Name: ______________________________

CHAPTER 2: LINEAR FUNCTIONS

This packet contains notes from Sections 2.4 & 2.5.

2.4.D1 – Systems of Two Linear Equations

OBJECTIVE:

- Solve a system of two linear equations graphically and algebraically using the substitution and elimination methods

❖ Systems of Two Linear Equations

- ➢ Two linear equations that relate the same two variables are called a **system of linear equations**.
  - ▪ The solution of a system is the SET OF ORDERED PAIRS that satisfy both equations.
    - If the system has exactly one solution, the system is called consistent.
    - If the system has no solution, the system is called inconsistent.

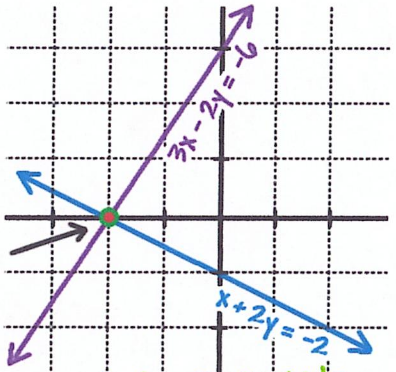
❖ **Graphical Method**

- ➢ Graph both equations on the same grid
- ➢ If the two lines intersect, the coordinates of the point of intersection represent the solution of the system: (x, y)
  - ▪ If the lines are parallel, the system has no solution

$x + 2y = -2$

$3x - 2y = -6$

These equations are given in standard form. You can graph these by finding the intercepts OR by rewriting the equation in slope-intercept form.



Check your solution by plugging the point of intersection into BOTH equations.

$x + 2y = -2 \rightarrow -2 + 2(0) = -2$

$3x - 2y = -6 \rightarrow 3(-2) - 2(0) = -6$

Yep! So, the answer is $(-2, 0)$.

convert to $y=mx+b$

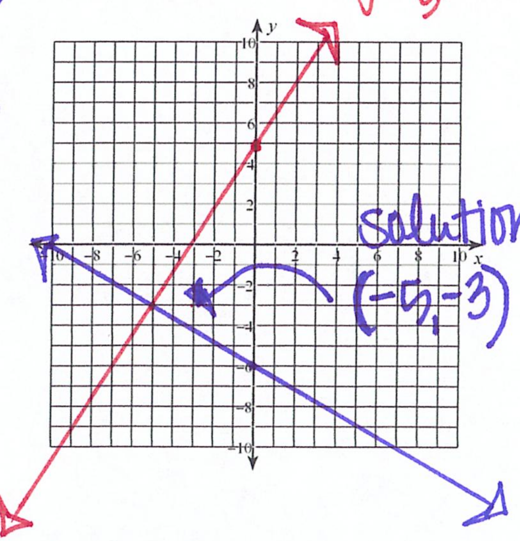
EXAMPLES:

1. $8x - 5y = -25$ $\rightarrow$ $-5y = -8x - 25$, $y = \frac{8}{5}x + 5$
   $3x + 5y = -30$

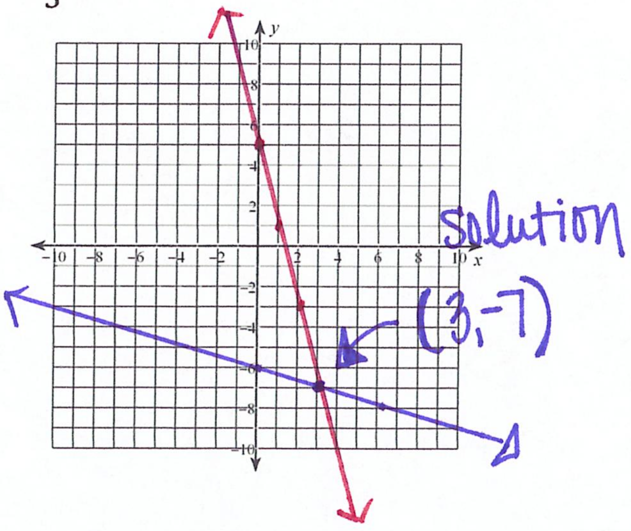
$3x = -30$
$x = -10$

$5y = -30$
$y = -6$

2. $y = -4x + 5$
   $y = -\frac{1}{3}x - 6$



Solution $(-5, -3)$



Solution $(3, -7)$

❖ Substitution Method

- ➢ Replace (or substitute) the variable in one equation with its algebraic expression in terms of the other variable from the other equation and solve
- ➢ Substitute this value into either function rule to determine the corresponding variable's value

①

$$3x - 7y = -14$$
$$x = 2y - 3$$

↑

Notice that one equation is solved for X...

Let's stick that X blob into the other equation in place of X:

$$3x - 7y = -14$$
$$x = \boxed{2y-3}$$

② This gives us

$$3(2y-3) - 7y = -14$$ Solve for y

$$6y - 9 - 7y = -14$$
$$-y - 9 = -14$$
$$+9 \quad +9$$
$$-y = -5$$
$$\boxed{y = 5}$$

③ OK, we've got y... Now, we need X... See the circled blob above?

Stick it in there! (That's why I circled it!)

$$x = \boxed{2y-3} \qquad y = \boxed{5}$$

$$x = 2(5) - 3 = 7$$
$$\boxed{x = 7}$$

EXAMPLES:

3. $y = -4x + 9$
$8x + 7y = 3$

$$8x + 7(-4x+9) = 3$$
$$8x - 28x + 63 = 3$$
$$-20x + 63 = 3$$
$$-20x = -60$$
$$x = 3$$

$x = 3$

therefore $y = -4(3) + 9$

$y = -12 + 9$

$y = -3$

solution: $(3, -3)$

❖ Addition/Elimination Method

$$2x + 3y = 20$$
$$-2x + y = 4$$

↑

See how these guys are the same, but with a different sign?

If we add the two equations -- straight down, those X critters are going to drop right out!

Just add "like terms" and drag the "=" down to:

$$
\begin{array}{r}
2x + 3y = 20 \\
+ \quad -2x + y = 4 \\
\hline
0 + 4y = 24 \\
4y = 24 \\
\boxed{y = 6}
\end{array}
$$

We've got one of them... Now, we just need to get the X. To do this, you can stick the y into either of the original equations...

The second equation is easier:

$$y = \boxed{6}$$
$$-2x + y = 4$$
$$-2x + 6 = 4$$
$$-2x = -2$$
$$\boxed{x = 1}$$

It looks like the answer is (1, 6).

Check it! (In BOTH equations!)

DOES ADDITION WORK?

Does adding the equations together result in the elimination of one of the variables?

EXAMPLES:

4.
$$\begin{array}{r} -5x + 3y = 33 \\ + \quad 4x - 3y = -21 \\ \hline -x = 12 \\ x = -12 \end{array}$$

$$\begin{array}{r} 4(-12) - 3y = -21 \\ -48 - 3y = -21 \\ -3y = -27 \\ y = 9 \end{array}$$

Solution $(-12, 9)$

IF NOT, DOES SUBTRACTION WORK?

Does subtracting the equations result in the elimination of one of the variables?

5.
$$\begin{array}{r} 9x - 8y = -26 \\ - \quad 9x + 9y = +36 \\ \hline y = 10 \end{array}$$

$$\begin{array}{r} 9x - 8(10) = -26 \\ 9x - 80 = -26 \\ 9x = 54 \\ x = 6 \end{array}$$

Solution: $(6, 10)$

❖ Elimination Method

- Multiplying one equation is necessary

Look at this one:

$$\begin{array}{r} 3x - 4y = -5 \\ 5x - 2y = -6 \end{array}$$

If we just add straight down, nothing's going to drop out and we'll just get a mess.

So, let's do it! Remember that we can multiply an equation by a number... So, let's multiply the second equation by a -2:

$$\begin{array}{r} 3x - 4y = -5 \\ -2(5x - 2y = -6) \end{array} \rightarrow \begin{array}{r} 3x - 4y = -5 \\ -10x + 4y = 12 \; + \\ \hline -7x + 0 = 7 \\ -7x = 7 \\ \boxed{x = -1} \end{array}$$

Remember to hit each guy!

Now, stick the x guy into either of the original equations. I'm going to go for the first one:

$$x = \textcircled{-1}$$

$$\begin{array}{r} 3x - 4y = -5 \\ 3(-1) - 4y = -5 \\ -3 - 4y = -5 \\ -4y = -2 \\ \boxed{y = \frac{1}{2}} \end{array}$$

The answer is $(-1, \frac{1}{2})$

Check it - and don't let that fraction freak you... These things happen!

- Multiplying both equations is necessary

Sometimes, you'll have to make adjustments to both equations to get something to drop out. When possible, always go after the easier numbers!

Let's do this one:

$$\begin{array}{r} 2x - 9y = 8 \\ -5x + 8y = -20 \end{array}$$

use opposite x's

These numbers are easier than the -9 and 8.

We want to make these 10x and -10x:

$$\begin{array}{r} 5(2x - 9y = 8) \\ 2(-5x + 8y = -20) \end{array} \rightarrow \begin{array}{r} 10x - 45y = 40 \\ -10x + 16y = -40 \\ \hline -29y = 0 \\ \boxed{y = 0} \end{array}$$

Remember to hit each guy!

*** It's easy to forget the last guys.**

Let's stick y = 0 into the first equation:

$$y = \textcircled{0}$$

$$\begin{array}{r} 2x - 9y = 8 \\ 2x - 9(0) = 8 \\ 2x = 8 \\ \boxed{x = 4} \end{array}$$

The answer is $(4, 0)$.

EXAMPLE:

6. $\begin{pmatrix}-4x - 3y = -40\end{pmatrix} \cdot 5$
$\begin{pmatrix}5x + 13y = 13\end{pmatrix} \cdot 4$

$$\begin{array}{r} -20x - 15y = -200 \\ + \quad 20x + 52y = 52 \\ \hline 37y = -148 \\ y = -4 \end{array}$$

$$5x + 13(-4) = 13$$
$$5x - 52 = 13$$
$$5x = 65$$
$$x = 13$$

Solution $(13, -4)$

2.4.D2 – Modeling w/Systems of Linear Equations

OBJECTIVES:

- Determine the solution to a system of equations and interpret the real-world meaning of the results
- Use the substitution and elimination methods to solve linear systems that model real-world scenarios

SLOPE-INTERCEPT FORM: $y = mx + b$

y = (RATE OF CHANGE) x + *initial value*

STANDARD FORM: $Ax + By = C$

combination = TOTAL NUMERICAL AMOUNT

EXAMPLES:

Define variables and write a system of equations to represent each situation.

1. Stella is trying to choose between two rental car companies. Speedy Trip Rental Cars charges a base fee of \$24 plus an additional fee of \$0.05 per mile. Wheels Deals Rental Cars charges a base fee of \$30 plus an additional fee of \$0.03 per mile.

Let x = miles *& y =* Rental cost

Equation 1: $y = 24 + 0.05x$ *& Equation 2:* $y = 30 + 0.03x$

2. Marcus is selling t-shirts at a fair. He brings 200 shirts to sell. He has long-sleeve and short-sleeved t-shirts for sale. On the first day he sells $^1/_2$ of his long-sleeved t-shirts and $^1/_3$ of his short-sleeved t-shirts for a total of 80 t-shirts sold.

Let x = long-seeeve t-shirts *& y =* short-sleeve shirts

Equation 1: $1/2x + 1/3y = 80$ *& Equation 2:* $x + y = 200$

3. Technology is now promising to bring light, fast, and beautiful wheelchairs to millions of people with disabilities. A company is planning to manufacture these radically different wheelchairs. Fixed costs will be \$500,000 and it will cost \$400 to produce each wheelchair. Each wheelchair will be sold of \$600. Determine the break-even point.

   a. Define variables and write a system of equations to represent the situation:

   Let x = the number of wheel chairs.

   Cost function: C = 500000 + 400X

   & Revenue function: R = 600X

Revenue & Cost Functions

A company produces and sells x units of a product.

REVENUE FUNCTION:

$$R(x) = (\text{unit price})x$$

COST FUNCTION:

$$C(x) = \text{fixed cost} + (\text{unit cost})x$$

500000 400

   b. Solve the system of equations:

BReak even: R=C

$600X = 500000 + 400X$

$200X = 500000$

$X = 2500$ $y = 1500000$

   c. Interpret the solution of the linear system in terms of the problem situation.

the company will break even if 2500 wheel chairs are sold. Revenue = cost = \$1500000

   d. Determine the profit function for the wheelchair business.

$P(x) = R(x) - C(x)$

$= 600X - (500000 + 400X)$

$P(x) = 200X - 500000$

The Profit Function

The profit generated after producing and selling x units of a product is given by the **PROFIT FUNCTION:**

$$P(x) = R(x) - C(x)$$

4. A chemist working on a flu vaccine needs to mix a 10% sodium-iodine solution with a 60% sodium-iodine solution to obtain 50 milliliters of a 30% sodium-iodine solution. How many milliliters of the 10% solutions and of the 60% solution should be mixed?

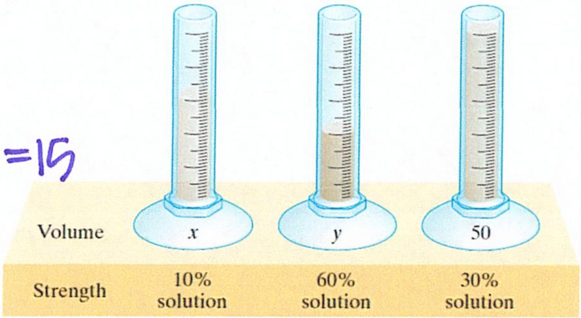
   a. Define variables and write a system of equations to represent the situation:

   Let x = mL of 10% solution

   & y = mL of 60% solution

   Equation 1: $0.10X + 0.60y = \boxed{0.30(50)}$ ← =15

   & Equation 2: $X + y = 50$ (mL)



   b. Solve the system of equations:

$10(0.10X + 0.60y = 15) \Rightarrow 1x + 6y = 150$

$-1(X + y = 50) \Rightarrow -X - y = -50$

$+$

$5y = 100$

$y = 20$ mL of 60% solution

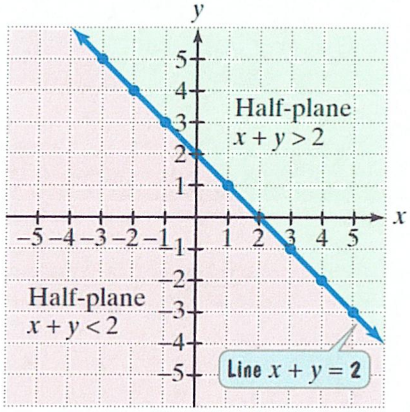
$X + 20 = 50$

$X = 30$ mL of 10% solution

2.5D1 – Linear Inequalities

OBJECTIVES:

- Graph linear inequalities given in slope-intercept or standard form
- Determine the solution region of a system of linear inequalities



- ❖ Linear Inequalities
  - ➢ The graph of a linear inequality is the set of all points whose coordinates satisfy the inequality.
  - ➢ The boundary line divides the coordinate plane into three sets: two half-planes and the line.
    - ▪ A half-plane is the graph of a linear inequality that involves **< or >** and the BOUNDARY LINE IS DASHED.
    - ▪ The graph of a linear inequality that involves **≤ or ≥** is a half-plane and a line. The BOUNDARY LINE IS SOLID as it IS part of the solution set.

❖ Graphing a Linear Inequality

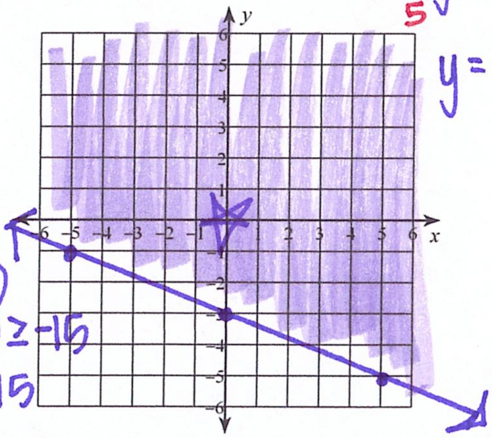
- ➢ Determine whether the boundary should be solid or dashed. Graph the boundary line.
- ➢ Select a point, not on the boundary line, and test it in the inequality. Substitute it into the linear inequality and simplify.
  - ▪ If the simplified statement is true, the point and all other points on the same side of the boundary line are in the solution region.
  - ▪ If the statement is false, all points on the opposite side of the boundary line are in the solution region.
- ➢ Shade the solution region.

EXAMPLE: GRAPHING A LINEAR INEQUALITY

1. $2x + 5y \geq -15$

$$\frac{5y}{5} = \frac{-2x}{5} - \frac{15}{5}$$

$$y = \frac{-2}{5}x - 3$$

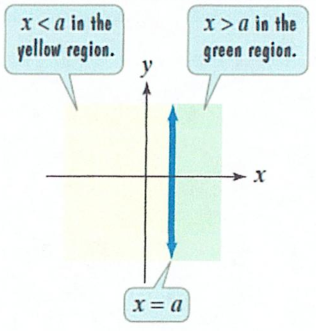


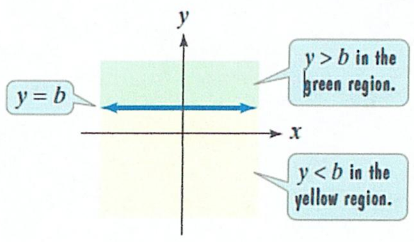
Test (0,0)

$2(0) + 5(0) \geq -15$

TRUE $0 \geq -15$

For the Vertical Line $x = a$:	For the Horizontal Line $y = b$:
• If $x > a$, shade the half-plane to the right of $x = a$.	• If $y > b$, shade the half-plane above $y = b$.
• If $x < a$, shade the half-plane to the left of $x = a$.	• If $y < b$, shade the half-plane below $y = b$.

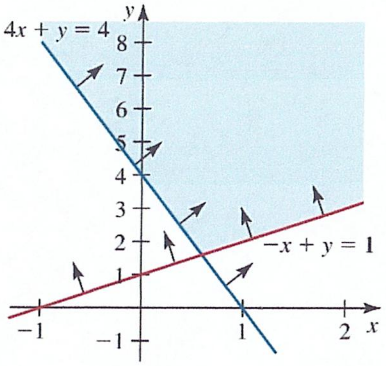




❖ Systems of Linear Inequalities

- ➢ Graph each linear inequality following the steps above.
- ➢ The Solution Region
  - ▪ The intersection of the solution regions of the individual inequalities.
  - ▪ Place arrows on the boundary lines to indicate which side of the line satisfies each inequality.
  - ▪ Once all the inequality graphs have been drawn, we shade the region that has arrows from all sides pointing into the interior of the region.
- ➢ *It is possible that two regions do not intersect. In such cases, no solution exists.*

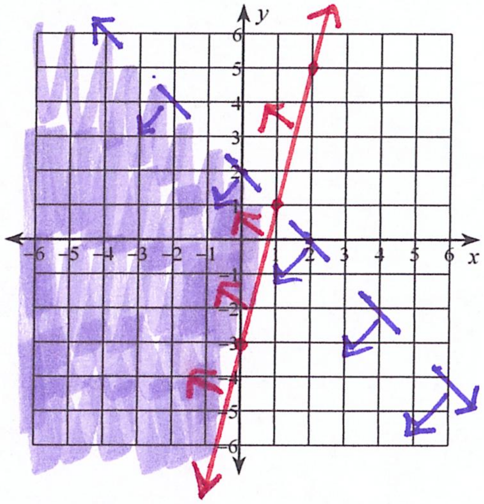
Example:
$4x + y \geq 4 \ \& \ -x + y \geq 1$



EXAMPLES: GRAPHING A SYSTEM OF LINEAR INEQUALITIES

2. $x + y < +2$ → $0 < 2$ TRUE
$y \geq 4x - 3$

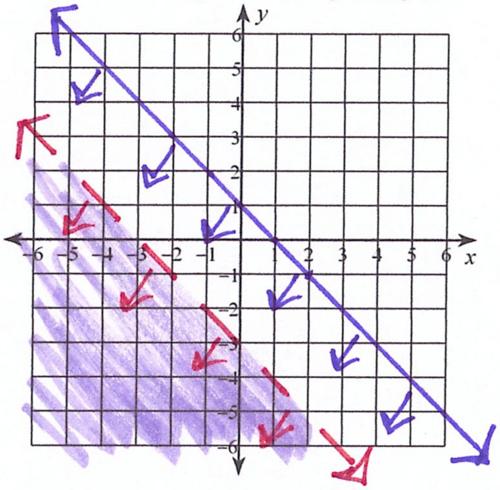
$0 \geq -3$
TRUE



3. $x + y \leq 1$ $0 \leq 1$ TRUE
$2x + 2y < -6$ $0 < -6$ False

$2x = -6$
$x = -3$

$2y = -6$
$y = -3$



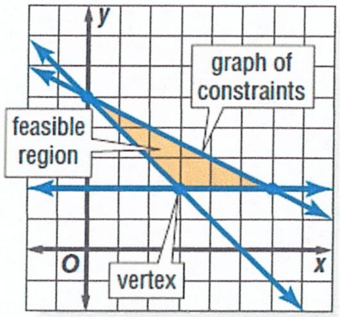
2.5D2 – Systems of Linear Inequalities

OBJECTIVES:

- **Graph linear inequalities given in slope-intercept or standard form**
- **Determine the corner points of a solution region of a system of linear inequalities**

❖ Solution Region

- ➢ The inequalities are called the constraints.
- ➢ The intersection of the graphs is called the feasible region.
- ➢ Corner Point/Vertex
  - ▪ The points of intersection of the boundary lines of a system of linear inequalities bordering the shaded solution region are called corner points or vertices.



❖ Maximum & Minimum Values

- ➢ The maximum or minimum value of a related function *always* occurs at one of the vertices of the feasible region.

EXAMPLES: GRAPHING A SYSTEM OF LINEAR INEQUALITIES

Graph the solution region to the system of linear inequalities. Find the coordinates of the corner points.

1. $x \le 3$
   $y < 2x + 1$
   $x + 3y \ge -18$

$0 < 1$ True

$0 \ge -18$ True

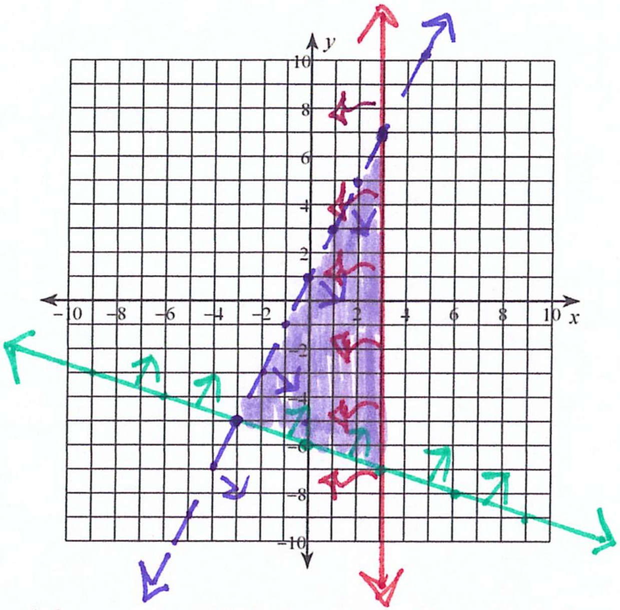
$3y = -x - 18$

$y = -\frac{1}{3}x - 6$

corner points

$(-3,-5)$

$(3,7)$ $(3,-7)$



EXAMPLE: FINDING MAXIMUM & MINIMUM VALUES

Graph the system of linear inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

2. $y \le x + 6$
   $1 \le y \le 5$
   $x + 2y \le 12$
   $f(x, y) = 3x + y$

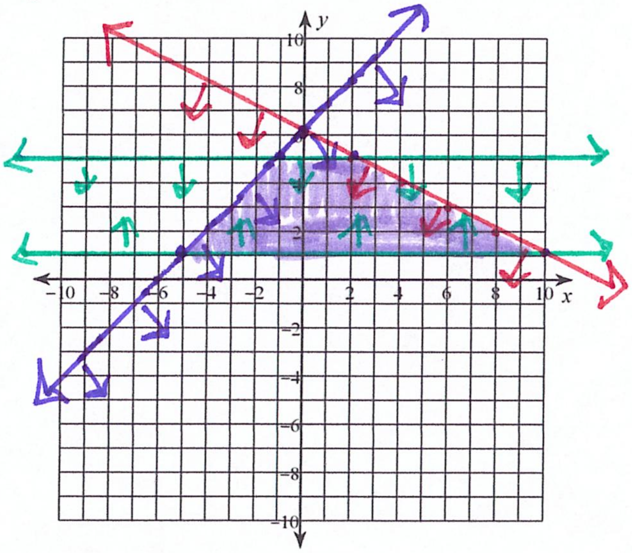
$0 \le 6$ True

$0 \le 12$ True

$\frac{2y}{2} = \frac{-x}{2} + \frac{12}{2}$

$y = -\frac{1}{2}x + 6$

(x,y)	$3x+y$
$(-5,1)$	$3(-5)+1 = -14$ min @ $(-5,1)$
$(-1,5)$	$3(-1)+5 = 2$
$(2,5)$	$3(2)+5 = 11$
$(10,1)$	$3(10)+1 = 31$ max @ $(10,1)$



2.5.D3 – Linear Programming

OBJECTIVES:

- Graph linear inequalities given in slope-intercept or standard form
- Determine the corner points of a solution region of a system of linear inequalities
- Explain the practical meaning of solutions of linear inequalities in real-world contexts

❖ Writing Inequalities

$<$	$\le$	$>$	$\ge$
Less than Fewer than	Less than or equal to At most No greater than As much as No more than	Greater than More than	Greater than or equal to At least No less than As little as No fewer than

EXAMPLE #1

An entrance exam has two sections: a verbal section and a mathematics section. You can score a maximum of 1600 points. For admission, the school of your choice requires a math score of at least 600. Write a system of inequalities to model scores that meet the school's requirements

a. Define the variables: Let x = the verbal score

Let y = the math score

b. Write a system of inequalities.

	x	y	Total
Givens	$x \geq 0$	$y \geq 0$	
Points	$x +$	y	≤ 1600
Math requirement		y	≥ 600

EXAMPLE #2

A student earns $8.00 per hour working fast food and $15.00 per hour babysitting. She has at most 20 hours per week to work and needs to earn at least $255.

a. Define the variables: Let x = number of hours working fast food

Let y = number of hours babysitting

b. Write a system of inequalities.

	x	y	Total
Givens	$x \geq 0$	$y \geq 0$	
Hours	$x +$	y	≤ 20
Earnings	$8x +$	$15y$	≥ 255

c. Use Desmos to graph the region showing all possible work-hour allocations that meet her time and income requirements. (on last page)

d. Give an example of a possible combination of hours that is a solution to this system.

(2, 17) babysitting

↑ Fast food

❖ **Linear Programming**

- ➢ The process of finding maximum or minimum values of a function for a region defined by inequalities is called <u>linear programming</u>.
- ➢ Linear programming can be used to solve many types of real-world problems. These problems have certain restrictions placed on the variables, and some function of the variable must be maximized or minimized.

❖ Linear Programming Procedure

- ➢ Define the variables.
- ➢ Write a system of inequalities.
- ➢ Graph the system of inequalities.
- ➢ Find the coordinates of the vertices of the feasible region.
- ➢ Write a function to be maximized or minimized.
- ➢ Substitute the coordinates of the vertices into the function.
- ➢ Select the greatest or least result; answer the problem.

EXAMPLE #3

Cho requires 1 hour of cutting and 2 hours of sewing to make a Batman costume. He requires 2 hours of cutting and 1 hours of sewing to make a Wonder Woman costume. At most 10 hours per day are available for cutting and at most 8 hours per day are available for sewing. At least one costume must be made each day to stay in business. Find Cho's maximum income from selling one day's costumes if a Batman costume profits $68 and a Wonder Woman costume profits $76.

a. Define the variables: Let x = number of Batman costumes

Let y = number of Wonder Woman costumes

b. Write a system of inequalities and a function to be maximized or minimized.

	x	y	Total
Givens	$x \geq 0$	$y \geq 0$	
Cutting time	1x +	2y	≤ 10
Sewing time	2x +	1x	≤ 8
Costumes	x +	y	≥ 1
Income	68x +	76y	

c. Use Desmos to graph the system of inequalities.

(on the last page)

d. Find the coordinates of the vertices of the feasible region and then substitute the coordinates of the vertices into the function to be maximized/minimized.

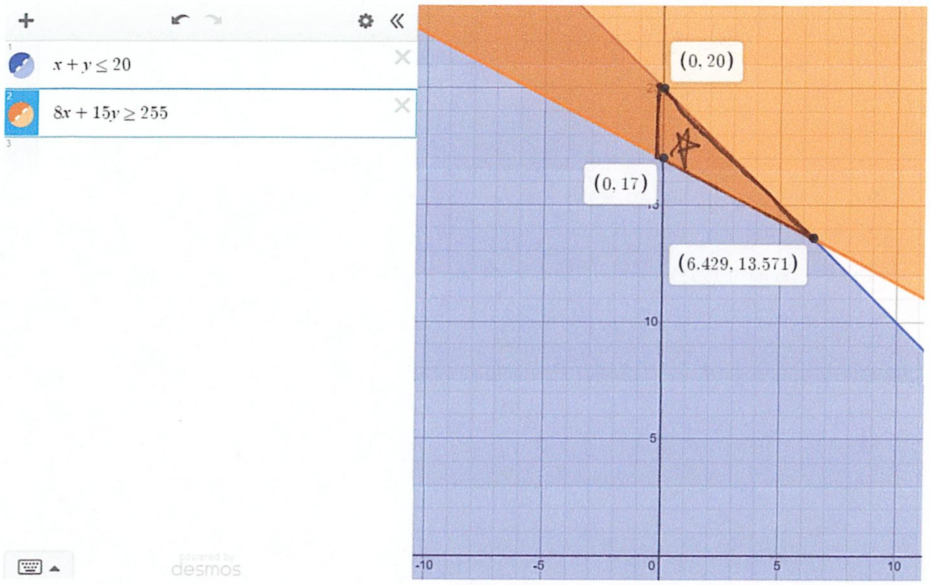
68x+76y

(x, y)	$f(x, y)$
(0,5)	380
(0,1)	76
(1,0)	68
(4,0)	272
(2,4)	440

e. Select the greatest or least result; answer the problem and interpret the solution in context of the problem situation.

max income of $440
for 2 Batman & 4 wonder Woman
costumes

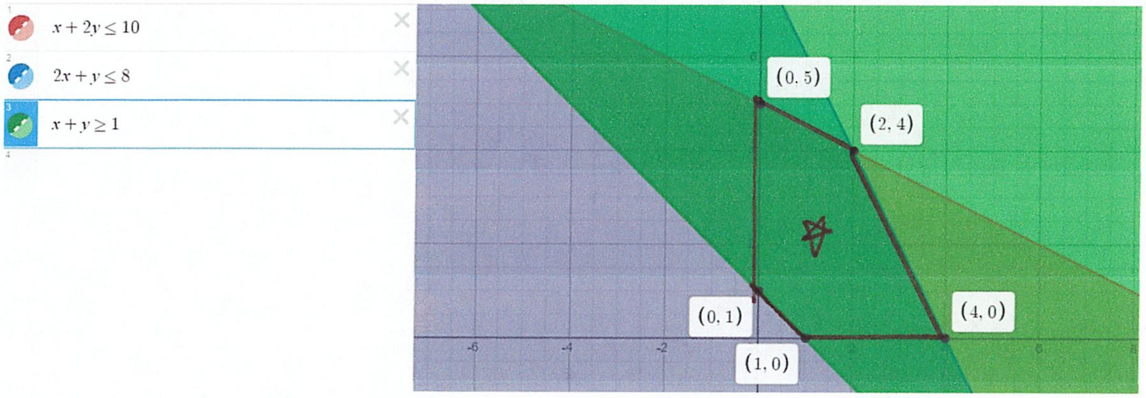
2.5.D3 Example #2



1 $x + y \leq 20$

2 $8x + 15y \geq 255$

2.5.D3 Example #3



1 $x + 2y \leq 10$

2 $2x + y \leq 8$

3 $x + y \geq 1$