P.1 Real Numbers

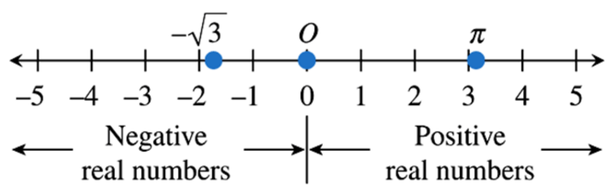
Objectives: Write inequalities; apply the basic properties of algebra; and work with exponents and scientific notation

Representing Real Numbers

- A real number –
  - Natural (or counting) numbers
  - Whole numbers
  - Integers

  } RATIONAL NUMBERS
- Rational numbers –
  - A decimal in rational form, terminates OR is infinitely repeating
    - e.g. $^7/_4 = 1.75$ (terminates)
    - e.g. $^4/_{11} = 0.3636363...$ (infinitely repeats)

    The real number line:



- Irrational numbers –
  - A decimal in irrational form, does not terminate NOR does it repeat

Order & Interval Notation

- The set of real numbers is ordered.
  - Trichotomy Property:
    - Let a & b be any two real numbers. Exactly one of the following is true: a < b, a = b, or a > 0
    - Inequalities describe intervals
- Bounded Intervals of Real Numbers
  - Let a & b be real numbers with a < b:

Interval Notation	Inequality Notation
$[a,b]$	$a \le x \le b$
(a,b)	$a < x < b$
$[a,b)$	$a \le x < b$
$(a,b]$	$a < x \le b$

a & b are the endpoints

Write your own summary of what you see here.

- Unbounded Intervals of Real Numbers
  - Let a & b be real numbers:

Interval Notation	Inequality Notation
$[a,\infty)$	$x \geq a$
(a, ∞)	$x > a$
$(-\infty,b]$	$x \leq b$
$(-\infty,b)$	$x < b$

Write your own summary of what you see here.

Each of these intervals has exactly one endpoint: a or b

Basic Properties of Algebra

- The following properties hold for real numbers, variables & algebraic expressions:

Let u, v, and w be real numbers, variables, or algebraic expressions.

1. Commutative property
Addition: $u + v = v + u$
Multiplication: $uv = vu$

2. Associative property
Addition:
$(u + v) + w = u + (v + w)$
Multiplication: $(uv)w = u(vw)$

3. Identity property
Addition: $u + 0 = u$
Multiplication: $u \cdot 1 = u$

4. Inverse property
Addition: $u + (-u) = 0$
Multiplication: $u \cdot \frac{1}{u} = 1,\ u \neq 0$

5. Distributive property
Multiplication over addition:
$u(v + w) = uv + uw$
$(u + v)w = uw + vw$

Multiplication over subtraction:
$u(v - w) = uv - uw$
$(u - v)w = uw - vw$

Integer Exponents

- Properties of Exponents:

$u^m u^n = u^{m+n}$

$u^0 = 1$

$(uv)^m = u^m v^m$

$(u^m)^n = u^{mn}$

$u^{-n} = \frac{1}{u^n}$

$\frac{u^m}{u^n} = u^{m-n}$

$\left(\frac{u}{v}\right)^m = \frac{u^m}{v^m}$

These properties are on your semester summary sheet.

Scientific Notation

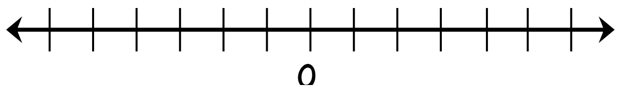
- Any positive number can be written in <u>scientific notation</u>,

  $c \times 10^m$, where $1 \le c < 10$ and m is an integer

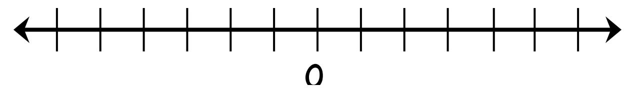
  - Positive exponent: moves the decimal point – of c – to the right m places
  - Negative exponent: moves the decimal point – of c – to the left m places

<u>Examples: Converting Between Intervals & Inequalities</u>
Convert interval notation to inequality notation or vice versa. Find the endpoints & state whether the interval is bounded & open or closed. Then graph the interval.

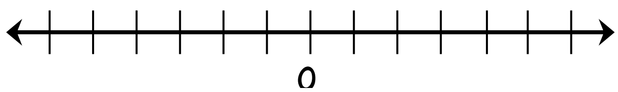
1. $[-2, 3]$



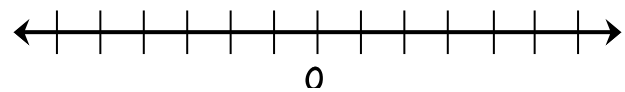
2. $(-\infty, 0)$



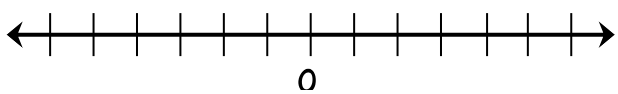
3. $-4 \le x < 4$



4. $2 < x < 5$



5. $[3, \infty)$



<u>Examples: Simplifying Expressions Involving Powers</u>
Use the properties of exponents to simplify the expression. Assume that the variables in denominators are nonzero.

6. $(2ab^3)(5a^2b^5)$

7. $\dfrac{u^2v^{-2}}{u^{-1}v^3}$

8. $\left(\dfrac{x^2}{2}\right)^{-3}$

<u>Examples: Using Scientific Notation</u>
Use the properties of exponents & simplify using scientific notation.

9. $\dfrac{(1.35 \times 10^{-7})(2.41 \times 10^8)}{1.25 \times 10^9}$

P.2 Cartesian Coordinate System

Objectives: Graph points, find distances and midpoints on a number line and in the coordinate plane, and write standard-form equations of circles.

- The Cartesian Coordinate System
  - An association between the points in a plane & ordered pairs of real numbers
    - AKA the rectangular coordinate system
- Absolute Value of a Real Number
  - The absolute value of a real number suggests its _______________ (size).
    - Definition: $|a| = \begin{cases} a, \text{ if } a > 0 \\ -a \text{ if } a < 0 \\ 0, \text{ if } a = 0 \end{cases}$
  - Properties of Absolute Value
    - Let a & b be real numbers:

      1. $|a| \geq 0$
      2. $|-a| = |a|$
      3. $|ab| = |a||b|$
      4. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|},\ b \neq 0$
- Distance Formulas
  - Distance Formula (Number Line)
    - Let a and b be real numbers. The distance between a and b is _____________.
      - Note that $|a - b| = |b - a|$.
  - Distance Formula (Coordinate Plane)
    - The distance d between points $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the coordinate plane is:
  - Using an Inequality to Express Distance
    - "the distance between x & -3 is less than 9"
      -
- Midpoint Formulas
  - Midpoint Formula (Number Line)
    - The midpoint of the line segment with endpoints a and b is:
  - Midpoint Formula (Coordinate Plane)
    - The midpoint of the segment with endpoints (a, b) and (c, d) is:

Examples: Finding Distance & the Midpoint between Two Points

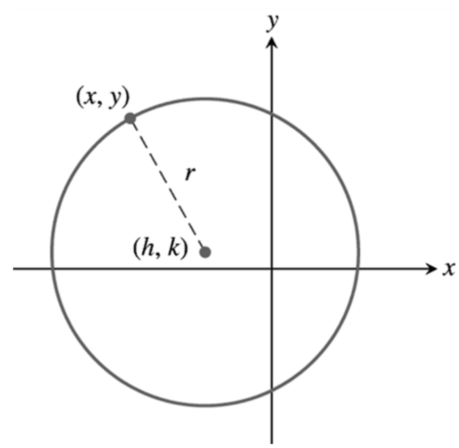
Find the distance between and the midpoint of the line segment with the given endpoints.

1. -9.3, 10.6

2. (-3, -1), (5, -1)

Equations of Circles

- A circle is...
  - The standard form equation of a circle with center (h, k) & radius r is:



Examples: Finding Standard Form Equations of Circles

Find the standard form equation of the circle.

3. Center (-1, -4), radius 3

Find the center and radius of the circle.

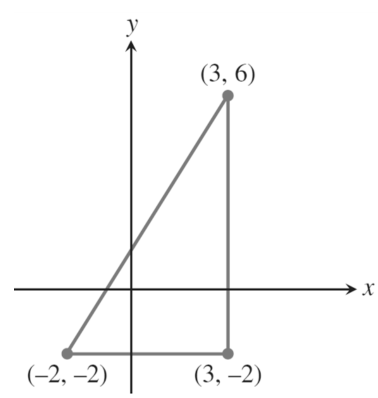
4. $(x-3)^2 + (y+1)^2 = 36$

Examples: Using Coordinates, Distance and Midpoint Formulas

5. Find the lengths of the sides of the triangle in the figure.

6. Show that the triangle is a right triangle.

7. Prove that the midpoint of the hypotenuse is equidistant from the three vertices.



P.3 Linear Equations & Inequalities

Objectives: Solve linear equations and inequalities in one variable.

- Solving Equations
  - Equation -
    - We can use the properties of equality – reflexive, symmetric, transitive, addition & multiplication – to solve equations algebraically.

- Linear Equations in One Variable
  - A linear equation in x is
    - Has exactly one solution
    - We solve such an equation by transforming it into an ____________________
      - Two or more equations are ______________ if they have the same solutions.

Examples: Solving Linear Equations

Solve the equation.

1. $2(3-4x)-5(2x+3)=x-17$

2. $\frac{a+5}{8}-\frac{a-2}{2}=\frac{1}{3}$

HOW DO YOU RID AN EQUATION OF FRACTIONS?

- Linear Inequalities in One Variable
  - A linear inequality in x is one that can be written in the form:

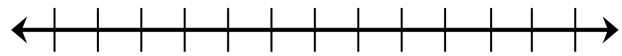
$$ax+b<0,\ ax+b\leq 0,\ ax+b>0,\ ax+b\geq 0$$

    - A solution of an inequality in x is a value of x for which the inequality is true → solution set
    - Fill in the blank: We reverse the direction of the inequality sign when we multiply or divide both sides of an inequality by a ___________________ number.

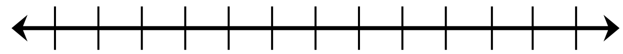
- Double Inequalities
  - Combination of two inequalities
  - Solution: a double inequality with x isolated as the middle term

<u>Examples: Solving Linear Inequalities</u>
Solve the inequality. Express your solution as an inequality and in interval notation. Graph the solution set.

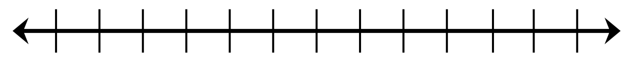
3. $2x - 1 \leq 4x + 3$



4. $0 \leq 2m + 5 < 8$



5. $\frac{2y-3}{2} + \frac{3y-1}{5} < y - 1$



P.4 Lines in the Plane

<u>Objectives</u>: Use the concepts of slope & y-intercept to graph and write linear equations in two variables.

Slope of a Line

- The slope of a non-vertical line through the points (x_1, y_1) and (x_2, y_2) is:
  - If the line is vertical, its slope is _______________.

Point-Slope Form Equation of a Line

- If we know the coordinates of one point on a line & the slope of the line, then we can find the equation of the line.

- The point-slope form of an equation of a line that passes through the point (x_1, y_1) and has slope m is:

Slope-Intercept Form of an Equation of a Line

- The y-intercept of a non-vertical line is…

- The slope-intercept form of an equation of a line with slope m and y-intercept $(0, b)$ is:

General Form for an Equation of a Line

- Every line has an equation that can be written in general form:

Other Forms for the Equation of a Lines

- Vertical lines:
- Horizontal lines:

Examples: Finding the Slope of and the Equation of a Line

1. Find the slope of the line that passes through (4, 6) and (-3, -2).

2. Find the general form equation of the line in #1.

Graphing Linear Equations in Two Variables

- A linear equation in x and y is one that can be written in the form $Ax + By = C$, where A and B are not both zero.
  - The graph of an equation in x and y consists of all pairs (x, y) that are solutions of the equation.
- Use Intercepts
  - Since two points determine a line you can use the x- and y-intercepts.
    - x-intercept - the point $(x', 0)$ where the graph intersects the x-axis
      - ~ Set $y = 0$ and solve for x to find the x-intercept
    - y-intercept - the point $(0, y')$
      - ~ Set $x = 0$ and solve for y to find the y-intercept
- Use the slope-intercept equation of a line: $y = mx + b$

Graphing with a Graphing Utility

To draw a graph of an equation using a grapher:

1. Rewrite the equation in the form $y =$ (an expression in x).
2. Enter the equation into the grapher.
3. Select an appropriate **viewing window** (see Figure P.26).
4. Press the "graph" key.

- Refer to "Basic Graphing" step one: http://www.mathbits.com/MathBits/TISection/General/BasicGraphing.htm

Parallel & Perpendicular Lines

- Parallel: slopes are equal →
- Perpendicular: slopes are the opposite reciprocals of each other:

Examples: Finding Equations of Parallel & Perpendicular Lines

3. Find an equation of a line through $P(2, -1)$ that is parallel to line L with equation: $3x + 2y = 5$

4. Find an equation of a line through $P(2, -3)$ that is perpendicular to the line L with equation: $3x - y = 3$

Examples: Applications with Linear Equations

5. A commercial jet airplane climbs at takeoff with slope $m = {}^{3}/_{8}$. How far in the horizontal direction will the airplane fly to reach an altitude of 12,000 feet above the takeoff point?

6. Camelot Apartments purchased a \$50,000 building and depreciates it \$2000 per year over a 25-year period.
   a. Write a linear equation giving the value y of the building in terms of the years x after the purchase.

   b. In how many years will the value of the building be \$24,500?

7. American's personal income in trillions of dollars is given in Figure P.30.
   a. Write a linear equation for Americans' income y in terms of the year x using the data given for 1998 and 1999.

Year	Amount (trillions of dollars)
1998	7.4
1999	7.8
2000	8.4
2001	8.7
2002	8.9
2003	9.2

FIGURE P.30 Americans' Personal Income.

   b. Use the equation in (a) to estimate American's income in 2001.

   c. Use the equation in (a) to estimate American's income in 2006.

   d. Superimpose a graph of the linear equation in (a) on a scatter plot of the data. What do you observe?

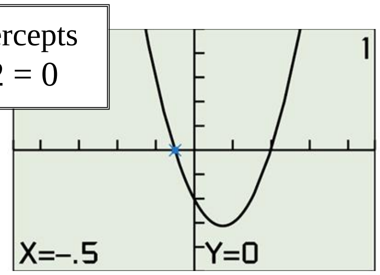
P.5 Solving Equations Graphically, Numerically & Algebraically

<u>Objectives</u>: Solve equations involving quadratic, absolute value & rational functions by: finding x-intercepts or intersection points on graphs and by using algebraic or numerical techniques.

Solving Equations Graphically

- Finding x-Intercepts
  - Recall that x-intercepts are where the graph crosses the x-axis <u>and</u> where $y = 0$

Finding x-intercepts
$2x^2 - 3x - 2 = 0$



<u>Examples: Solve by Finding x-Intercepts</u>

1. $5x^2 + 19x + 12 = 0$
2. $x(3x - 7) = 6$

Quadratic Equations

- A <u>quadratic equation in x</u> is…

Solving Quadratic Equations Algebraically

- Factoring
  - Factor and apply the <u>Zero Factor Property</u>
    -
- Extracting Square Roots
  - Use to solve quadratic equations of the form ____________________
  - <u>Square Root Principle</u>: __________________________________

<u>Examples: Solving Quadratic Equations Algebraically</u>

3. Solve by factoring: $4x^2 - 8x + 3 = 0$

4. Solve by extracting square roots: $3(x + 4)^2 - 8 = 0$

Solving Quadratic Equations Algebraically (continued)

- Completing the Square
  - To solve $x^2 + bx = c$, add ________ to both sides of the equation & ________ the left side of the new equation.
  - In cases where $\boldsymbol{a}x^2 + bx = c$, we ___________________________________ and then complete the square.
- Using the Quadratic Formula
  - The solutions of the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the quadratic formula:

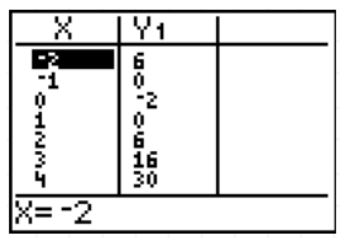
Examples: Solving Quadratic Equations Algebraically

5. Solve $x^2 + 6x = 7$ by completing the square.

6. Solve $3x + 4 = x^2$ using the quadratic formula.

Approximating Solutions of Equations Numerically w/Tables

- If you need to see values relating to a function (inputted into $y =$), go to the TABLE (2nd GRAPH).
  - Here you can find the value of x if the equation is equal to 0, -2, 6, 16, 30…



Example: Solving Equations Numerically

7. Use a table to find the solutions of the equation: $x^2 - x - 1 = 0$ Approximate to two decimal places.
   a. Press 2nd WINDOW
   b. TblStart = 0
   c. △Tbl = 0.1
   d. 2nd GRAPH and scroll until you find a $Y_1 \approx 0$; what is x?
   e. Return to (a); change (b) to x value and (c) to 0.01
   f. Return to the table and find the first of two solutions
   g. Repeat (a - f) to find the second solution

- Solving Equations by Finding Intersections
  - We can rewrite an equation and solve it graphically by finding the ___________ ______________________________ of two graphs
    - When solving an equation by finding intersections, the value of y is irrelevant.

Examples: Solving Equations Graphically

8. Solve $|0.5x + 3| = x^2 - 4$ graphically by finding intersections. Approximate answers to the nearest 100th.

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P.6 Complex Numbers

Objectives: Add, subtract, multiply & divide complex numbers; and find complex roots of quadratic equations.

- Complex Numbers
    - Imaginary unit:
  - A complex number is…

  - **FACT**: All real numbers are also complex numbers.
    - Why?

  - Two complex numbers are equal if the real & imaginary parts are equal.
    - For example: $x + yi = 2 + 5i$ if and only if $x = 2$ & $y = 5$

Examples: Using Complex Numbers

1. Write the expression in the form bi, where b is a real number.

   a. $\sqrt{-16}$ b. $\sqrt{-3}$

2. Find the real numbers x and y that make the equation true:

   $(5 - 2i) - 7 = x - (3 + yi)$
~~~~~~

- Operations with Complex Numbers
  - Addition/Subtraction:
  - Multiplication:
  - Division
    - Multiply the numerator & denominator by the denominator's ______________
      - The product of the complex numbers $a + bi$ & $a - bi$ is a positive real number: $a^2 + b^2$

Examples: Performing Operations with Complex Numbers

Write the sum, difference or product in standard form.

3. $(2 - i) + (3 - \sqrt{-3})$
4. $(i^2 + 3) - (7 + i^3)$
5. $-i(3 + 2i)^2$

Write the expression in standard form.

6. $\frac{5+i}{2-3i}$

- Complex Solutions of Complex Equations
  - The Discriminant: $\boldsymbol{b^2 - 4ac}$
    - If $b^2 - 4ac > 0 \rightarrow$ 2 distinct real solutions
    - If $b^2 - 4ac = 0 \rightarrow$ one repeated real solution
    - If $b^2 - 4ac < 0 \rightarrow$ complex conjugate pair of solutions

Examples: Finding Complex Solutions of Quadratic Equations

7. Solve $x^2 + 2x + 5 = 0$

P.7 Solving Inequalities Algebraically & Graphically

Objectives: Solve inequalities involving absolute value, quadratic polynomials and expressions with fractions.

Solving Absolute Value Inequalities

Let u be an algebraic expression in x and let a be a real number with $a \geq 0$.

1. If $|u| < a$, then u is in the interval $(-a, a)$. That is,

$$|u| < a \quad \text{if and only if} \quad -a < u < a.$$

2. If $|u| > a$, then u is in the interval $(-\infty, -a)$ or (a, ∞), that is,

$$|u| > a \quad \text{if and only if} \quad u < -a \text{ or } u > a.$$

The inequalities $<$ and $>$ can be replaced with $\leq$ and $\geq$, respectively. S

UNION OF TWO SETS

The **union of two sets A and B**, denoted by $A \cup B$, is the set of all objects that belong to A or B or both.

Examples: Solving Absolute Value Inequalities

Solve the inequality algebraically. Write the solution in interval notation.

1. $|x+4| \geq 5$
2. $|4-3x| - 2 < 4$

Solving Quadratic Inequalities

- First, set the corresponding quadratic equation equal to zero and solve
  - Then find the interval(s) for which it is greater than or less than zero
    OR
  - … determine the values of x for which the graph lives above the x-axis (if greater than) or below the x-axis (if less than)

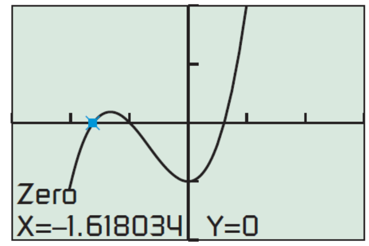
Examples: Solving Quadratic Inequalities

Solve the inequality algebraically. Write the solution in interval notation.

3. $2x^2 + 7x \geq 15$
4. $2x^2 + 17x + 21 < 0$

Solving Inequalities Graphically

- Set the corresponding equation equal to zero and the graph.
- Approximate the zeros.
  - Determine the intervals that satisfy the inequality.



[–3, 3] by [–2, 2]

Examples: Solving Inequalities Graphically

Solve the cubic inequality graphically. Write the solution in interval notation. Approximate your solutions to the nearest hundredth.

5. $3x^3 - 12x + 2 \geq 0$
6. $|2x^2 + 7x - 15| < 10$

Projectile Motion

- Suppose an object is launched vertically from a point s_0 feet above the ground with an initial velocity of v_0 feet per second. The vertical position s (in feet) of the object t seconds after it is launched is…

Examples: Applications Involving Inequalities

7. A projectile is launched straight up from ground level with an initial velocity of 140 ft/sec.
   a. How long is the projectile aloft?
   b. When is the projectile's height above ground at least 165 feet?
   c. When is it less than or equal to 165 feet?
8. Barb wants to drive to a city 105 mi from her home in no more than 2 hours. What is the lowest average speed she must maintain on the drive?
9. A stuntman will jump off a 20-meter building. A high-speed camera is ready to film him between 15 m and 10 m above the ground. When should the camera film him? Solve $d = 20 - 5t^2$.