Because we have found two expressions for h, we can set these expressions equal to each other.

$$a \sin B = b \sin A \quad \text{Equate the expressions for } h.$$

$$\frac{a \sin B}{\sin A \sin B} = \frac{b \sin A}{\sin A \sin B} \quad \text{Divide both sides by } \sin A \sin B.$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} \quad \text{Simplify.}$$

This proves part of the Law of Sines. If we use the same process and draw an altitude of length h from vertex A, we obtain the following result:

$$\frac{b}{\sin B} = \frac{c}{\sin C}.$$

When this equation is combined with the previous equation, we obtain the Law of Sines. Because the sine of an angle is equal to the sine of 180° minus that angle, the Law of Sines is derived in a similar manner if the oblique triangle contains an obtuse angle.

1 Use the Law of Sines to solve oblique triangles.

Solving Oblique Triangles

Solving an oblique triangle means finding the lengths of its sides and the measurements of its angles. The Law of Sines can be used to solve a triangle in which one side and two angles are known. The three known measurements can be abbreviated using SAA (a side and two angles are known) or ASA (two angles and the side between them are known).

EXAMPLE 1 Solving an SAA Triangle Using the Law of Sines

Solve the triangle shown in **Figure 6.3** with $A = 46°$, $C = 63°$, and $c = 56$ inches. Round lengths of sides to the nearest tenth.

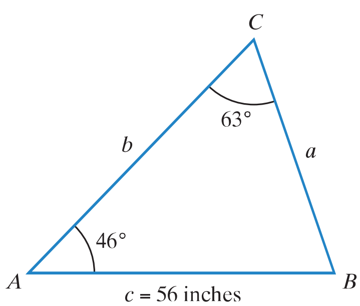

FIGURE 6.3 Solving an oblique SAA triangle

SOLUTION

We begin by finding B, the third angle of the triangle. We do not need the Law of Sines to do this. Instead, we use the fact that the sum of the measures of the interior angles of a triangle is 180°.

$$A + B + C = 180°$$

$$46° + B + 63° = 180° \quad \text{Substitute the given values: } A = 46° \text{ and } C = 63°.$$

$$109° + B = 180° \quad \text{Add.}$$

$$B = 71° \quad \text{Subtract } 109° \text{ from both sides.}$$

When we use the Law of Sines, we must be given one of the three ratios. In this example, we are given c and C: $c = 56$ and $C = 63°$. Thus, we use the ratio $\frac{c}{\sin C}$, or $\frac{56}{\sin 63°}$, to find the other two sides. Use the Law of Sines to find a.

$$\frac{a}{\sin A} = \frac{c}{\sin C} \quad \text{The ratio of any side to the sine of its opposite angle equals the ratio of any other side to the sine of its opposite angle.}$$

$$\frac{a}{\sin 46°} = \frac{56}{\sin 63°} \quad A = 46°, c = 56, \text{ and } C = 63°.$$

$$a = \frac{56 \sin 46°}{\sin 63°} \quad \text{Multiply both sides by } \sin 46° \text{ and solve for } a.$$

$$a \approx 45.2 \text{ inches} \quad \text{Use a calculator.}$$

GREAT QUESTION!

Do I have to set up the Law of Sines with the unknown side in the upper left position?

No. However, many students find it easier to solve for the unknown sides when they are placed in the upper left position.

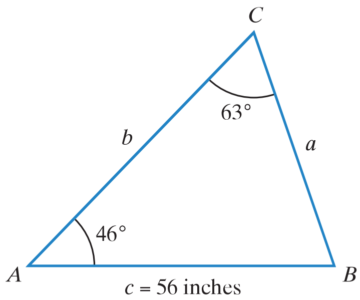

FIGURE 6.3 (repeated)

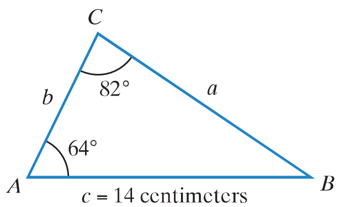

FIGURE 6.4

Use the Law of Sines again, this time to find b.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$ We use the given ratio, $\frac{c}{\sin C}$, to find b.

$$\frac{b}{\sin 71°} = \frac{56}{\sin 63°}$$ We found that $B = 71°$. We are given $c = 56$ and $C = 63°$.

$$b = \frac{56 \sin 71°}{\sin 63°}$$ Multiply both sides by sin 71° and solve for b.

$$b \approx 59.4 \text{ inches}$$ Use a calculator.

The solution is $B = 71°$, $a \approx 45.2$ inches, and $b \approx 59.4$ inches. ●●●

Check Point 1 Solve the triangle shown in **Figure 6.4** with $A = 64°$, $C = 82°$, and $c = 14$ centimeters. Round as in Example 1.

EXAMPLE 2 Solving an ASA Triangle Using the Law of Sines

Solve triangle ABC if $A = 50°$, $C = 33.5°$, and $b = 76$. Round measures to the nearest tenth.

SOLUTION

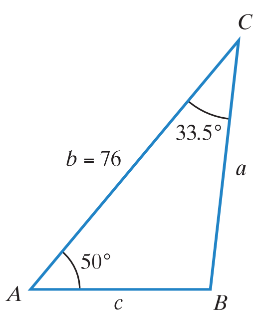

FIGURE 6.5 Solving an ASA triangle

We begin by drawing a picture of triangle ABC and labeling it with the given information. **Figure 6.5** shows the triangle that we must solve. We begin by finding B.

$$A + B + C = 180°$$ The sum of the measures of a triangle's interior angles is 180°.

$$50° + B + 33.5° = 180°$$ $A = 50°$ and $C = 33.5°$.

$$83.5° + B = 180°$$ Add.

$$B = 96.5°$$ Subtract 83.5° from both sides.

Keep in mind that we must be given one of the three ratios to apply the Law of Sines. In this example, we are given that $b = 76$ and we found that $B = 96.5°$. Thus, we use the ratio $\frac{b}{\sin B}$, or $\frac{76}{\sin 96.5°}$, to find the other two sides. Use the Law of Sines to find a and c.

Find a: | **Find c:**

This is the known ratio.

$$\frac{a}{\sin A} = \frac{b}{\sin B} \qquad \frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\frac{a}{\sin 50°} = \frac{76}{\sin 96.5°} \qquad \frac{c}{\sin 33.5°} = \frac{76}{\sin 96.5°}$$

$$a = \frac{76 \sin 50°}{\sin 96.5°} \approx 58.6 \qquad c = \frac{76 \sin 33.5°}{\sin 96.5°} \approx 42.2$$

The solution is $B = 96.5°$, $a \approx 58.6$, and $c \approx 42.2$. ●●●