}{7-3} = \frac{25000}{4} = 6250$$

Housing prices are increasing by $6250 each year.

   b. Write a linear function equation that represents this situation.

$y=mx+b$

$150000=6250(7)+b$

$106250=b$

$y=106250+6250x$

   c. What is the vertical intercept? What is the practical meaning of the vertical intercept in this situation?

(0, 106250) In 2002, the housing price was $106,250.

4. The cost for a part-time student is determined by an activity fee plus a fixed tuition amount per credit. The cost for a student taking 6 credits is $1520. The cost for a student taking 9 credits is $2255. The total cost, C, is a function of the number of credits taken, n. (6, 1520) (9, 2255)

   a. Write a linear equation, in slope-intercept form, that represents the total cost, C, as a function of the number of credits, n.

$$m = \frac{2255-1520}{9-6} = \frac{735}{3} = \underline{245}$$

$y=mx+b$

$1520=245(6)+b$

$50=b$

$C=50+245n$

   b. What is the vertical intercept and what does it represent?

(0, 50)

the Activity fee is $50.

7.2.D1 ~ Piecewise Functions

OBJECTIVES:

- **Determine function values of piecewise functions from an equation and/or its graph**
- **Graph piecewise functions**

❖ WHAT IS A PIECEWISE FUNCTION?

- ➢ A function that is a combination of one or more functions.
  - ▪ The rule for a piecewise function is different for different parts/pieces of the domain.

EXAMPLES: EVALUATING A PIECEWISE-DEFINED FUNCTION

1. Evaluate $f(x)$ for the given input value.

$$f(x) = \begin{cases} x-5 & \text{for } x \le 3 \\ 1 & \text{for } 3 < x < 5 \\ x+1 & \text{for } x \ge 5 \end{cases}$$

$f(-1) =$ -6

$f(4.9) =$ 1

$f(3) =$ -2

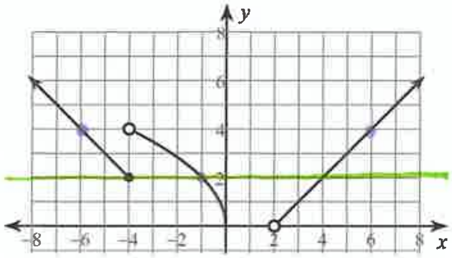
$f(27) =$ 28

$f(5) =$ 6

$f(3.1) =$ 1

2. Use the piecewise function equation and/or its graph to determine the following:

$$f(x)=\begin{cases}|x+2|, & x\le -4\\ \sqrt{-4x}, & -4<x\le 2\\ x-2, & x>2\end{cases}$$



a. Evaluate $f(-6)$ 4

b. Evaluate $f(-1)$ 2

c. Evaluate $f(6)$ 4

d. Evaluate $f(20)$ = 20-2=18

e. Solve $f(x)=2$ $x=-4,-1,4$

f. Solve $f(x)=4$.

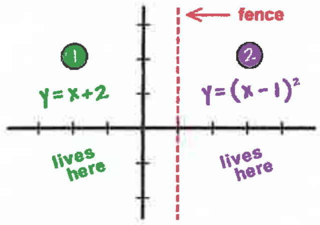
$x=-6$ & 6

❖ GRAPHING PIECEWISE FUNCTIONS

$$y=\begin{cases}x+2 \; ; \; x<1 \leftarrow ①\\ (x-1)^2 \; ; \; x\ge 1 \leftarrow ②\end{cases}$$ It's in two pieces!

Each piece must live ONLY in its own neighborhood.

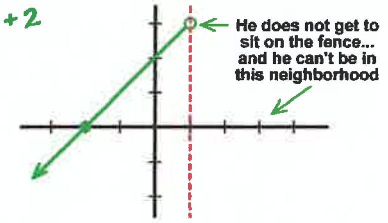
Let's put up a fence, so we don't make any mistakes:



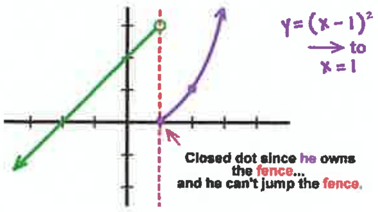
$$y=\begin{cases}x+2 \; ; \; x<1\\ (x-1)^2 \; ; \; x\ge 1 \leftarrow\end{cases}$$ **This guy has the "=", so he gets to live ON the fence.**

Let's graph part ① :

$y=x+2$



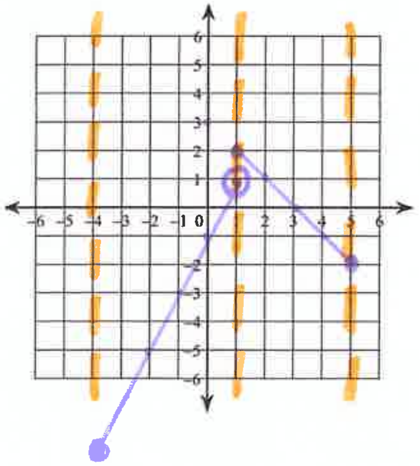
Now, let's graph part ② :



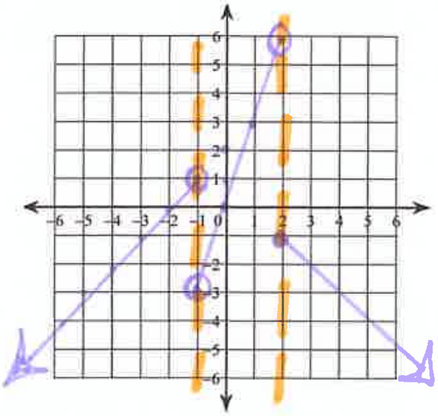
EXAMPLES: GRAPHING PIECEWISE-DEFINED FUNCTIONS

Graph the piecewise-defined function.

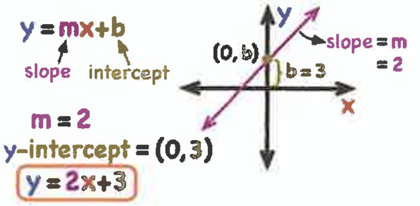
3. $f(x)=\begin{cases}2x-1, & -4\le x<1\\ -x+3, & 1\le x\le 5\end{cases}$



4. $f(x)=\begin{cases}x+2, & x<-1\\ 3x, & -1\le x<2\\ -x+1, & x\ge 2\end{cases}$



7.2.D2 ~ Piecewise Functions



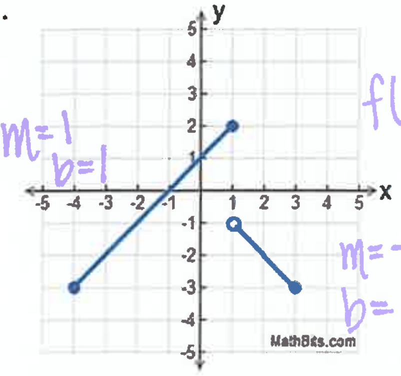
OBJECTIVES:

- Define piecewise functions using equations, tables, graphs, and words
- Write a piecewise function given its graph or a contextual situation

EXAMPLES: WRITING PIECEWISE-DEFINED FUNCTIONS

Write a piecewise defined function for the function shown.

1.

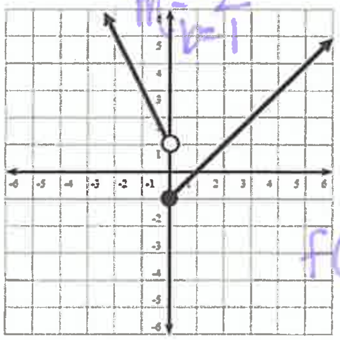


m=1
b=1

m=−1
b=0

$$f(x)=\begin{cases} x+1 & [-4,1] \\ -x & (1,3] \end{cases}$$

2.



m=−2
b=1

m=1
b=−1

$$f(x)=\begin{cases} -2x+1 & (-\infty,0) \\ x-1 & [0,\infty) \end{cases}$$

EXAMPLES: INTERPRETING & WRITING PIECEWISE-DEFINED FUNCTIONS FOR A CONTEXTUAL SITUATION

3. The function, $V(t)$, models the volume of water (in gallons) in a swimming pool t hours as it is filled. Evaluate $V(24)$ and explain what the numerical value represents in the real-world context.

$$V(t)=\begin{cases} 100t & \text{if } 0 \le t < 20 \\ 2000 & \text{if } 20 \le t < 23 \\ 1500t - 28000 & \text{if } 23 \le t < 38 \\ 32000 & \text{if } 40 \le t \le 48 \end{cases}$$

$V(24) = 1500(24) - 28000 = 8000$

there are 8000 gallons of water in the swimming pool after 24 hours.

4. A cellular phone company offers the following plan:
   - $20 per month buys 60 minutes
   - Additional time costs $0.40 per minute
   - Customers are limited to 2000 total minutes each month.

The total monthly cost, C, is a function of the number of calling minutes, t.

a. Evaluate $C(20)$ & $C(100)$.

$C(20) = \$20$

$C(100) = 20 + (100-60)\cdot 0.40 = \$36$

b. Represent this plan with a piecewise defined function in which the total monthly cost, C, is a function of the number of calling minutes, t.

$$C=\begin{cases} 20 & [0,60] \\ 20+0.40(t-60) & (60,2000] \end{cases}$$

c. What is the practical domain and the practical range of this function?

D = [0, 2000] minutes

R = [20, 796] dollars