Name: _____ FaCtoRiNg!

Factoring Polynomials

❖ Factoring: An Introduction
- ➢ Factoring is the reverse of multiplying.
  - ▪ To factor an expression means to write an equivalent expression that is a product of two or more expressions (called factors).

❖ Factoring: The Process
- ➢ Always look for COMMON FACTORS
  - ▪ Use Divisibility Rules to spot the GCF!
    - • These rules let you test if one number is divisible by another – with little, to no, calculation!

A number is divisible by…	if…
2	The last digit is even: 0, 2, 4, 6, 8
3	The sum of the digits is divisible by 3
4	The number formed by the last two digits is divisible by 4
5	The last digit is 0 or 5
6	The number is even and divisible by 3
8	The number formed by the last three digits is divisible by 8
9	The sum of the digits is divisible by 9
10	The last digit is 0

- ➢ Factor out the GCF – greatest common factor
  - ▪ The GCF appears in – or is common to – EVERY term in the expression

Examples:

1. $12u^3v^2 + 16uv^4$

2. $18y^4 - 6y^3 + 12y^2$

❖ Factoring: The Process (continued)
- ➢ Look for Special Cases Involving Binomials
  - ▪ Difference of Squares: $a^2 - b^2 = (a + b)(a - b)$
    - • Example:

$$4x^2 - 36$$
$$4(x^2 - 9) \quad \text{Always look for a common factor.}$$
$$4(x^2 - 3^2) \quad \text{Write as } a^2 - b^2$$
$$4(x + 3)(x - 3) \quad \text{Factoring using pattern}$$

- Sum of Cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
  - Example:

$8x^3 + 27$	
$(2x)^3 + 3^3$	Write as $a^3 + b^3$
$(2x + 3)(4x^2 - 6x + 9)$	Factor using pattern

- Difference of Cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
  - Example:

$2x^3 - 128$	
$2(x^3 - 64)$	Always look for a common factor
$2(x^3 - 4^3)$	Write as $a^3 - b^3$
$2(x - 4)(x^2 + 4x + 16)$	Factor using pattern

Examples:

3. $9x^2 - 25y^2$

4. $81m^4 - 16$

5. $3n^3 - 24$

6. $27a^3 + 64b^3$

- Look for Special Cases Involving Trinomials
  - Perfect Square Trinomials
    - $a^2 + 2ab + b^2 = (a + b)^2$

$x^2 + 6x + 9$	
x^2: a perfect square	Is the first term a perfect square?
$9 = 3^2$: a perfect square	Is the last term a perfect square?
$6x = 2(x)(3)$	Is the middle term the product of 2 times the square roots of the first term and the last term?
$(x + 3)^2$	Factor using pattern

    - $a^2 - 2ab + b^2 = (a - b)^2$

$5x^2 - 20x + 20$	
$5(x^2 - 4x + 4)$	Always look for a common factor
x^2: a perfect square	Is the first term a perfect square?
$4 = 2^2$: a perfect square	Is the last term a perfect square?
$4x = 2(x)(2)$	Is the middle term the product of 2 times the square roots of the first term and the last term?
$5(x - 2)^2$	Factor using pattern

Examples:

7. $9a^2 + 24a + 16$

8. $4x^2 - 12xy + 9y^2$

❖ Factoring: The Process (continued)

- Look for expressions with 4 terms & factor by grouping
  - Group the first two and the last two terms
  - Take out the GCF from each group
    - Example:

$$6x^4 + 5x^3 - 24x - 20$$

$(6x^4 + 5x^3)(-24x - 20)$ — Group the first two and the last two terms

$x^3(6x - 5) - 4(6x - 5)$ — Take out the GCF from each group

$$(6x - 5)(x^3 - 4)$$

Example:

9. $6x^2 + 3xy + 8x + 4y$

❖ Factoring: The Process (continued)

- DEFOILING – method for factoring any trinomial
  - Multiply the first and last terms
  - Find the factors that add up to be the middle term
  - Replace the middle term with these factors
  - Factor by grouping
    - Example:

$$8x^2 + 14x - 15$$

$(8x^2)(-15) = -120x^2$ — Multiply the first and last terms

$-120x^2$	
$20x \ \& \ -6x$	$14x$

Find the factors that add up to be the middle term

$8x^2 + 20x - 6x - 15$ — Replace the middle term with these factors

$(8x^2 + 20x)(-6x - 15)$ — Factor by grouping

$$4x(2x + 5) - 3(2x + 5)$$

$$(2x + 5)(4x - 3)$$

Examples:

10. $6x^2 + 7x + 2$

11. $3x^2 - 21x + 36$

12. $8x^2 - 2$

13. $6m^2 + 15mn - 9n^2$

FACTORING FLOW CHART

Always look for a common factor first!

↓

Then look at the number of terms.

↙ ↓ ↘

Binomial

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Is it a difference of squares OR a sum/difference of cubes?

↙ ↘

No.

Yes.

↓

$a^2 - b^2 = (a + b)(a - b)$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Four Terms

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Try factoring by grouping.

Trinomial

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Is it a perfect square trinomial?

↙ ↘

Yes.

No.

↓

Look @ the middle term.

↓

Defoil.

↙ ↘

Positive.

Negative.

↓

$a^2+2ab+b^2=(a+b)^2$

↓

$a^2-2ab+b^2=(a-b)^2$