CHAPTER 6 – EXPONENTIAL & LOGARITHMIC FUNCTIONS

6.1.D1 ~ Exponential Functions & Their Graphs

OBJECTIVES

- Demonstrate an understanding of the characteristics, behaviors, formulas, graphs, and applications of exponential functions
- Identify the properties of the graph of an exponential function
- Describe the effects of parameters in the formula for an exponential function

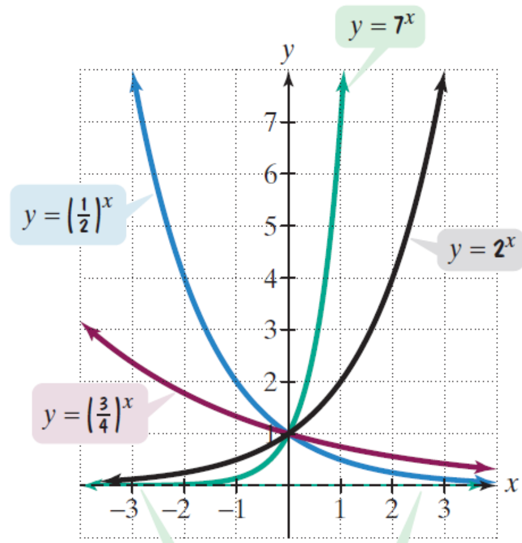
Exploration:

The graphs of four exponential functions are graphed.

Compare the graphs of those functions with $b > 1$ to those with $0 < b < 1$.

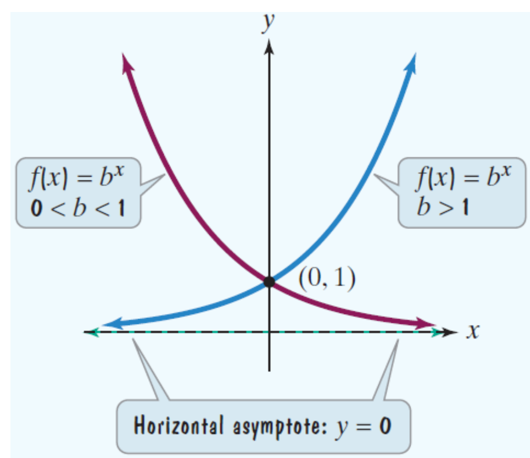
How are they similar?

How are they different?



❖ **Graphs of Exponential Functions**

➢ $y = a(b)^x$
- Domain: all real numbers
- Range: $y > 0$
- Vertical/y-intercept: $(0, a)$ or evaluate: $f(0)$
- If $b > 1$, b is the growth factor
  - The function is increasing
- If $0 < b < 1$, b is the decay factor
  - The function is decreasing
- There is NO horizontal/x-intercept
- Horizontal asymptote: $y = 0$



❖ The Effect of the Parameter a

➢ Tells us where the graph crosses the y-axis: the y-intercept

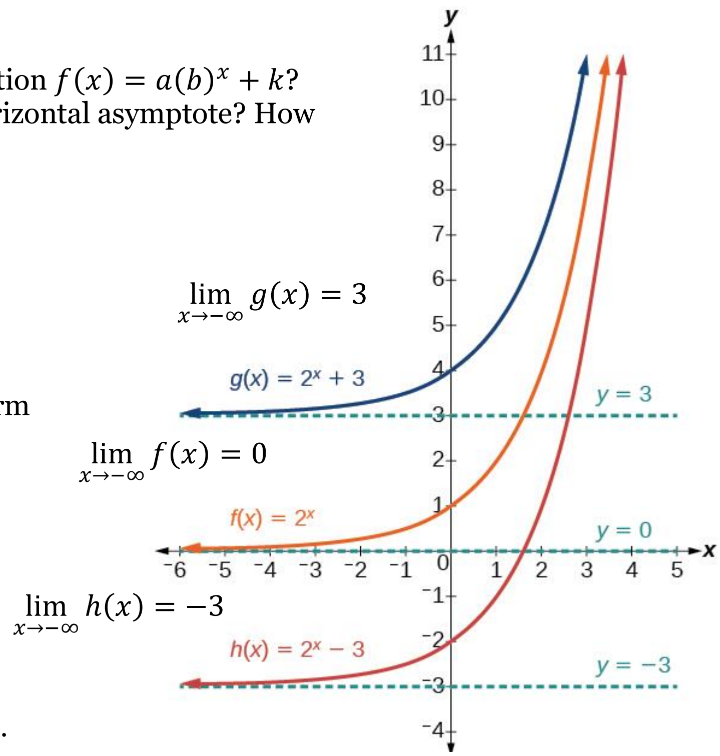
❖ The Effect of the Parameter b

➢ If $b > 1$, the graph increases from left to right
- The greater the value of b, the more rapidly the graph rises

➢ If $0 < b < 1$, the graph decreases from left to right
- The smaller the value of b, the more rapidly the graph falls

Think About It?

- What happens to the graph of an exponential function $f(x) = a(b)^x + k$? What changes: domain, range, the y-intercept, horizontal asymptote? How would it change?



❖ Vertical Intercept

- ➢ How would you find the vertical intercept of an exponential function with an equation of the form $f(x) = a(b)^x + k$?

❖ Horizontal Asymptote

- ➢ The horizontal line $y = k$ is a horizontal asymptote of a function, $f(x) = a(b)^x + k$
  - ▪ The horizontal asymptote also is used to describe the range of the exponential function.

❖ End Behavior

- ➢ Exponential Growth
  - ▪ $\lim_{x\to-\infty} f(x) = k$ & $\lim_{x\to\infty} f(x) = \infty$
- ➢ Exponential Decay
  - ▪ $\lim_{x\to-\infty} f(x) = \infty$ & $\lim_{x\to\infty} f(x) = k$

Examples:

1. $Q(t) = 6(1.03)^t$
   a. What is the vertical intercept of the graph?
   b. Is the graph of Q increasing or decreasing?
   c. What is the equation of the horizontal asymptote of the graph?
   d. $\lim_{t\to-\infty} Q(t) =$
   e. $\lim_{t\to\infty} Q(t) =$
   f. Range:

2. $Q(t) = 6(0.97)^t + 3$
   a. What is the vertical intercept of the graph?
   b. Is the graph of Q increasing or decreasing?
   c. What is the equation of the horizontal asymptote of the graph?
   d. $\lim_{t\to-\infty} Q(t) =$
   e. $\lim_{t\to\infty} Q(t) =$
   f. Range:

6.1.D2 ~ Exponential Functions

OBJECTIVES

- Demonstrate an understanding of the characteristics, behaviors, formulas, graphs, and applications of exponential functions
- Construct exponential models algebraically from tables
- Determine the growth or decay factor of an exponential function

❖ Exponential Functions

- Functions whose equations contain a **VARIABLE IN THE EXPONENT**.
- The exponential function f with base b is defined by $\boldsymbol{f(x) = a(b)^x}$ **or** $\boldsymbol{y = a(b)^x}$ where a is nonzero, $b > 0$ & $b \neq 1$, and x is any real number.
  - a is the **INITIAL VALUE** of f (the value at $x = 0$), & b is the *base*, aka the **CHANGE FACTOR**
    - If $b > 1$, b is the growth factor
    - If $0 < b < 1$, b is the decay factor

Examples:

1. The balance $B(t)$, in dollars, of an investment account is defined by $B(t) = 5500(1.12)^t$, where t is the number of years.
   a. What is the initial balance of the investment account?
   b. Identify b. Is it a growth factor or a decay factor?
   c. What is the balance of the investment account after 10 years?

❖ **Finding an Exponential Function from its Table of Values**

- In an exponential function equally spaced input values yield output values whose successive ratios are constant.
  - In each case, the **CHANGE FACTOR** is the base, b, of the exponential function.
    - Change factor = what you multiply by

INPUT	OUTPUT
a	$f(a)$
b	$f(b)$

$$b = \text{ratio} = \frac{f(b)}{f(a)}$$

- Observe the patterns in the $g(x)$ and $h(x)$ columns of Table 3.2.

Table 3.2 Values for Two Exponential Functions

x	$g(x)$		$h(x)$	
−2	4/9		128	
		$\times 3$		$\times \frac{1}{4}$
−1	4/3		32	
		$\times 3$		$\times \frac{1}{4}$
0	4		8	
		$\times 3$		$\times \frac{1}{4}$
1	12		2	
		$\times 3$		$\times \frac{1}{4}$
2	36		1/2	

The function $g(x)$ has an initial value of _____; its output values increase by a factor of _____.

Therefore, $g(x) =$ _______________

The function $h(x)$ has an initial value of _____; its output values decrease by a factor of _____.

Therefore, $h(x) =$ _______________

Examples:

Determine a formula for each exponential function.

2. $f(x)$

3. $g(x)$

x	$f(x)$	$g(x)$
-2	6	108
-1	3	36
0	3/2	12
1	3/4	4
2	3/8	4/3

4. The population of Russia in selected years can be approximated by the following table:

Year	1995	1996	1997	2000	2006	2009	2011
Population (millions)	148.0	147.6	146.9	146.0	143.0	142.0	140.2

a. Let b be the ratio between the population of Russia in 1996 & 1995. Find b. *Round to 3 decimal places.* Then determine an exponential function of the form $y = a(b)^t$ to represent the population of Russia with t representing the years since 1995.

b. Use the function to predict the population of Russia in 2020.

6.1.D3 ~ Constant Percent Rate

OBJECTIVES

- Demonstrate an understanding of the characteristics, behaviors, formulas, graphs, and applications of exponential functions
- Determine the growth or decay factor (and rate) of an exponential function
- Write and analyze the formula for an exponential function given numerically or verbally

❖ **Exponential functions represent quantities that change at a constant percent rate, r.**

➢ Exponential Growth: $f(x) = a(1+r)^x$
- Growth Factor vs. Growth Rate
  - The growth factor, $b = 1 + r$
  - The growth rate, r = (base – 1) × 100

➢ Exponential Decay: $f(x) = a(1-r)^x$
- Decay Factor vs. Decay Rate
  - The decay factor, $b = 1 - r$
  - The decay rate, r = (base – 1) × 100
    Ignore the negative.

Examples:

1. Determine the growth and decay factors and/or the growth and decay rates (written as a percent) in the following tables.

GROWTH FACTOR	GROWTH RATE
1.002	
	2.9%
2.23	

DECAY FACTOR	DECAY RATE
0.77	
	68%
0.953	

Tell whether the population model is an exponential growth function or an exponential decay function, and find the growth rate or decay rate.

3. San Jose: $P(t) = 898{,}759(1.0064)^t$

   GROWTH OR DECAY?

   RATE:

4. Detroit: $P(t) = 1{,}203{,}368(0.9858)^t$

   GROWTH OR DECAY?

   RATE:

Write the formula for the exponential function, $f(x) = a(b)^x$, described.

5. Determine the exponential function with an initial value of 12; increasing at a rate of 8%

6. Determine the exponential function with an initial value of 15; decreasing at a rate of 4.6%

7. You have just purchased a new automobile for $22,000. Much to your dismay, you have just learned that the value of your car will depreciated by 30% per year! Let $V(t)$ represent the automobile's value (in thousands of dollars) after *t* years. Determine the exponential function that models the value. Then determine the value of the car after three years of ownership.

6.2.D1 ~ Exponential Function Modeling

OBJECTIVES

- Demonstrate an understanding of the characteristics, behaviors, formulas, graphs, and applications of exponential functions
- Determine the growth or decay factor (and rate) of an exponential function
- Write and analyze the formula for an exponential function given numerically or verbally

❖ Finding Change Factors

- *If the input values are 1 unit apart, the change factor is equal to the ratio of consecutive output values.*
- What if the input values aren't 1 unit apart?
  - To calculate the change factor of an exponential function with points (x_1, y_1) and (x_2, y_2), raise the ratio of the output values to 1 over the difference in the input values:

$$b = \left(\frac{y_2}{y_1}\right)^{\frac{1}{x_2 - x_1}}$$

Example:

1. Calculate the change factor for an exponential function that passes through $(10, 415)$ & $(15, 677)$. *Round b to three decimal places.* Is the function growth or decay? What is the percent rate of change?

Backer Breaks It Down:

WHAT IS B?

(the base, change factor, growth factor, decay factor)

WHAT WERE YOU GIVEN?

TABLE

ARE THE INPUT VALUES 1 UNIT APART?

Yes

Look at the output values: What number are you multiplying by to obtain the next output value?

No

2 DATA VALUES

Use the change factor formula:

$$b = \left(\frac{y_2}{y_1}\right)^{\frac{1}{x_2 - x_1}}$$

PERCENT RATE

Growth

$1 + r$

Decay

$1 - r$

Examples:

2. On March 31, 2007, USAA Federal Savings Bank advertised a 5-year certificate of deposit (CD) with an annual percentage yield of 5.0%. Find an exponential function that models the value of a $1000 investment in the CD as a function of the number of years the money has been invested.

3. You purchase a new Toyota Camry for $21,500 and want to estimate its worth over the next 5 years assuming it will lose 15% of its value each year. Find an exponential function model for the value of the car.

4. A news article titled "Study Indicates Volunteerism Rises Among Collegians" made the claim that "The number of college students volunteering grew more than 20 percent, from 2.7 million to 3.3 million, between 2002 and 2005." (*Source: East Valley Tribune*: October 16, 2006)
   a. Find the annual growth factor. *Round b to three decimal places*
   b. What is the annual growth rate?
   c. Find an exponential model for the number of college-aged volunteers.

6.2.D2 ~ Comparing Exponential and Linear Functions

OBJECTIVES

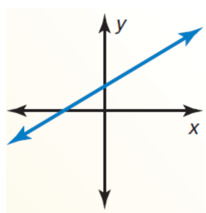
- Compare the characteristics, behaviors, formulas, graphs, and applications of linear and exponential functions
- Fit a formula to a function that is given numerically, graphically or verbally
- Calculate rates of change and change factors from tables and graphs

❖ Identifying Linear & Exponential Functions

➢ For a table of data that gives y as a function of x and in which Δx is constant:

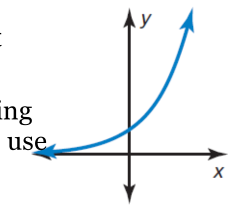
▪ LINEAR

- The difference of consecutive y-values is constant
- Pattern: addition or subtraction
- $y = b + mx$
- m represents the constant difference (aka slope)
  - ♦ m is positive for addition; negative for subtraction
- b is the initial value



▪ EXPONENTIAL

- The ratio of consecutive y-values is constant
- Pattern: multiplication or division
- $y = a(b)^x$
- b represents the constant ratio
  - ♦ This is the number being multiplied; if division, use the reciprocal
- a is the initial value



Examples:

Examine the output pattern to determine which of the following data sets is linear and which is exponential. For the linear set, write a linear equation of the form $y = mx + b$; for the exponential set, write an exponential equation of the form $y = a(b)^x$.

1.

x	-2	-1	0	1	2	3	4
y	-5	-1	3	7	11	15	19

2.

x	-2	-1	0	1	2	3	4
y	$\frac{1}{8}$	$\frac{1}{2}$	2	8	32	128	512

3.

x	-2	-1	0	1	2	3	4
y	81	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$

❖ Writing Linear & Exponential Functions

Linear Functions

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y = mx + b$$

① Calculate the change value.

② Substitute and solve for the initial value.

③ Write the function.

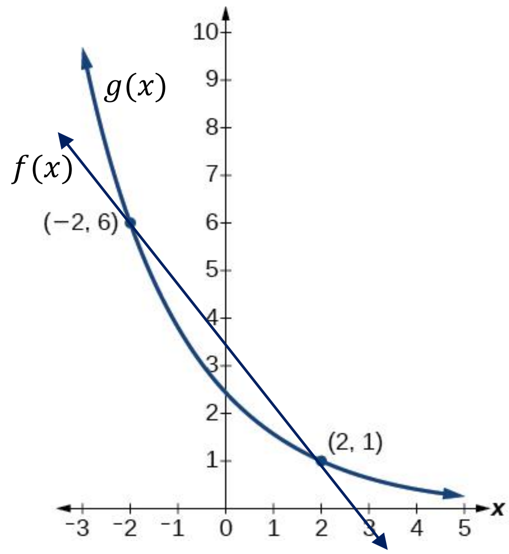
Exponential Functions

$$b = \left(\frac{y_2}{y_1}\right)^{\frac{1}{x_2 - x_1}}$$

$$y = a(b)^x$$

Examples:

4. Find a formula, of the form $y = mx + b$, for the linear function $f(x)$.



5. Find a formula, of the form $y = a(b)^x$, for the exponential function $g(x)$. *Round the values of a to 2 decimal places & b to 3 decimal places.*

6.2.D3 ~ Modeling with Exponential and Linear Functions

OBJECTIVES

- Compare the characteristics, behaviors, formulas, graphs, and applications of linear and exponential functions
- Fit a formula to a function that is given numerically, graphically or verbally

❖ **Identifying Linear & Exponential Functions**

- Linear functions have a constant rate of change.
  - $y = mx + b$
    - m represents the constant rate of change; b is the initial value (when $x = 0$)
- Exponential functions have a percent rate of change.
  - $y = a(1 + r)^x$ OR $y = a(1 - r)^x$
    - r represents the percent rate of change; a is the initial value (when $x = 0$)

Examples:

Decide whether the situation can best be represented with a linear or an exponential function.

1. Ali reads at a rate of 25 pages each hour.
2. An investment account, with an initial balance of $100, earns 3% interest each year.
3. Connor receives a monthly allowance of $75 and spends $15 each week.

Write the appropriate function – linear or exponential – that best represents the scenario.

4. In 2007, Lucky Brand Jeans advertised their Socialite Jean for $118. Write a function that represents the cost, C, in terms of w weeks if the price is reduced by the given amount per week.
   a. $10 per week?
   b. 10% per week?

❖ Writing Linear & Exponential Functions

Linear Functions		Exponential Functions
$m = \dfrac{y_2 - y_1}{x_2 - x_1}$	① Calculate the change value.	$b = \left(\dfrac{y_2}{y_1}\right)^{\frac{1}{x_2 - x_1}}$
	② Substitute and solve for the initial value.	
$y = mx + b$	③ Write the function.	$y = a(b)^x$

Example:

5. The value of a one-year old luxury automobile is $50,000. Seven years later it is worth $22,000.
   a. If the value of the car decreases linearly, by how much does the value of the car depreciate each year?
   b. Write a function that represents the value of the car, V, if it has decreased at a constant rate each year, t.
   c. If the value of the car decreases exponentially, what is the percent decay rate?
   d. Write a function that represents the value of the car, V, if it has decreased at a constant percent rate each year, t.

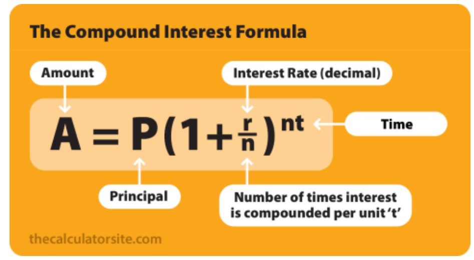
6.3.D1 ~ Compound Interest

OBJECTIVES

- Demonstrate an understanding of applications of exponential functions to compound interest
- Identify and analyze the nominal and effective annual rates for compounded interest

Calculating w/Compound Interest:

1. Suppose you deposit \$10,000 in an account that has a 6.5% annual interest rate (APR). If the interest is compounded monthly (n = 12). What is your balance, A, at the end of 10 years?



❖ Nominal vs. Effective Rate

- ➢ 12% compounded monthly
  - ▪ Interest is added 12 times per year; 12%/12 = 1% of the current balance is added each time
- ➢ 12% is referred to as the **NOMINAL RATE**
- ➢ when the interest is compounded more frequently than once a year, the account effectively earns more than the nominal rate.
  - ▪ The effective annual rate tells you how much interest the investment actually earns.
  - ▪ AKA annual percentage yield (APY)
  - ▪ **Effective Rate** = APY = $\left(1+\frac{r}{n}\right)^n - 1$ *then multiply by 100%*
    - *The effective rate can be used to compare investments.*

Examples:

2. What are the nominal and effective annual rates of an account paying 12% interest, compounded monthly?

3. Which investment is more attractive, one that pays 8.75% compounded quarterly or another that pays 8.7% compounded monthly?

6.3.D2 ~ Continuous Growth & Decay

OBJECTIVES

- Demonstrate an understanding of the number e
- Identify the continuous growth or decay rate of an exponential function
- Write and analyze a formula for an exponential function with a base of e that is given graphically, numerically, or verbally
- Identify and compare nominal, effective annual interest rates and continuous growth rates

The Number e
Base e – the natural base.
$e \approx 2.71828 \ldots$

❖ Exponential Functions with Base e

- ➢ Any positive base b can be written as a power of e: $\boldsymbol{b = e^k}$
- ➢ Any exponential function $\boldsymbol{Q = a(b)^t}$ can be rewritten in terms of e: $\boldsymbol{Q = ae^{kt}}$
  - If $b > 1$, then k is positive — Q is an exponential growth function & Q is increasing
  - If $0 < b < 1$, then k is negative — Q is an exponential decay function & Q is decreasing
  - The constant k is called the **CONTINUOUS GROWTH/DECAY RATE.**

Non-Continuous	Vs.	Continuous
$Q = a(b)^t$		$Q = a(e)^{kt}$
$b = 1 + r$	Growth	$k = +r$
$b = 1 - r$	Decay	$k = -r$
Annual % Rate = (base – 1) × 100%		Continuous % Rate = k × 100%
$B = P\left(1 + \frac{r}{n}\right)^{nt}$	Compound Interest	$B = Pe^{rt}$
$\left[\left(1 + \frac{r}{n}\right)^n - 1\right] \times 100\%$	APY	$(e^r - 1) \times 100\%$

Examples:

Analyze the formula of the exponential function, Q, over time t.

1. $Q = 0.01e^{-0.2t}$		2. $Q = 10(1.22)^t$
	What is the quantity at time $t = 0$?	
	Is the quantity increasing or decreasing over time?	
	What is the percent per unit time growth or decay rate?	
	Is the rate continuous?	
	What is the quantity at time $t = 10$?	

3. Find the effective annual rate if $1000 is deposited at 5% annual interest, compounded continuously. *Round to 3 decimal places.*

4. If \$5000 is deposited in an account paying a nominal interest rate of 4% per year, how much is in the account 10 years later if interest is compounded…
   a. Monthly?
   b. Continuously?

5. A town has population 3000 people at year $t = 0$.
   Determine whether the population can be expressed with…
   - a linear function $P = b + mt$
   - a non-continuous exponential function $P = a(b)^t$
   - a continuous exponential function $P = ae^{kt}$

   Then write a formula for the population, P, in year t if the town…
   a. Shrinks at a continuous rate of 4% per year.
   b. Grows by 200 people per year
   c. Minimizes by 50 people every 52 weeks.
   d. Escalates at a continuous rate of 6% annually.
   e. Diminishes by 4% every 365 days.
   f. Boosts by 6% every 12 months.

6.4.D1 ~ Logarithms & Exponents

OBJECTIVES
- Demonstrate an understanding of logarithms & their properties
- Convert from logarithmic form to exponential form (and vice versa)
- Evaluate & simplify expressions with logarithms
- Solve exponential equations using properties of exponents

❖ Logarithms
➢ An exponential function, $f(x) = b^x$, has an inverse that is a function. This inverse is the logarithmic function with base b, denoted $\log_b x$.

❖ Converting Between Logarithmic & Exponential Form

$$y = \log_b x \leftrightarrow b^y = x$$

Examples:
Write each equation in its equivalent exponential form.

1. $\log_b 64 = 3$
2. $y = \log_3 7$
3. $\log_5 x = 2$

- A logarithm is an EXPONENT
  - We can evaluate simple logarithmic expressions using our understanding of exponents.

One-to-One Property of Exponents

- If $b^M = b^N$, then $M = N$.
  - Express each side of the equation as a power of the same base.
  - Set the exponents equal to each other & then solve for the variable.

Properties of Exponents

- Let a, b, x, and y be real numbers with a & $b > 0$,

$b^0 = 1$ $\quad$ $b^{-x} = \frac{1}{b^x}$ $\quad$ $(a^m)^n = a^{mn}$ $\quad$ $\sqrt{x} = x^{\frac{1}{2}}$ $\quad$ $\sqrt[n]{x^m} = x^{\frac{m}{n}}$

Logarithmic Expression	Question Needed for Evaluation	Logarithmic Expression Evaluated
a. $\log_2 16$	2 to what power gives 16? $2^? = 16$	$\log_2 16 = 4$ because $2^4 = 16$.
b. $\log_7 \frac{1}{49}$	7 to what power gives $\frac{1}{49}$? $7^? = \frac{1}{49}$	$\log_7 \frac{1}{49} = -2$ because $7^{-2} = \frac{1}{7^2} = \frac{1}{49}$.
c. $\log_{25} 5$	25 to what power gives 5? $25^? = 5$	$\log_{25} 5 = \frac{1}{2}$ because $25^{\frac{1}{2}} = \sqrt{25} = 5$.
d. $\log_2 \sqrt[5]{2}$	2 to what power gives $\sqrt[5]{2}$, or $2^{\frac{1}{5}}$? $2^? = \sqrt[5]{2}$	$\log_2 \sqrt[5]{2} = \frac{1}{5}$ because $2^{\frac{1}{5}} = \sqrt[5]{2}$.

Examples:

Evaluate the logarithm by converting it to exponential form. Use the properties of exponents to simplify the logarithm.

4. $\log_3 \sqrt{3} = y$

5. $\log_5 \frac{1}{25} = y$

Use the One-to-One Properties of Exponents to solve the exponential equation.

6. $27^{-2x} = \frac{1}{81}$

7. $64^{3-x} = 4^{x-3}$

6.4.D2 ~ Solving Logarithmic Equations

OBJECTIVES

- Demonstrate an understanding of logarithms & their properties
- Solve logarithmic equations using properties of logarithms and the definition of a logarithm

❖ What is a logarithm?

- ➢ The inverse function of the exponential function with base b is called the logarithmic function with base b: $f(x) = \log_b x$.
  - For $x > 0$ & $0 < b \neq 1$, then $y = \log_b x$ is equivalent to $b^y = x$.

❖ Using the Definition of a Logarithm to Solve Logarithmic Equations

- ➢ Express the equation in the form: $\log_b M = c$
  (*Get the logarithm alone.*)
- ➢ Use the definition of a logarithm to rewrite the equation in exponential form: $b^c = M$
- ➢ Solve for the variable.
- ➢ Check for extraneous solutions: $M > 0$.
  - Exclude from the solution set any proposed solution that produces the logarithm of a negative number or the logarithm of 0.

1. $2\log_2(4x+1) = 10$

❖ Using the One-to-One Property of Logarithms to Solve Logarithmic Equations

- ➢ Express the equation in the form: $\log_b M = \log_b N$
- ➢ Use the one-to-one property to rewrite the equation without logarithms:
  - If $\log_b M = \log_b N$, then $M = N$.
- ➢ Solve for the variable.
- ➢ Check for extraneous solutions: $M > 0$ & $N > 0$

2. $\log_6(4x+9) = \log_6 21$

IT IS SOMETIMES NECESSARY TO USE PROPERTIES OF LOGARITHMS TO CONDENSE LOGARITHMS INTO A SINGLE LOGARITHM.

❖ Properties of Logarithms

- ➢ Let b, R, and S be positive real numbers with $b \neq 1$, and c any real number,
  - Product rule: $\log_b R + \log_b S = \log_b(RS)$
  - Quotient rule: $\log_b R - \log_b S = \log_b\left(\frac{R}{S}\right)$
  - Power rule: $\log_b(R^c) = c\log_b R$

Examples: Use the properties of logarithms to condense the expression.

3. $5\log_9 a + \log_9 b$

4. $6\log_8 x - 2\log_8 y$

Examples:

Use a property of logarithms to condense the left side of the equation. Then use the appropriate method to solve the logarithmic equation. *If necessary, round to two decimal places. Remember to check for extraneous solutions.*

5. $\log_9 8 + \log_9(x-10) = \log_9 26$

6. $\log_2(x+6) - \log_2 x = 5$

6.4.D3 ~ Solving Exponential Equations

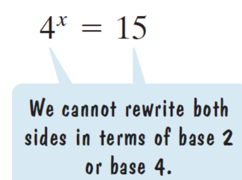
OBJECTIVE

- Solve exponential equations using logarithms

Recall: Logarithms "undo" exponentials, so logs are used to solve exponential equations.

MOST EXPONENTIAL EQUATIONS CANNOT BE REWRITTEN SO THAT EACH SIDE HAS THE SAME BASE.

❖ **Using Logarithms to Solve Exponential Equations**



- Isolate the exponential expression.
- Take the natural logarithm of both sides.
- Simplify using one of the following properties:
  - $\ln b^x = x \ln b$
  - $\ln e^x = x$
- Solve for the variable.

Examples:

Solve the equation using a logarithm. Provide an exact answer, solve for x, and an approximate answer, rounded to three decimal places.

1. $15(2.3)^x = 63$

2. $7^{x+2} = 410$

3. $10e^{4x+1} = 20$ **Recall:** $\ln e^x = x$

6.5.D1 ~ Logarithms & Exponential Models

OBJECTIVES

- Analyze logarithms & exponential models
- Set up exponential equations given verbally
- Solve applied problems involving logarithmic functions

❖ Doubling Time

➢ Eventually any exponentially growing quantity doubles, or increases by 100%. The time it takes a quantity to do this is known as doubling time.

- If $Q = a(1+r)^t$, the initial quantity, a, doubles, therefore: $Q = 2a$.
- Then $2a = a(1+r)^t$, or simply: $2 = (1+r)^t$.

	Non-Continuous		Continuous
Doubling Time	$2 = b^t$	$2 = (1+r)^t$	$2 = e^{kt}$

Example:

1. In 2005, Sudan's population was 40.19 million and has been increasing by 2.6% every year. Estimate the number of years it will take for the population to double.

❖ Half Life

➢ The half-life of a substance is the amount of time it takes for half of the substance to decay.

- If $Q = a(1-r)^t$, the initial quantity, a, halves, therefore: $Q = {}^1\!/_2\, a$.
- Then ${}^1\!/_2\, a = a(1-r)^t$, or simply: ${}^1\!/_2 = (1-r)^t$.

	Non-Continuous		Continuous
Half-Life	$\frac{1}{2} = b^t$	$\frac{1}{2} = (1-r)^t$	$\frac{1}{2} = e^{-kt}$

Examples:

2. Tritium decays at a continuous rate of 5.471%. Find the half-life of Tritium.

3. The half-life of a Twinkie is 14 days. (I think this is after it has been removed from its plastic packaging because, according to *Zombieland,* Twinkies will survive the apocalypse.) Find the daily decay rate of the Twinkie.

6.5.D2 ~ Logarithms & Exponential Models

OBJECTIVES:

- Analyze logarithms & exponential models
- Set up exponential equations given verbally & solve using logarithms
- Rewrite any exponential in the form $Q = ae^{kt}$

❖ Converting Between $Q = a(b)^t$ & $Q = ae^{kt}$

	Non-Continuous	**Continuous**
Function Formula	$Q = a(b)^t$	$Q = ae^{kt}$
Convert	$b = e^k$	$k = \ln b$
Rate Rule	$(b - 1) \times 100\%$	$k \times 100\%$

- If k is positive, then Q is a continuous growth function
- If k is negative, then Q is a continuous decay function

Examples:

1. Fill in the table below, assuming that t is measured in years. Round k and b values to 4 decimal places.

FORMULA		GROWTH OR DECAY?	RATE	CONTINUOUS RATE
$Q = a(b)^t$	$Q = ae^{kt}$			
	$Q = 6e^{-0.04t}$			
$Q = 5(1.2)^t$				
$Q = 10(0.91)^t$				

2. The population of a bacteria colony starts at 100 and grows by 30% per hour.
   a. Find an exponential function model for the number of bacteria, B, after t hours.

   b. What is the continuous growth rate of the colony?

3. A population doubles in size every 20 years. Assuming exponential growth, find the…
   a. Annual growth rate.

   b. Continuous growth rate.

4. The population of Spring Grove grew from 11,000 to 13,000 in three years.
   a. The population of Spring Grove, can be represented by an exponential function. Calculate the change factor. *Round the change factor to 3 decimal places.*

$$\boxed{b = \left(\frac{y_2}{y_1}\right)^{\frac{1}{x_2 - x_1}}}$$

   b. What is the annual growth rate?

   c. What is the continuous growth rate?

Backer Breaks it Down: Non-Continuous vs. Continuous

	Non-Continuous		Continuous
Function Formula	$Q = a(b)^t$		$Q = ae^{kt}$
Growth	$b = 1 + r$		$k = +r$
Decay	$b = 1 - r$		$k = -r$
Compound Interest	$B = P\left(1 + \frac{r}{n}\right)^{nt}$		$B = Pe^{rt}$
Effective Rate	$APY = \left[\left(1 + \frac{r}{n}\right)^n - 1\right] \times 100\%$		$APY = [e^r - 1] \times 100\%$
Convert	$b = e^k$		$k = \ln b$
Rate Rule	$(b - 1) \times 100\%$		$k \times 100\%$
Doubling Time	$2 = b^t$	$2 = (1 + r)^t$	$2 = e^{kt}$
Half-Life	$\frac{1}{2} = b^t$	$\frac{1}{2} = (1 - r)^t$	$\frac{1}{2} = e^{-kt}$
Other	$b = \left(\frac{y_2}{y_1}\right)^{\frac{1}{x_2 - x_1}}$		