Calculus: Formulas & Techniques

Evaluating Limits

- The limit at a hole is the height of the hole.
- The limit at infinity is the height of the horizontal asymptote.
- Before trying other techniques, plug in the arrow number. If the result is
  1. A number, you're done.
  2. A number over zero or infinity over zero, the answer is infinity.
  3. A number over infinity, the answer is zero.
  4. $\frac{0}{0}$ or $\frac{\infty}{\infty}$, use L'Hôpital's Rule.

     (a) If you get $\infty \cdot 0$ or $\infty - \infty$, tweak into $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

     (b) If you get 1^{∞}, 0^{0}, or ∞^{0}, use logarithmic differentiation.

Other Limit Knowledge

- ✓ $\lim_{x\to c} x = c$
- ✓ $\lim_{x\to c} k = k$
- ✓ Limits of polynomials can be found by substitution.
- ✓ The limit of a rational function can be found by substitution when the denominator is different from zero.
- ✓ Limits Involving Infinity
  1. If N˚< D˚, then limit = 0.
  2. If N˚= D˚, then limit = the ratio of the leading coefficients.
  3. If N˚> D˚, then limit = ±∞
- ✓ $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$

The Continuity Test: A function $y = f(x)$ is continuous at $x = c$ iff it meets <u>all three</u> of the following conditions:

1. $f(c)$ exists — (f is defined @ c)
2. $\lim_{x\to c} f(x)$ exists — (f has a limit as $x \to c$)
3. $\lim_{x\to c} f(x) = f(c)$ — (limit value equals function value)

Doing Derivative Problems

Definition of Derivative

The derivative of a function f is the function f' whose value at x is:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

(if the limit exists).

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Power Rule

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Trig Derivatives

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\csc x = -\csc x \cot x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

Product Rule

$$\frac{d}{dx}[f(x) \times g(x)] = f(x) \times g'(x) + g(x) \times f'(x)$$

Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) \times f'(x) - f(x) \times g'(x)}{[g(x)]^2}$$

Implicit Differentiation

1. Differentiate both sides of the equation with respect to x.
2. Collect the terms with *dy/dx* on one side of the equation.
3. Factor out *dy/dx*.
4. Solve for *dy/dx* by dividing.

Fundamental Theorem of Calculus

If f is continuous on every point [a, b] and F is any antiderivative of f on [a, b], then:

$$\int_a^b f(x)dx = F(b) - F(a)$$

General Antiderivatives

$$x^n = \frac{x^{n+1}}{n+1} + C$$

$$\sin x = -\frac{\cos(kx)}{k} + C$$

$$\cos x = \frac{\sin(kx)}{k} + C$$

Techniques of Integration

- **Guess and Check:** Works when integrand is close to a simple backward derivative.
- **Trig Integrals**
  1. Use Pythagorean identities.
     (1) $\sin^2 x + \cos^2 x = 1$
     (2) $\tan^2 x + 1 = \sec^2 x$
     (3) $1 + \cot^2 x = \csc^2 x$
  2. Use half-angle formulas.
     (1) $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$
     (2) $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

Differential Equations & Initial Value Problems

1. Find the general antiderivative of f.
2. Use the initial condition to find the value of C.

$$y = F(x) + C$$

Integral Rules of Trig Functions

$$\int \cos x\, dx = \sin x + C$$

$$\int \sin x\, dx = -\cos x + C$$

$$\int \sec^2 x\, dx = \tan x + C$$

$$\int \csc x \cot x\, dx = -\csc x + C$$

$$\int \sec x \tan x\, dx = \sec x + C$$

$$\int \csc^2 x\, dx = -\cot x + C$$

Basic Integral Rules

$$\int 0\, dx = C$$

$$\int k\, dx = kx + C$$

$$\int k\, f(x)dx = k\int f(x)\, dx$$

$$\int [f(x) \pm g(x)]dx = \int f(x)\, dx \pm \int g(x)dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq 1$$

Algebra Tricks

$$\frac{\#}{x^n} = \#x^{-n}$$

$$\sqrt[n]{x^m} = x^{m/n}$$