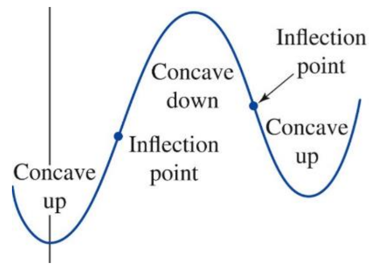
Name: ______________________________

CHAPTER 4: QUADRATIC FUNCTIONS

4.1 • Variable Rates of Change

Objectives:

- Determine if a quadratic equation represents a data set
- Determine the concavity and increasing/decreasing behavior of a function from a table



Variable Rate of Change

- Any function whose rate of change is not constant is said to have a variable rate of change.

$$\text{ROC} = \frac{y_2 - y_1}{x_2 - x_1}$$

Successive Differences of Functions

- Analyzing First Differences
  - ***LINEAR*** – constant rate of change (first differences)
  - ***CONCAVE UP*** – rate of change (first differences) increases
  - ***CONCAVE DOWN*** – rate of change (first differences) decreases
- Analyzing Second Differences
  - ***QUADRATIC*** – constant second differences

x	$R(x) = -x^2 + 40.50x$	First Differences	Second Differences
0	0		
		39.50	
1	39.50		−2
		37.50	
2	77.00		−2
		35.50	
3	112.50		

EQUALLY SPACED INPUT VALUES

CONSTANT SECOND DIFFERENCES THE FUNCTION IS QUADRATIC

OUTPUTS ARE INCREASING – R(X) IS AN INCREASING FUNCTION

THE FUNCTION IS CONCAVE DOWN BECAUSE THE 1ST DIFFERENCES ARE DECREASING

Examples: Numerical representations of a function are shown in a table. Identify whether the function is linear or quadratic; its increasing/decreasing intervals; and its concavity.

1. $f(x)$

x	$f(x)$	First Differences	Second Differences
-2	20		
		______	
-1	2		______
		______	
0	0		
1	14		

LINEAR OR QUADRATIC?

INCREASING INTERVAL: ____ $< x <$ ____

DECREASING INTERVAL: ____ $< x <$ ____

CONCAVE UP / CONCAVE DOWN / NEITHER

2. $g(x)$

x	$g(x)$	First Differences	Second Differences
-3	8		
		______	
-2	6		______
		______	
-1	4		
0	2		

LINEAR OR QUADRATIC?

INCREASING INTERVAL: ____ $< x <$ ____

DECREASING INTERVAL: ____ $< x <$ ____

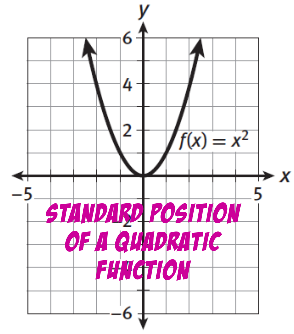
CONCAVE UP / CONCAVE DOWN / NEITHER

4.3D1 • Intro to Quadratic Functions

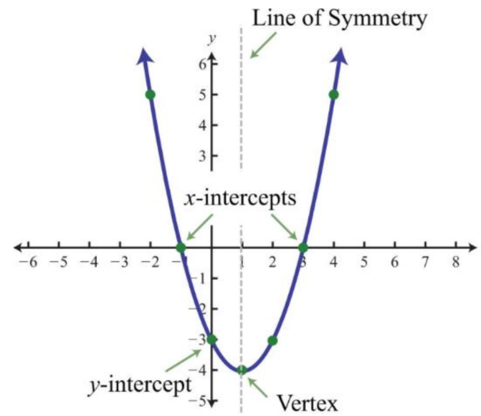
Objectives:

- ❖ Determine the properties and characteristics of a quadratic function given graphically, numerically, and algebraically

❖ Quadratic Functions



- ➢ The graph of any quadratic function is called a **PARABOLA**.
  - Its graph is either concave up ($a > 0$) or concave down ($a < 0$).
  - The **VERTEX** (h, k) is the turning point and, also, where a maximum or minimum value of the function occurs.
    - *The y-value of the vertex is the maximum or minimum of the function.*
  - The vertical line that passes through the vertex of a parabola and divides the parabola into two symmetrical parts is the **AXIS OF SYMMETRY**.
  - **Intercepts**
    - Quadratic functions may have two, one, or no **X-INTERCEPTS**. The x-intercept(s) can be found algebraically by solving $f(x) = 0$ or identified from the graph where the parabola crosses or touches the x-axis.
    - The **Y-INTERCEPT** can be found by evaluating $f(0)$ or identified from the graph where the parabola crosses the y-axis.



❖ Formulas for Quadratic Functions

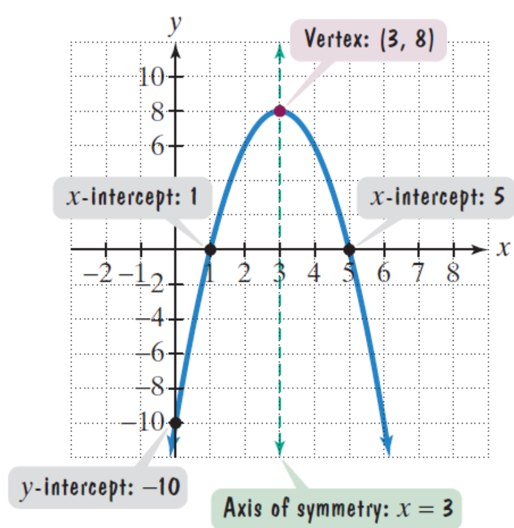
Standard Form	Factored/Intercept Form	Vertex Form
$f(x) = ax^2 + bx + c$	$f(x) = a(x - p)(x - q)$	$f(x) = a(x - h)^2 + k$

$$f(x) = -2x^2 + 12x - 10$$

$$f(x) = -2(x - 1)(x - 5)$$

$$f(x) = -2(x - 3)^2 + 8$$

x	$f(x)$
0	−10
1	0
2	6
3	8
4	6
5	0
6	−10



The same quadratic function is given graphically, numerically (table), and algebraically with a function formula in standard, factored, and vertex form. Look for the graph's key characteristics – vertex, x-intercepts, y-intercept, and axis of symmetry – in the table and function formulas. Where do you see these characteristics within the table and function formulas?

Identify the vertex, x-intercepts, y-intercept, and axis of symmetry of each quadratic function.

1.

x	$f(x)$
-8	6
-6	0
-5	-1.5
-4	-2
-3	-1.5
-2	0
0	6

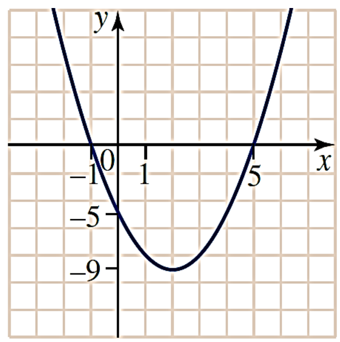
VERTEX:

X-INTERCEPTS:

Y-INTERCEPT:

AXIS OF SYMMETRY:

2.



VERTEX:

X-INTERCEPTS:

Y-INTERCEPT:

AXIS OF SYMMETRY:

SUMMARIZE: Identify the features of the graph of a quadratic function that can be determined from the quadratic function's formula given in…

STANDARD FORM $f(x) = ax^2 + bx + c$	VERTEX FORM $f(x) = a(x-h)^2 + k$	INTERCEPT FORM $f(x) = a(x-p)(x-q)$

FEATURES INCLUDE: DIRECTION OF OPENING, ZEROS/X-INTERCEPTS, Y-INTERCEPT, AXIS OF SYMMETRY & VERTEX.

<<Prep Work>> INVESTIGATION: Quadratic Function Forms

Consider the following quadratic functions: Identify the feature(s) you KNOW the graph will have based on the function formula ALONE.

$y = x^2 + 2x - 35$	$y = (x+4)^2 - 1$	$y = (x-4)(x-4)$
$y = -(x-4)^2 + 1$	$y = -x^2 - 6x - 16$	$y = -\frac{1}{2}(x-3)(x-5)$

4.3D2 • Intercepts of Quadratic Functions

Objectives:
- ❖ Determine the intercepts of a quadratic function given its equation
- ❖ Write the formula of a quadratic function given analytically, graphically, numerically, and verbally

❖ Factored/Intercept Form: $f(x) = a(x - p)(x - q)$

- ➢ The x-intercepts are $(p, 0)$ & $(q, 0)$
  - ▪ Use the **ZERO PRODUCT PROPERTY**
    - Let a and b be real numbers. If $ab = 0$, then $a = 0$ or $b = 0$.
    - Set each factor equal to 0 and solve.
- ➢ Find the y-intercept by computing $f(0)$.

Example: For the quadratic function, determine the direction of opening and find the zeros (if any) and the y-intercept.

1. $h(x) = -3(x - 2)(4x + 3)$ DIRECTION OF OPENING: ZERO(S): Y-INTERCEPT:

❖ Writing Equations of Quadratic Functions

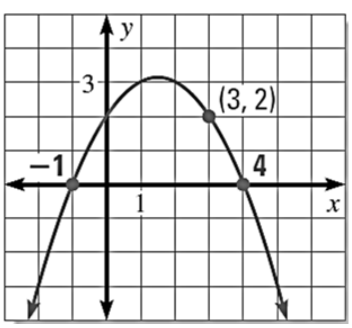
- ➢ If the x-intercepts, p and q, and one other point on the graph (x, y) of a parabola are known, you can write the equation of the parabola in factored form.
  - ▪ Substitute these values into the factored form of the equation and solve for a.

$$y = a(x - p)(x - q)$$

  - ▪ Write the equation using only a, p, and q.

Examples: Write a quadratic function, in factored form, for the parabola shown or described.

2.



3. x-intercepts are at $x = -1$ & $x = 2$ and $(-2, 16)$ is on the function's graph.

❖ Standard Form of a Quadratic Function: $f(x) = ax^2 + bx + c$

- Finding Intercepts of Quadratic Functions in Standard Form
  - Find any x-intercepts by solving $f(x) = 0$
    - Solve using the **QUADRATIC FORMULA**
- Vertical/y-intercept: $(0, c)$

$$ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Examples: For the quadratic function, find the x-intercepts (*rounded to 2 decimal places, if necessary*) & the y-intercept.

4. $f(x) = -x^2 + 8x + 22$

5. $g(x) = 3x^2 + 3x - 16$

X-INTERCEPTS: Y-INTERCEPT: X-INTERCEPTS: Y-INTERCEPT:

❖ Vertex Form of a Quadratic Function: $f(x) = a(x - h)^2 + k$

- Finding Intercepts of Quadratic Functions in Vertex Form
  - Find any x-intercepts by solving $f(x) = 0$
  - Use the **SQUARE ROOT PROPERTY**
    - Set the function equal to 0: $a(x - h)^2 + k = 0$
      Isolate the quadratic expression on one side of the equation
      Take the square root of both sides. **Don't forget the ±!**
      Solve for x and simplify.

$$(x-5)^2 = 9$$
$$x - 5 = \pm\sqrt{9}$$
$$x - 5 = \pm 3$$
$$x = 5 \pm 3$$
$$x = 8 \text{ or } x = 2$$

Examples: For the quadratic function, find the x-intercepts (*rounded to 2 decimal places, if necessary*) & the y-intercept.

6. $f(x) = 2(x + 4)^2 - 50$

7. $f(x) = -2(x + 5)^2 + 10$

X-INTERCEPTS: Y-INTERCEPT: X-INTERCEPTS: Y-INTERCEPT:

4.3D3 • Intercepts & Factoring

Objectives:

- Convert quadratic functions in standard form to intercept form and find x-intercepts
- Factor trinomials of the form $x^2 + bx + c$ and $ax^2 + bx + c$

- **Factoring a Difference of Two Squares:** $\boldsymbol{a^2 - b^2}$
  - $\boldsymbol{a^2 - b^2 = (a + b)(a - b)}$

REMEMBER TO **ALWAYS** LOOK FOR COMMON FACTORS FIRST!

Examples: Write the quadratic function in factored form.

1. $q(x) = x^2 - 64$
2. $u(x) = -2x^2 + 162$
3. $a(x) = 4x^2 - 36$

- Finding zeros via Factoring: $y = \boldsymbol{x^2 + bx + c}$
  - An example:

REMEMBER TO **ALWAYS** LOOK FOR COMMON FACTORS FIRST!

$y = x^2 + 5x - 24$

-24	
$8\ \&-3$	5

FIND THE FACTORS OF C THAT ADD UP THE TO BE THE MIDDLE TERM B

$y = (x + 8)(x - 3)$

WRITE AS A PRODUCT OF TWO BINOMIALS

$x + 8 = 0$ $\quad$ $x - 3 = 0$

$x = -8$ $\quad$ $x = 3$

USE THE ZERO PRODUCT PROPERTY TO FIND THE ZEROS.
SET EACH FACTOR EQUAL TO ZERO & SOLVE FOR X.

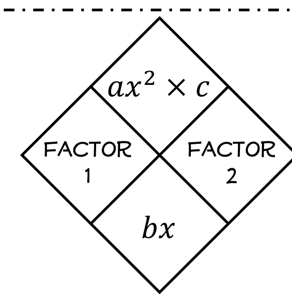
Examples: Write the quadratic function in factored form and then find its x-intercepts using the Zero Product Property.

4. $d(x) = x^2 + 8x - 20$
5. $r(x) = -x^2 - 7x - 10$
6. $t(x) = 2x^2 - 20x + 50$

- Finding zeros via Factoring: $y = \boldsymbol{ax^2 + bx + c}$
  - ***DEFOIL/A-C Method***
    - Multiply the first and last terms: $ax^2 \times c$
    - Find the factors that multiply to be the product (in step 1) and that add to be the middle term: bx
    - Replace the middle term with these factors
    - Factor by grouping

REMEMBER TO **ALWAYS** LOOK FOR COMMON FACTORS FIRST!

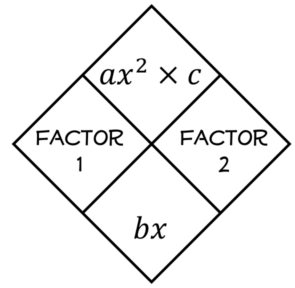
Organize this information with an X-box →



➢ THE BOX METHOD

1. Multiply the first and last terms: $ax^2 \times c$
2. Find the factors that multiply to be the product (in step 1) and that add to be the middle term: bx

Organize this information with an X-box →



3. Draw a 2×2 square
4. Put the first term of the trinomial ax^2 in the upper-left corner and the constant term, c, in the lower-right corner.
5. Put the factors (from step 2) in the two remaining squares.
6. Find the GCF of each row & each column
7. Write the result as a product of two binomials.

ax^2	FACTOR 1	GCF of row 1
FACTOR 2	c	GCF of row 2
GCF of column 1	GCF of column 2	

AN EXAMPLE: $3x^2 - 4x - 7$

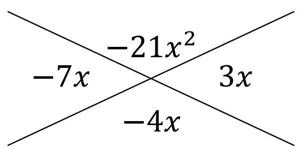
BOX METHOD

$3x^2$	$-7x$	x
$3x$	-7	1
$3x$	-7	

$(3x - 7)(x + 1)$

1st 2 steps are the same

1. MULTIPLY THE FIRST & LAST TERMS
2. FIND THE FACTORS (OF THE PRODUCT IN STEP 1) THAT ADD UP THE TO BE THE MIDDLE TERM



DEFOIL/A-C METHOD

3. REPLACE THE MIDDLE TERM WITH THESE FACTORS
4. FACTOR BY GROUPING

$$3x^2 - 7x + 3x - 7$$
$$(3x^2 - 7x)(+3x - 7)$$
$$x(3x - 7) + 1(3x - 7)$$
$$(3x - 7)(x + 1)$$

$$3x - 7 = 0$$
$$x = \frac{7}{3}$$

$$x + 1 = 0$$
$$x = -1$$

USE THE ZERO PRODUCT PROPERTY TO FIND THE ZEROS. SET EACH FACTOR EQUAL TO ZERO & SOLVE FOR X.

REMEMBER TO **ALWAYS** LOOK FOR COMMON FACTORS FIRST!

Examples: Write the quadratic function in factored form and then find its x-intercepts using the Zero Product Property.

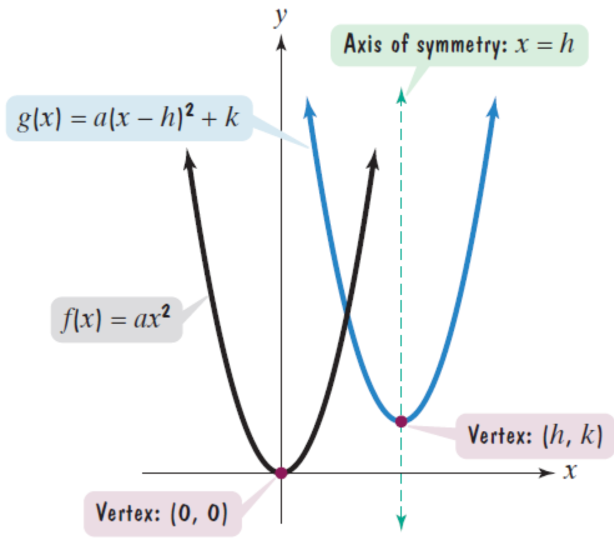
7. $q(x) = 2x^2 + 13x + 21$

8. $c(x) = 6x^2 - 21x + 9$

4.3D4 • The Vertex of a Quadratic Function

Objectives:
- Determine the properties and characteristics of a quadratic function given its equation in vertex form
- Write equations of quadratic functions using its vertex and a point on its graph

Vertex Form of a Quadratic Function: $f(x) = a(x - h)^2 + k$, where $a \neq 0$



$$f(x) = a(x - h)^2 + k$$

VERTEX: (h, k)

AXIS OF SYMMETRY: $x = h$

Y-INTERCEPT: evaluate $f(0)$

DIRECTION OF OPENING/MAX OR MIN/RANGE:

$a > 0$	$a < 0$
Opens up	Opens down
Min of k @ $x = h$	Max of k @ $x = h$
range = $[k, \infty)$	range = $(-\infty, k]$

Example 1: Consider the quadratic function $f(x) = -2(x + 5)^2 + 10$.

a. What is the function's axis of symmetry?

b. What is the function's vertex?

c. What is the function's range?

d. Does the function have a maximum or minimum value? What is that value and where is it located?

e. On what interval is the function increasing?

f. On what interval is the function decreasing?

Writing Equations of Quadratic Functions

- If the vertex (h, k) and one other point on the graph (x, y) of a parabola are known, you can write the equation of the parabola in vertex form.
  - Substitute these values into the vertex form of the equation and solve for a.

$$y = a(x - h)^2 + k$$

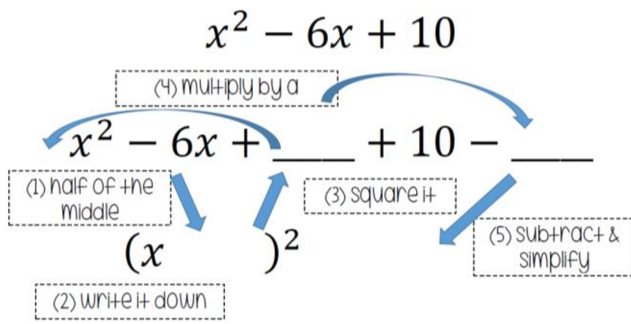
  - Write the equation using only a, h, and k.

Example 2: Write an equation for the parabola whose vertex is at $(-1, 4)$ and passes through $(2, 1)$.

❖ Standard Form of a Quadratic Function

- ➢ Complete the square to write the function in its corresponding vertex form.

❖ Standard Form → Vertex Form: Completing the Square ($a = 1$)



Getting Ready:
Sort/organize leaving a positive blank after the linear term and a negative blank after the constant term AND set up the binomial square.

Completing the Square:
1. Half the middle
2. Write it down
3. Square it
4. Multiply by a
5. Subtract & simplify

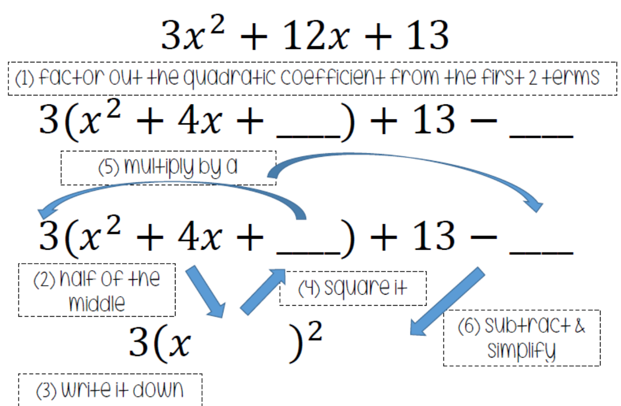
Example: Write the quadratic function in vertex form. Then determine the following characteristics: the direction in which the parabola opens, the coordinates of the vertex, the y-intercept, and the range.

3. $f(x) = x^2 + 18x + 84$

DIRECTION OF OPENING: VERTEX:

Y-INTERCEPT: RANGE:

❖ Standard Form → Vertex Form: Completing the Square ($a \neq 1$)



Getting Ready:
Factor out the quadratic coefficient from the first two terms.
Sort/organize leaving a positive blank after the linear term and a negative blank after the constant term AND set up the binomial square.

Completing the Square:
1. Half the middle
2. Write it down
3. Square it
4. Multiply by a
5. Subtract & simplify

Examples: Write the quadratic function in vertex form. Then determine the vertex and the range.

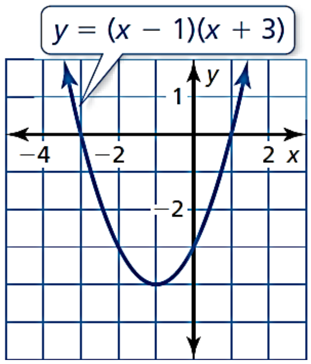
4. $f(x) = 2x^2 - 16x + 27$

5. $g(x) = -4x^2 + 8x + 2$

VERTEX: RANGE: VERTEX: RANGE:

❖ Factored/Intercept Form of a Quadratic Function

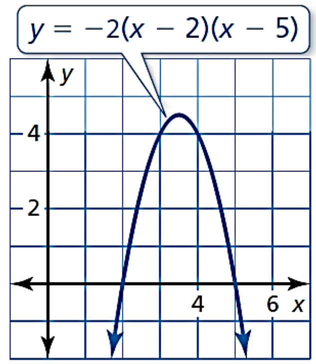
➢ Identify the *x*-intercepts, the coordinates of the vertex and the equation of the axis of symmetry from the graph.



X-INTERCEPTS:

VERTEX:

AXIS OF SYMMETRY:



X-INTERCEPTS:

VERTEX:

AXIS OF SYMMETRY:

Summarize your findings: How can we find the axis of symmetry and vertex of a parabola if we know it's x-intercepts: p & q?

Examples: Determine the following characteristics: the direction in which the parabola opens, the equation of the axis of symmetry, the coordinates of the vertex, and the range.

6. $g(x) = 5(x + 1)(x + 2)$

Direction of opening:

Axis of symmetry:

Vertex:

Range:

7. $h(x) = -4(x - 7)(x - 3)$

Direction of opening:

Axis of symmetry:

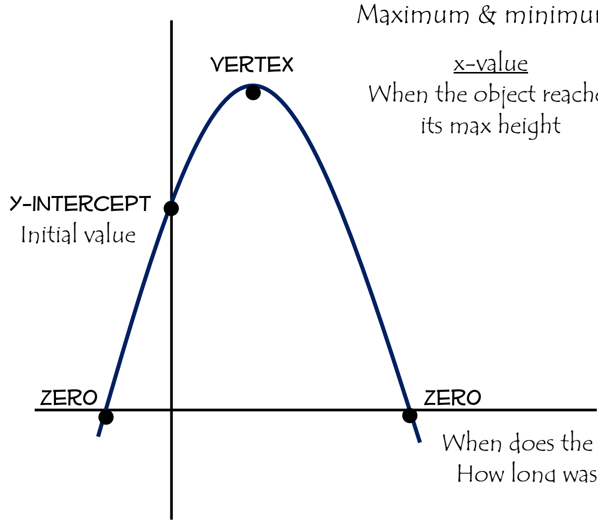
Vertex:

Range:

4.2 • Modeling with Quadratic Functions

Objective:

- Construct and use quadratic models to predict unknown results and interpret these findings in a real-world context



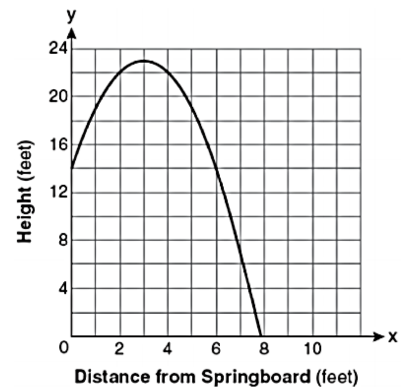
Maximum & minimum values occur @ the vertex:

x-value	y-value
When the object reaches its max height	The maximum height of the object

When does the object hit the ground?
How long was the object in the air?

Examples:

1. A swim team member performs a dive from a 14-foot-high springboard. The quadratic function $h = -x^2 + 6x + 14$ and the parabola (at right) shows the path of her dive.



a. What is the diver's maximum height? At what distance from the springboard does the diver reach that height?

What parabolic property would you need to identify?

x-intercepts y-intercept vertex

How would you identify this feature, algebraically, using the function formula?

Answer the problem:

b. What is the diver's horizontal distance from the springboard when she hits the surface of the water?

What parabolic property would you need to identify?

x-intercepts y-intercept vertex

How would you identify this feature, algebraically, using the function formula?

Answer the problem:

c. Identify a reasonable domain – the x-values that make sense for the given scenario.

What parabolic property would you need to identify?

x-intercepts y-intercept vertex

Answer the problem:

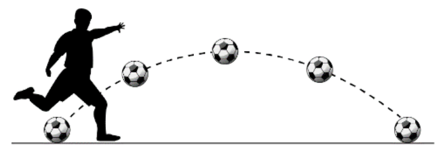
d. Identify a reasonable range – the y-values that make sense for the given scenario.

What parabolic property would you need to identify?

x-intercepts y-intercept vertex

Answer the problem:

❖ Vertical Motion Problems



- Height of a **DROPPED OBJECT**: $h = -16t^2 + h_0$
  - h_0 is the initial height
- Height of an **OBJECT LAUNCHED OR THROWN:** $h = -16t^2 + v_0 t + h_0$
  - v_0 is the initial velocity

Examples:

2. A soccer player passes the ball to a teammate with an initial velocity of 16 feet per second, and the teammate kicks the ball from a height of 4 feet.

   a. Write a function that represents the height of the ball (in feet) t seconds after it is kicked.

   b. Calculate the time at which the ball reaches its maximum height.

   c. What is the maximum height of the ball?

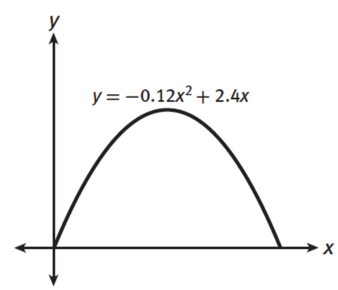
   d. Assuming no one touches the ball after it is kicked, determine the time when the ball falls to the ground.

   e. Identify a reasonable domain for the function.

   f. Identify a reasonable range for the function.

   g. Determine the times when the ball is higher than 6 feet.

3. An architect is designing a tunnel and is using the function $y = -0.12x^2 + 2.4x$ to determine the shape of the tunnel's entrance, as shown in the figure. In this model, y is the height of the entrance (in feet) and x is the distance (in feet) from one end of the entrance.



   a. How wide is the tunnel's entrance at its base?

   b. What is the vertex? What does it represent?

   c. Could a truck that is 14 feet tall pass through the tunnel? Explain.