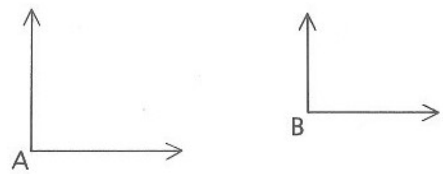
Theorem 1 ***If two angles are right angles, then they are congruent.***

Given: ∠A is a right ∠.
∠B is a right ∠.

Prove: ∠A ≅ ∠B



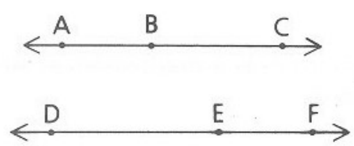
Proof:

Statements	Reasons
1 ∠A is a right angle.	1 Given
2 m∠A = 90	2 If an angle is a right angle, then its measure is 90.
3 ∠B is a right angle.	3 Given
4 m∠B = 90	4 Same as 2
5 ∠A ≅ ∠B	5 If two angles have the same measure, then they are congruent. (See steps 2 and 4.)

Theorem 2 ***If two angles are straight angles, then they are congruent.***

Given: ∠ABC is a straight angle.
∠DEF is a straight angle.

Prove: ∠ABC ≅ ∠DEF



Proof:

Statements	Reasons
1 ∠ABC is a straight angle.	1 Given
2 m∠ABC = 180	2 If an angle is a straight angle, then its measure is 180.
3 ∠DEF is a straight angle.	3 Given
4 m∠DEF = 180	4 Same as 2
5 ∠ABC ≅ ∠DEF	5 If two angles have the same measure, then they are congruent. (See steps 2 and 4.)

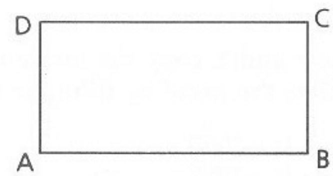
Now that we have presented and proved two theorems, we are ready to use them to help prove some sample problems.

We will use the theorems themselves as reasons in our proofs. You should also use the theorems as reasons in your homework problems.

Remember, the purpose of a theorem is to shorten your work. Therefore, when doing homework problems, do not use the proofs of theorems as a guide. Use the sample problems as a guide.

Part Two: Sample Problems

Problem 1 *Given: ∠A is a right angle.*
∠C is a right angle.
Conclusion: ∠A ≅ ∠C

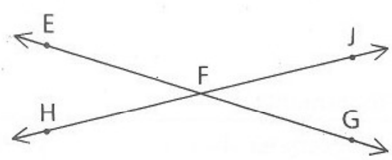


Proof

Statements	Reasons
1 ∠A is a right angle.	1 Given
2 ∠C is a right angle.	2 Given
3 ∠A ≅ ∠C	3 If two angles are right angles, then they are congruent.

You probably recognize that reason 3 is Theorem 1. Although it may seem easier merely to write "Theorem 1," *do not do so!* Eventually, such a shortcut would make it harder for you to learn the concepts of geometry.

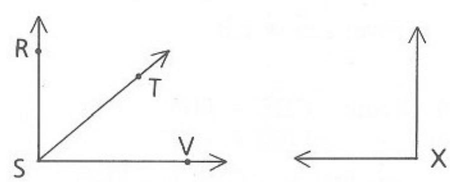
Problem 2 *Given: Diagram as shown*
Conclusion: ∠EFG ≅ ∠HFJ



Proof

Statements	Reasons
1 Diagram as shown	1 Given
2 ∠EFG is a straight angle.	2 Assumed from diagram
3 ∠HFJ is a straight angle.	3 Assumed from diagram
4 ∠EFG ≅ ∠HFJ	4 If two angles are straight angles, then they are congruent.

Problem 3 *Given: ∠RST = 50°,*
∠TSV = 40°;
∠X is a right angle.
Prove: ∠RSV ≅ ∠X



Proof

Statements	Reasons
1 ∠RST = 50°	1 Given
2 ∠TSV = 40°	2 Given
3 ∠RSV = 90°	3 Addition (50° + 40° = 90°)
4 ∠RSV is a right angle.	4 If an angle is a 90° angle, it is a right angle.
5 ∠X is a right angle.	5 Given
6 ∠RSV ≅ ∠X	6 If two angles are right angles, then they are congruent.

2.1 PERPENDICULARITY

Objectives

After studying this chapter, you will be able to

- Recognize the need for clarity and concision in proofs
- Understand the concept of perpendicularity

Part One: Introduction

A Look Back and a Look Ahead

If you feel somewhat confused at this time, you need not feel discouraged. Some confusion is inevitable at the start of geometry. Be patient! Read the lessons carefully, study the sample problems closely, and the confusion will begin to go away. Also, see your teacher for help as you need it.

In Chapter 1, you concentrated on two-column proofs but were also exposed to paragraph proofs. When writing either type, remember that understanding what you are trying to say is the most important element.

From now on, when you write a two-column proof, try to state each reason in a single sentence or less. To help you, the problems in Problem Set A of this section and the next will include a hint when a proof requires more than two steps.

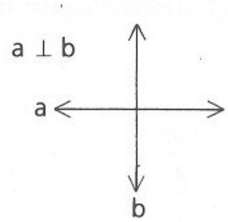
This chapter contains more definitions and theorems for you to memorize and use. Toward the end of the chapter, the proofs will begin to get a little longer. As the proofs become more challenging, you will find more satisfaction in completing them.

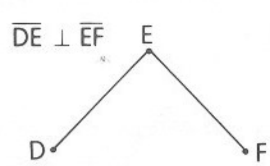
Perpendicular Lines, Rays, and Segments

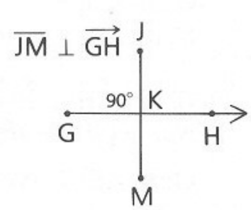
Perpendicularity, right angles, and 90° measurements all go together.

Definition Lines, rays, or segments that intersect at right angles are ***perpendicular*** ($\perp$).

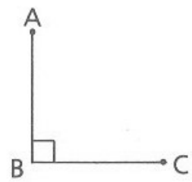
Below are some examples of perpendicularity.



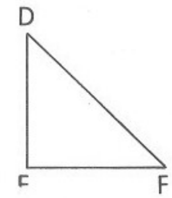




In the figure at the right, the mark inside the angle (⌐) indicates that $\angle B$ is a right angle. It is also true that $\overline{AB} \perp \overline{BC}$ and $\angle B = 90°$.

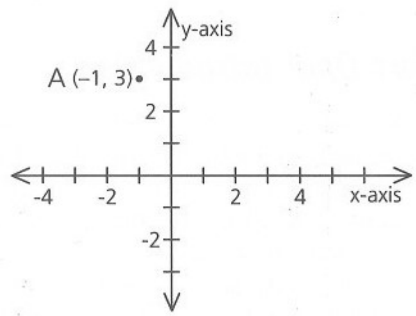


Do not assume perpendicularity from a diagram! In $\triangle DEF$ it *appears* that $\overline{DE} \perp \overline{EF}$, but we may not assume so.

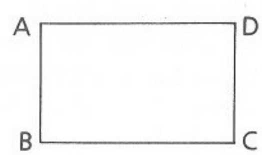


In your algebra studies, you learned that two perpendicular number lines form a two-dimensional coordinate system, or coordinate plane. (The horizontal line is called the ***x-axis***; the vertical line, the ***y-axis***.) Each point on the plane can be represented by an ordered pair in the form (x, y). The values of x and y in the pair, called the point's ***coordinates***, represent the point's distances from the y-axis and the x-axis respectively. In the diagram, point A is represented by (−1, 3).

The intersection of the axes is called the ***origin***. Its coordinates are (0, 0).



Part Two: Sample Problems



Problem 1 *Given:* $\overline{AB} \perp \overline{BC}$, $\overline{DC} \perp \overline{BC}$

Conclusion: $\angle B \cong \angle C$

Proof

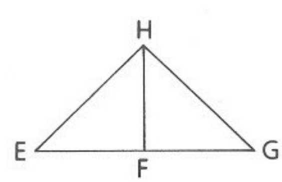
Statements	Reasons
1 $\overline{AB} \perp \overline{BC}$	1 Given
2 $\angle B$ is a right angle.	2 If two segments are ⊥, they form a right angle.
3 $\overline{DC} \perp \overline{BC}$	3 Given
4 $\angle C$ is a right angle.	4 Same as 2
5 $\angle B \cong \angle C$	5 If angles are right angles, they are ≅.

The braces joining steps 1 and 2 emphasize the logical flow of reasoning from one step to the other. There is a similar logical flow from step 3 to step 4.

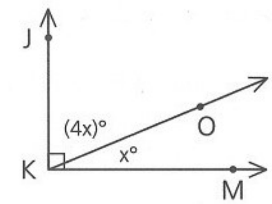
Problem 2 *Given:* $\overleftrightarrow{EH} \perp \overleftrightarrow{HG}$

Name all the angles you can prove to be right angles.

Answer Only $\angle EHG$ (Why not $\angle EFH$ and $\angle HFG$?)



Problem 3 *Given:* $\overrightarrow{KJ} \perp \overrightarrow{KM}$;
$\angle JKO$ *is four times as large as* $\angle MKO$.
Find: $m\angle JKO$

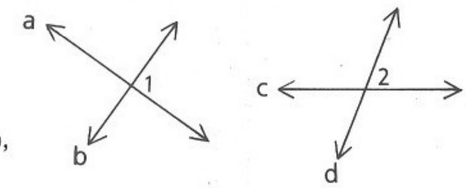


Solution Since $\overrightarrow{KJ} \perp \overrightarrow{KM}$, $m\angle JKO + m\angle MKO = 90$.

$$4x + x = 90$$
$$5x = 90$$
$$x = 18$$

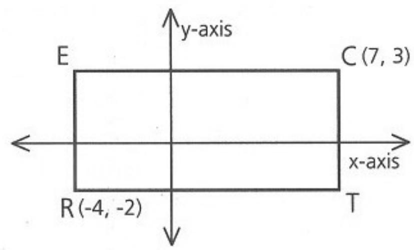
Substituting 18 for x, we find that $m\angle JKO = 72$.

Problem 4 *Given:* $a \perp b$,
$c \not\perp d$ (c *is not* $\perp$ *to* d.)
Conclusion: $\angle 1 \cong \angle 2$



Solution This conclusion is false. Since $a \perp b$, $\angle 1 = 90°$. Since $c \not\perp d$, $\angle 2 \neq 90°$. Since $\angle 1$ and $\angle 2$ have different measures, $\angle 1 \not\cong \angle 2$.

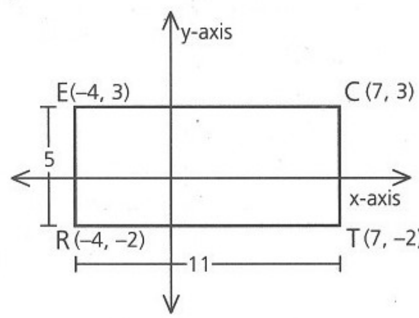
Problem 5 *Given:* $\overline{EC} \parallel$ *to x-axis*
$\overline{RT} \parallel$ *to x-axis*
Find: Area of rectangle RECT



Solution The remaining coordinates are $T = (7, -2)$ and $E = (-4, 3)$. So $RT = 11$ and $TC = 5$ as shown. We shall concentrate on area in Chapter 12, but from previous courses you should know how to find a rectangle's area.

$$\text{Area of rectangle} = \text{base} \times \text{height}$$
$$A = bh$$
$$= 11(5)$$
$$= 55$$

The area of RECT is 55 square units.

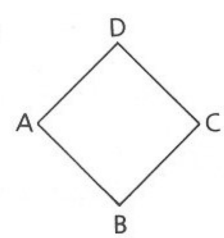


Part Three: Problem Sets

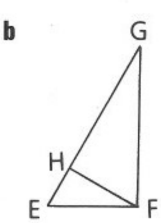
Problem Set A

1 Name all the angles in the figures to the right that appear to be right angles.

a



b



2.4 Congruent Supplements and Complements

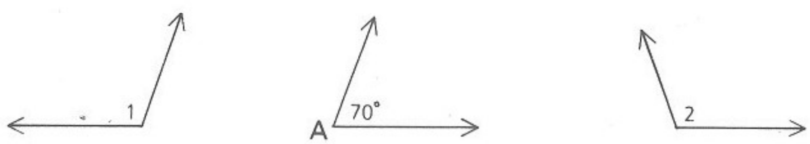
Objective

After studying this section, you will be able to

- Prove angles congruent by means of four new theorems

Part One: Introduction

In the diagram below, $\angle 1$ is supplementary to $\angle A$, and $\angle 2$ is also supplementary to $\angle A$.

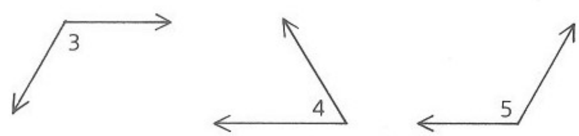


How large is $\angle 1$? Now calculate $\angle 2$. How does $\angle 1$ compare with $\angle 2$? Your results will illustrate (but not prove) the following theorem.

Theorem 4 ***If angles are supplementary to the same angle, then they are congruent.***

Given: $\angle 3$ is supp. to $\angle 4$.
$\angle 5$ is supp. to $\angle 4$.

Prove: $\angle 3 \cong \angle 5$

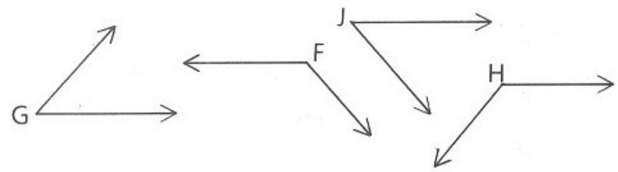


Proof: $\angle 3$ is supp. to $\angle 4$, so $m\angle 3 + m\angle 4 = 180$.
Therefore, $m\angle 3 = 180 - m\angle 4$.
$\angle 5$ is supp. to $\angle 4$, so $m\angle 5 + m\angle 4 = 180$.
Therefore, $m\angle 5 = 180 - m\angle 4$.
Since $\angle 3$ and $\angle 5$ have the same measure, $\angle 3 \cong \angle 5$.

A companion to Theorem 4 follows.

Theorem 5 ***If angles are supplementary to congruent angles, then they are congruent.***

Given: ∠F is supp. to ∠G.
∠H is supp. to ∠J.
∠G ≅ ∠J
Conclusion: ∠F ≅ ∠H



The proof of Theorem 5 is similar to that of Theorem 4.
Two similar theorems apply to complementary angles.

Theorem 6 ***If angles are complementary to the same angle, then they are congruent.***

Theorem 7 ***If angles are complementary to congruent angles, then they are congruent.***

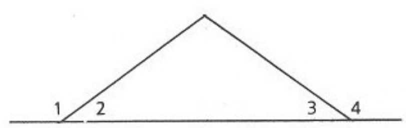
When studying the definitions of such terms as *right angle, bisect, midpoint,* and *perpendicular,* you will master the concepts more quickly if you try to understand the ideas involved without memorizing the definitions word for word. The theorems in this section, however, are different. Unless you *memorize* Theorems 4–7, you will have difficulty remembering the concepts they contain.

Therefore, before you begin your homework,

1 Memorize Theorems 4–7
2 Read the sample problems carefully, so that you understand which of the theorems is used in each type of problem

Part Two: Sample Problems

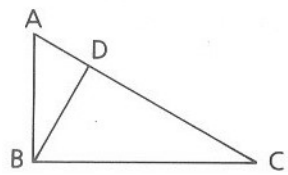
Problem 1 *Given:* ∠1 *is supp. to* ∠2.
∠3 *is supp. to* ∠4.
∠1 ≅ ∠4
Conclusion: ∠2 ≅ ∠3



Proof

Statements	Reasons
1 ∠1 is supp. to ∠2.	1 Given
2 ∠3 is supp. to ∠4.	2 Given
3 ∠1 ≅ ∠4	3 Given
4 ∠2 ≅ ∠3	4 If angles are supplementary to ≅ angles, they are ≅. (Short form: Supplements of ≅ ∠s are ≅.)

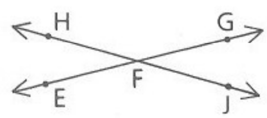
Problem 2 *Given:* $\angle A$ *is comp. to* $\angle C$.
$\angle DBC$ *is comp. to* $\angle C$.
Conclusion: ___?___



Proof

Statements	Reasons
1 $\angle A$ is comp. to $\angle C$.	1 Given
2 $\angle DBC$ is comp. to $\angle C$.	2 Given
3 $\angle A \cong \angle DBC$	3 If angles are complementary to the same angle, they are $\cong$. (Short form: Complements of the same $\angle$ are $\cong$.)

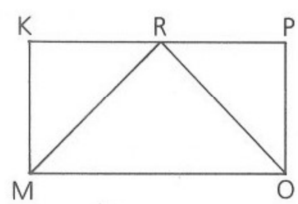
Problem 3 *Given: Diagram as shown*
Prove: $\angle HFE \cong \angle GFJ$



Proof

Statements	Reasons
1 Diagram as shown.	1 Given
2 $\angle EFG$ is a straight $\angle$.	2 Assumed from diagram
3 $\angle HFE$ is supp. to $\angle HFG$.	3 If two $\angle$s form a straight $\angle$, they are supplementary.
4 $\angle HFJ$ is a straight $\angle$.	4 Same as 2
5 $\angle GFJ$ is supp. to $\angle HFG$.	5 Same as 3
6 $\angle HFE \cong \angle GFJ$	6 If angles are supplementary to the same angle, they are $\cong$. (Short form: Supplements of the same $\angle$ are $\cong$.)

Problem 4 *Given:* $\overline{KM} \perp \overline{MO}$,
$\overline{PO} \perp \overline{MO}$,
$\angle KMR \cong \angle POR$
Prove: $\angle ROM \cong \angle RMO$



Proof

Statements	Reasons
1 $\overline{KM} \perp \overline{MO}$	1 Given
2 $\angle KMO$ is a right $\angle$.	2 If segments are $\perp$, they form right $\angle$s.
3 $\angle RMO$ is comp. to $\angle KMR$.	3 If two $\angle$s form a right $\angle$, they are complementary.
4 In a similar manner, $\angle ROM$ is comp. to $\angle POR$.	4 Reasons 1–3
5 $\angle KMR \cong \angle POR$	5 Given
6 $\angle ROM \cong \angle RMO$	6 If angles are complementary to $\cong$ angles, they are $\cong$. (Short form: Complements of $\cong$ $\angle$s are $\cong$.)

2.5 ADDITION AND SUBTRACTION PROPERTIES

Objectives

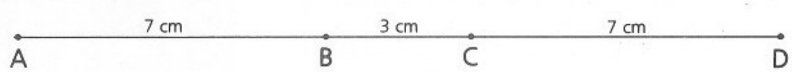
After studying this section, you will be able to

- Apply the addition properties of segments and angles
- Apply the subtraction properties of segments and angles

Part One: Introduction

Addition Properties

In the diagram below, AB = CD. Do you think that AC = BD? Suppose that BC were 3 cm. Would AC = BD? If AB = CD, does the length of BC have any effect on whether AC = BD?



Your answers should be that AC = BD in each case and the length of BC does not effect that equality. This is a geometric application of the algebraic Addition Property of Equality (AB + BC = CD + BC).

Theorem 8 ***If a segment is added to two congruent segments, the sums are congruent. (Addition Property)***

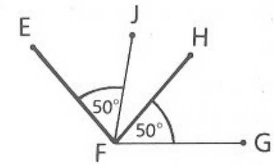
Given: $\overline{PQ} \cong \overline{RS}$

Conclusion: $\overline{PR} \cong \overline{QS}$

Proof: $\overline{PQ} \cong \overline{RS}$, so by definition of congruent segments, PQ = RS. Now, the Addition Property of Equality says that we may add QR to both sides, so PQ + QR = RS + QR. Substituting, we get PR = QS. Therefore, $\overline{PR} \cong \overline{QS}$ by the definition of *congruent segments* (reversed).

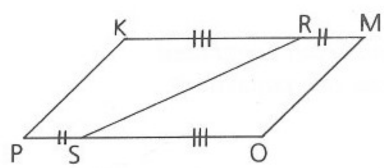
Does a similar relationship hold for angles? Is ∠EFH necessarily congruent to ∠JFG?

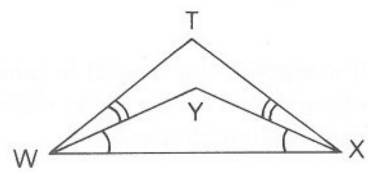
The next theorem confirms that the answer is yes. Its proof is like that of Theorem 8.



Theorem 9 ***If an angle is added to two congruent angles, the sums are congruent. (Addition Property)***

In the figures below, identical tick marks indicate congruent parts.





Do you think that $\overline{KM}$ is necessarily congruent to $\overline{PO}$? In the right-hand diagram, is ∠TWX necessarily congruent to ∠TXW? The answer to these questions is yes.

These congruencies are established by the following two theorems. Their proofs are similar to that of Theorem 8.

Theorem 10 ***If congruent segments are added to congruent segments, the sums are congruent. (Addition Property)***

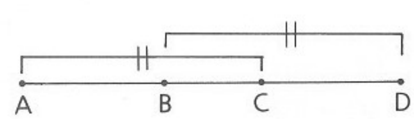
Theorem 11 ***If congruent angles are added to congruent angles, the sums are congruent. (Addition Property)***

Subtraction Properties

We now have four addition properties. Because subtraction is equivalent to addition of an opposite, we can expect four corresponding subtraction properties.

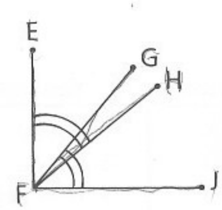
If AC = BD, is AB = CD?

Let AC = 12 and BC = 3.
How long is $\overline{BD}$?
Is AB = CD?



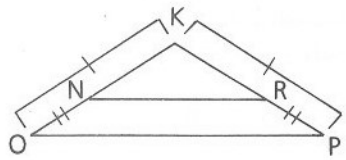
If ∠EFH ≅ ∠GFJ, is ∠EFG ≅ ∠HFJ?

Let m∠EFH = 50 and m∠GFH = 10.
How large is ∠GFJ?
Is ∠EFG ≅ ∠HFJ?



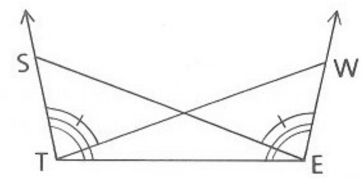
If KO = KP and NO = RP, is KN = KR?

Try this on your own and see what you think.



If ∠STE ≅ ∠WET and ∠STW ≅ ∠WES, is ∠WTE ≅ ∠SET?

Try this on your own.



Your results should agree with the next two theorems.

Theorem 12 ***If a segment (or angle) is subtracted from congruent segments (or angles), the differences are congruent. (Subtraction Property)***

Theorem 13 ***If congruent segments (or angles) are subtracted from congruent segments (or angles), the differences are congruent. (Subtraction Property)***

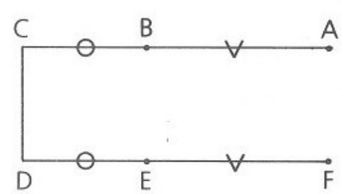
Using the Addition and Subtraction Properties in Proofs

1. An addition property is used when the segments or angles in the conclusion are *greater than* those in the given information.
2. A subtraction property is used when the segments or angles in the conclusion are *smaller than* those in the given information.

Part Two: Sample Problems

Problem 1 *Given:* $\overline{AB} \cong \overline{FE}$, $\overline{BC} \cong \overline{ED}$

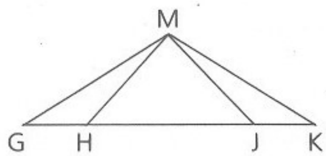
Prove: $\overline{AC} \cong \overline{FD}$



Proof

Statements	Reasons
1 $\overline{AB} \cong \overline{FE}$	1 Given
2 $\overline{BC} \cong \overline{ED}$	2 Given
3 $\overline{AC} \cong \overline{FD}$	3 If ≅ segments are added to ≅ segments, the sums are ≅. (Addition Property)

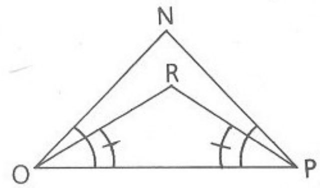
Problem 2 *Given:* $\overline{GJ} \cong \overline{HK}$
Conclusion: $\overline{GH} \cong \overline{JK}$



Proof

Statements	Reasons
1 $\overline{GJ} \cong \overline{HK}$	1 Given
2 $\overline{GH} \cong \overline{JK}$	2 If a segment ($\overline{HJ}$) is subtracted from $\cong$ segments, the differences are $\cong$. (Subtraction Property)

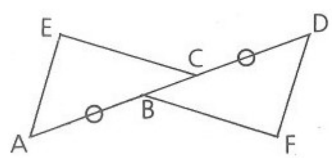
Problem 3 *Given:* $\angle NOP \cong \angle NPO$,
$\angle ROP \cong \angle RPO$
Prove: $\angle NOR \cong \angle NPR$



Proof

Statements	Reasons
1 $\angle NOP \cong \angle NPO$	1 Given
2 $\angle ROP \cong \angle RPO$	2 Given
3 $\angle NOR \cong \angle NPR$	3 If $\cong$ angles are subtracted from $\cong$ angles, the differences are $\cong$. (Subtraction Property)

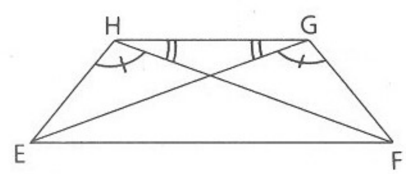
Problem 4 *Given:* $\overline{AB} \cong \overline{CD}$
Conclusion: ___?___



Proof

Statements	Reasons
1 $\overline{AB} \cong \overline{CD}$	1 Given
2 $\overline{AC} \cong \overline{BD}$	2 If a segment ($\overline{BC}$) is added to $\cong$ segments, the sums are $\cong$. (Addition Property)

Problem 5 *Given:* $\angle HEF$ *is supp. to* $\angle EHG$.
$\angle GFE$ *is supp. to* $\angle FGH$.
$\angle EHF \cong \angle FGE$,
$\angle GHF \cong \angle HGE$
Conclusion: $\angle HEF \cong \angle GFE$



Proof

Statements	Reasons
1 $\angle HEF$ is supp. to $\angle EHG$.	1 Given
2 $\angle GFE$ is supp. to $\angle FGH$.	2 Given
3 $\angle EHF \cong \angle FGE$	3 Given
4 $\angle GHF \cong \angle HGE$	4 Given
5 $\angle EHG \cong \angle FGH$	5 If $\cong$ angles are added to $\cong$ angles, the sums are $\cong$. (Addition Property)
6 $\angle HEF \cong \angle GFE$	6 Supplements of $\cong$ $\angle$s are $\cong$.

2.6 Multiplication and Division Properties

Objective

After studying this section, you will be able to

- Apply the multiplication and division properties of segments and angles

Part One: Introduction

In the figure below, B, C, F, and G are trisection points.

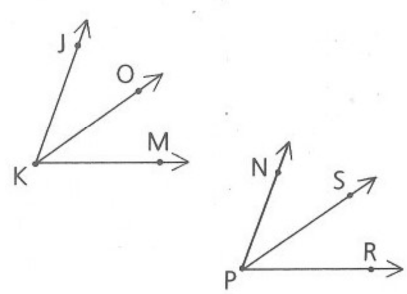
If AB = EF = 3, what can we say about $\overline{AD}$ and $\overline{EH}$?

If $\overline{AB} \cong \overline{EF}$, is $\overline{AD}$ congruent to $\overline{EH}$?

In the figure at the right, $\overrightarrow{KO}$ and $\overrightarrow{PS}$ are angle bisectors.

If m∠JKO = m∠NPS = 25, what can we say about ∠JKM and ∠NPR?

If ∠JKO ≅ ∠NPS, is ∠JKM congruent to ∠NPR?



The examples above illustrate a property whose proof is similar to the proof of Theorem 8.

Theorem 14 ***If segments (or angles) are congruent, their like multiples are congruent. (Multiplication Property)***

Also, because division is equivalent to multiplication by the reciprocal of the divisor, it is easy to prove the next theorem.

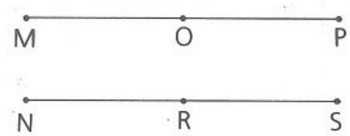
Theorem 15 ***If segments (or angles) are congruent, their like divisions are congruent. (Division Property)***

Using the Multiplication and Division Properties in Proofs

1 Look for a double use of the word *midpoint* or *trisect* or *bisects* in the given information.
2 The Multiplication Property is used when the segments or angles in the conclusion are *greater than* those in the given information.
3 The Division Property is used when the segments or angles in the conclusion are *smaller than* those in the given information.

Part Two: Sample Problems

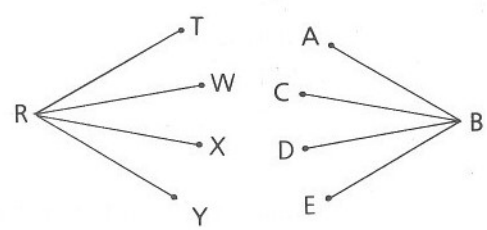
Problem 1 *Given:* $\overline{MP} \cong \overline{NS}$;
O is the midpoint of $\overline{MP}$.
R is the midpoint of $\overline{NS}$.
Prove: $\overline{MO} \cong \overline{NR}$



Proof

Statements	Reasons
1 $\overline{MP} \cong \overline{NS}$	1 Given
2 O is the midpoint of $\overline{MP}$.	2 Given
3 R is the midpoint of $\overline{NS}$.	3 Given
4 $\overline{MO} \cong \overline{NR}$	4 If segments are ≅, their like divisions (halves) are ≅. (Division Property)

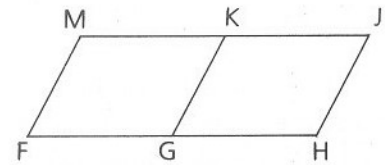
Problem 2 *Given:* $\angle TRY \cong \angle ABE$;
$\overrightarrow{RW}$ *and* $\overrightarrow{RX}$ *trisect* $\angle TRY$.
$\overrightarrow{BC}$ *and* $\overrightarrow{BD}$ *trisect* $\angle ABE$.
Conclusion: $\angle TRW \cong \angle CBD$



Proof

Statements	Reasons
1 $\angle TRY \cong \angle ABE$	1 Given
2 $\overrightarrow{RW}$ and $\overrightarrow{RX}$ trisect $\angle TRY$.	2 Given
3 $\overrightarrow{BC}$ and $\overrightarrow{BD}$ trisect $\angle ABE$.	3 Given
4 $\angle TRW \cong \angle CBD$	4 If angles are ≅, their like divisions (thirds) are ≅. (Division Property)

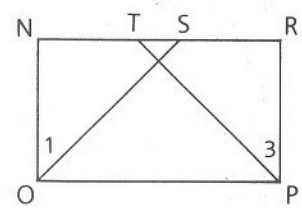
Problem 3 *Given:* $\overline{MK} \cong \overline{FG}$;
$\overline{KG}$ *bisects* $\overline{MJ}$ *and* $\overline{FH}$.
Prove: $\overline{MJ} \cong \overline{FH}$



Proof

Statements	Reasons
1 $\overline{MK} \cong \overline{FG}$	1 Given
2 $\overline{KG}$ bisects $\overline{MJ}$ and $\overline{FH}$.	2 Given
3 $\overline{MJ} \cong \overline{FH}$	3 If segments are ≅, their like multiples (doubles) are ≅. (Multiplication Property)

Problem 4 *Given:* $\angle NOP \cong \angle RPO$;
$\overrightarrow{PT}$ *bisects* $\angle RPO$.
$\overrightarrow{OS}$ *bisects* $\angle NOP$.
$\angle NSO$ *is comp. to* $\angle 1$.
$\angle RTP$ *is comp. to* $\angle 3$.
Prove: $\angle NSO \cong \angle RTP$



Proof

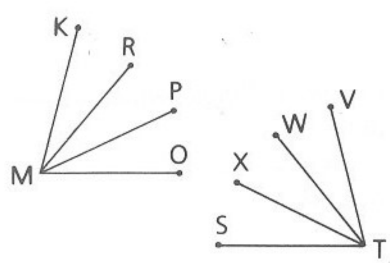
Statements	Reasons
1 $\angle NOP \cong \angle RPO$	1 Given
2 $\overrightarrow{PT}$ bisects $\angle RPO$.	2 Given
3 $\overrightarrow{OS}$ bisects $\angle NOP$.	3 Given
4 $\angle 1 \cong \angle 3$	4 Halves of ≅ angles are ≅. (An alternative form of the Division Property)
5 $\angle NSO$ is comp. to $\angle 1$.	5 Given
6 $\angle RTP$ is comp. to $\angle 3$.	6 Given
7 $\angle NSO \cong \angle RTP$	7 Complements of ≅ ∠s are ≅.

Part Three: Problem Sets

Problem Set A

Before starting the proofs in this problem set, reread the chart on page 90.

1 Given: $\angle KMR \cong \angle VTW$;
$\overrightarrow{MR}$ and $\overrightarrow{MP}$ trisect $\angle KMO$.
$\overrightarrow{TX}$ and $\overrightarrow{TW}$ trisect $\angle STV$.
Prove: $\angle KMO \cong \angle STV$



2.7

Transitive and Substitution Properties

Objectives

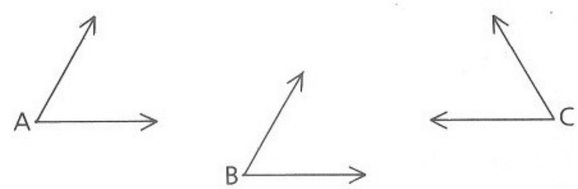
After studying this section, you will be able to

- Apply the transitive properties of angles and segments
- Apply the Substitution Property

Part One: Introduction

Transitive Properties

Suppose that $\angle A \cong \angle B$ and $\angle A \cong \angle C$. Is $\angle B \cong \angle C$?



The transitive property of algebra can be used to prove this general rule.

Theorem 16 ***If angles (or segments) are congruent to the same angle (or segment), they are congruent to each other. (Transitive Property)***

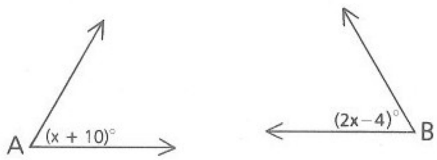
Theorem 16 can be used twice to prove the next theorem.

Theorem 17 ***If angles (or segments) are congruent to congruent angles (or segments), they are congruent to each other. (Transitive Property)***

Substitution Property

In your algebra studies and in some of the problems you have worked this year, you have solved for a variable such as x and then ***substituted*** the value you found for that variable.

Example *If* $\angle A \cong \angle B$, *find* $m\angle A$.

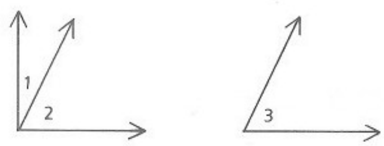


$$2x - 4 = x + 10$$
$$x = 14$$

We can now substitute 14 for x in $m\angle A = x + 10$ to find that $m\angle A = 14 + 10 = 24$.

The Substitution Property can also be applied when no variables are involved.

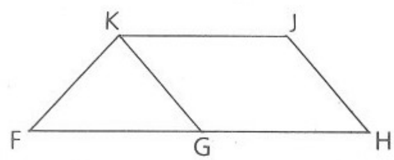
If $\angle 1$ is comp. to $\angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1$ is comp. to $\angle 3$ by ***substitution.***



Part Two: Sample Problems

Problem 1 *Given:* $\overline{FG} \cong \overline{KJ}$, $\overline{GH} \cong \overline{KJ}$

Prove: $\overleftrightarrow{KG}$ *bisects* $\overline{FH}$.

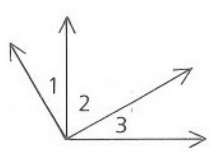


Proof

Statements	Reasons
1 $\overline{FG} \cong \overline{KJ}$	1 Given
2 $\overline{GH} \cong \overline{KJ}$	2 Given
3 $\overline{FG} \cong \overline{GH}$	3 If segments are ≅ to the same segment, they are ≅. (Transitive Property)
4 $\overleftrightarrow{KG}$ bisects $\overline{FH}$.	4 If a line divides a segment into two ≅ segments, it bisects the segment.

Problem 2 *Given:* $\angle 1 + \angle 2 = 90°$, $\angle 1 \cong \angle 3$

Prove: $\angle 3 + \angle 2 = 90°$



Proof

Statements	Reasons
1 $\angle 1 + \angle 2 = 90°$	1 Given
2 $\angle 1 \cong \angle 3$	2 Given
3 $\angle 3 + \angle 2 = 90°$	3 Substitution (step 2 in step 1)

2.8 VERTICAL ANGLES

Objectives

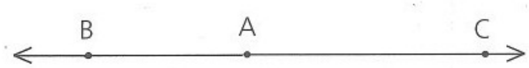
After studying this section, you should be able to

- Recognize opposite rays
- Recognize vertical angles

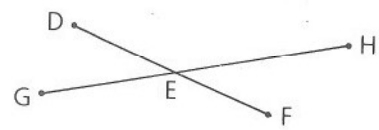
Part One: Introduction

Opposite Rays

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are ***opposite rays.***



$\overrightarrow{ED}$ and $\overrightarrow{EF}$ are also opposite rays, as are $\overrightarrow{EG}$ and $\overrightarrow{EH}$.

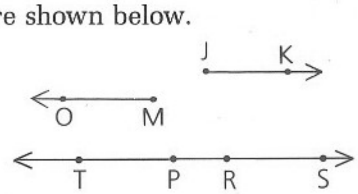


Definition Two collinear rays that have a common endpoint and extend in different directions are called ***opposite rays***.

Some pairs of rays that are *not* opposite rays are shown below.

$\overrightarrow{JK}$ and $\overrightarrow{MO}$ are not parts of the same line.

$\overrightarrow{PT}$ and $\overrightarrow{RS}$ are not opposite, since they do not have a common endpoint.

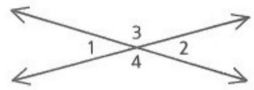


Vertical Angles

Whenever two lines intersect, two pairs of ***vertical angles*** are formed.

Definition Two angles are ***vertical angles*** if the rays forming the sides of one and the rays forming the sides of the other are opposite rays.

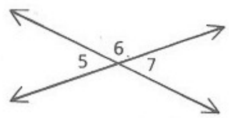
∠1 and ∠2 are vertical angles.
∠3 and ∠4 are vertical angles.



Are ∠3 and ∠2 vertical angles? How do vertical angles compare in size?

Theorem 18 ***Vertical angles are congruent.***

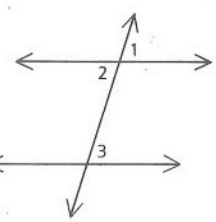
Given: Diagram as shown
Prove: ∠5 ≅ ∠7



We proved Theorem 18 in Section 2.4, sample problem 3.

Part Two: Sample Problems

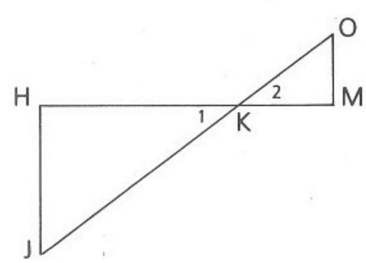
Problem 1 *Given: ∠2 ≅ ∠3*
Prove: ∠1 ≅ ∠3



Proof

Statements	Reasons
1 ∠2 ≅ ∠3	1 Given
2 ∠1 ≅ ∠2	2 Vertical angles are congruent.
3 ∠1 ≅ ∠3	3 If ∠s are ≅ to the same ∠, they are ≅. (Transitive Property)

Problem 2 *Given: ∠O is comp. to ∠2.*
∠J is comp. to ∠1.
Conclusion: ∠O ≅ ∠J



Proof

Statements	Reasons
1 ∠O is comp. to ∠2.	1 Given
2 ∠J is comp. to ∠1.	2 Given
3 ∠1 ≅ ∠2	3 Vertical angles are congruent.
4 ∠O ≅ ∠J	4 Complements of ≅ ∠s are ≅.