Name: ______________________________

Getting to Know Your Grapher

Table Building

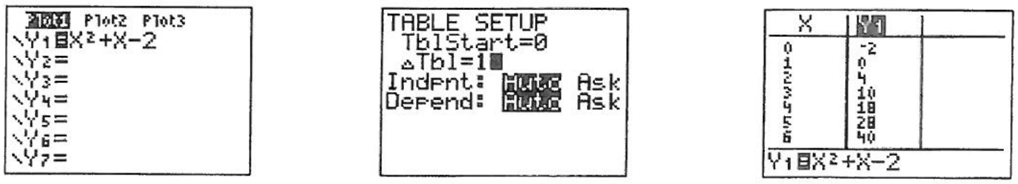
A grapher feature even more powerful than the ability to store variables is the ability to store functions. This feature is the basis of graphing and table building. In either case the *Y= edit screen* is used to store the symbolic expressions (rules) for functions. A table can be used to evaluate a function for several different x-values.

For the TI-83 and TI-84,

- press [Y=] (or [SYMB]) to go to the Y = screen. Then press [X,T,θ,n] [x^2] [+] [X,T,θ,n] [−] **2** [ENTER]. See Figure 1.8a.
- press [TBLSET] **0** [ENTER] **1** [ENTER] [ENTER]. See Figure 1.8b. Then press [TABLE]. See Figure 1.8c.

For the TI-89 Titanium, enter the Y = Editor from the home screen, then press [X] [^] **2** [+] [X] [−] **2** [ENTER].

- Press [TBLSET] **0** [▼] **1** [ENTER]. Then press [TABLE].



(a) (b) (c)

Figure 1.8 The steps in the table-building process on a grapher.

- Use the cursor keys ([▶], [◀], [▲], and [▼]) to move around the table and explore. Pay attention to the readout at the bottom of the screen as you move to different "cells" in the table. What happens when you try to move the cursor off the top or bottom of the screen?

Function Graphing

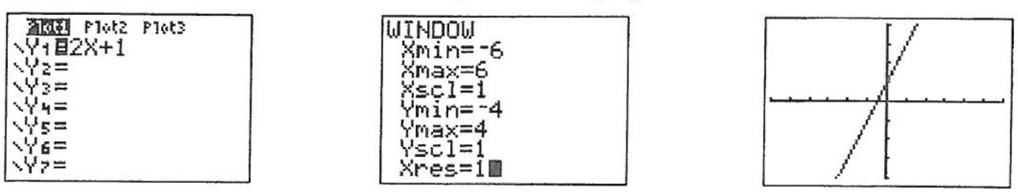
Graphing and Exploring a Function

Most graphers have several graphing modes. Be sure that your grapher is in FUNCTION mode. The algebraic form for the function needs to be $f(x) = \ldots$, or $y = \ldots$.

Example Graphing and Tracing Along a Function

Use the FUNCTION mode to graph $f(x) = 2x + 1$. Explore the ordered pairs of the graph with the TRACE feature.

Solution Figure 1.9 illustrates the procedure for graphing f in the window $[-6, 6]$ by $[-4, 4]$. Window is called range on some graphers.



(a) (b) (c)

Figure 1.9 The steps in the graphing process.

"Friendly" Windows

Square Windows

The graph of $f(x) = 2x + 1$ is a straight line with a slope of 2. You have seen several views of this graph in different windows. The apparent steepness of the graph can be quite different even though the slope is always 2.

- Graph $f(x) = 2x + 1$ in the window $[-9, 9]$ by $[-2, 2]$ and then in the window $[-9, 9]$ by $[-20, 20]$. Compare the apparent steepness of the graph in the two windows.

In general, to obtain a graph that suggests the graph's true shape, you must choose viewing dimensions that are proportional to the dimensions of your grapher screen. Most grapher screens have a width-to-height ratio of roughly 3 : 2. Windows whose dimensions are proportional to the physical dimensions of the grapher screen are called *square windows.* Square windows yield true shapes: They make perpendicular lines look perpendicular, squares look square, and circles look circular.

Example   Rounding Out a Circle

Use your grapher to plot the circle $x^2 + y^2 = 1$.

Solution First, you will need to do some algebra:

$$x^2 + y^2 = 1$$
$$y^2 = 1 - x^2$$
$$y = \pm\sqrt{1 - x^2}$$

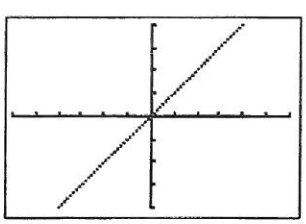
So the graph of the circle $x^2 + y^2 = 1$ is the union of the graphs of the functions $y_1 = \sqrt{1 - x^2}$ and $y_2 = -\sqrt{1 - x^2}$.

- Graph $y_1 = \sqrt{(1 - x^2)}$ and $y_2 = -\sqrt{(1 - x^2)}$ in several windows with different x-y dimension ratios. (We used parentheses around $1 - x^2$ because you will need them to enter the functions onto the Y= edit screen.) Continue until you obtain a graph that appears circular. *Note:* Gaps may appear near the x-axis.
- Graph $y_1 = \sqrt{1 - x^2}$ and $y_2 = -\sqrt{1 - x^2}$ in a square, friendly window.

Grapher Note: Some graphers have a ZSquare feature that adjusts the window dimensions to make them match the physical proportions of the screen.

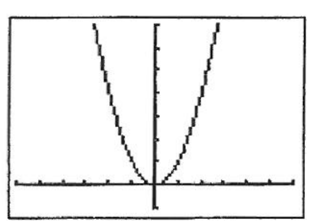
A Gallery of Basic Functions

Plot and explore the 12 graphs shown in Figure 1.13.



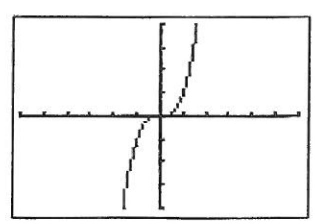
$[-6, 6]$ by $[-4, 4]$

$y = x$



$[-6, 6]$ by $[-1, 7]$

$y = x^2$



$[-6, 6]$ by $[-4, 4]$

$y = x^3$

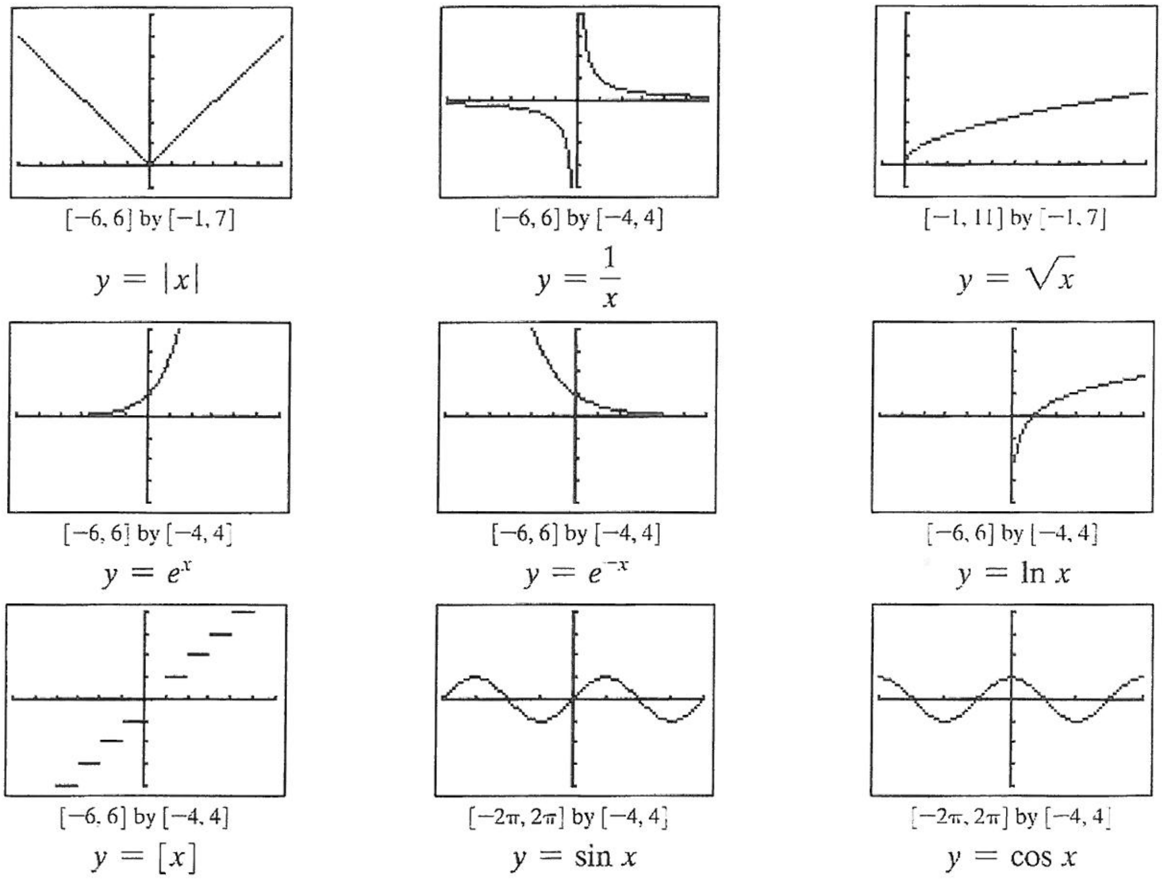


Figure 1.13 Gallery of Basic Functions

- Graph all but the sine and cosine functions with the windows indicated in Figure 1.13. Regraph each function for other window dimensions, including standard and square. Explore with TRACE.

- Graph the greatest integer function in CONNECTED mode and then in DOT mode. Which produces the better graph? Why?

- Graph the sine and cosine functions in DEGREE mode and then in RADIAN mode. Note that the graphs in Figure 1.13 are based on the use of RADIAN mode.

Grapher Note: Some graphers have ZTrig that automatically sets desirable window dimensions for viewing trigonometric functions.

Graphical Problem Solving

In this section we explore various grapher methods for solving equations and analyzing the graphical behavior of functions, so you should set your grapher to FUNCTION mode. We begin by showing how to solve equations graphically, using the example

$$|x| = x^2 + x - 2,$$

first by graphing

$$f(x) = |x| \qquad \text{and} \qquad g(x) = x^2 + x - 2$$

separately and then by investigating the related function

$$h(x) = f(x) - g(x) = |x| - (x^2 + x - 2).$$

Solving an Equation by Finding Intersections

We can solve an equation by graphing each side as a function and locating the points of intersection.

- Enter each side of the equation $|x| = x^2 + x - 2$ onto the Y= edit screen as shown in Figure 1.14.
- Graph the equations in a friendly window. See Figure 1.15.

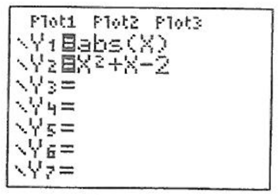
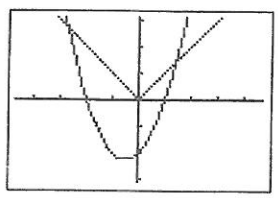


Figure 1.14



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

Figure 1.15 The dimensions of a friendly window vary from grapher to grapher. Possible x dimensions are given in Table 1.2. ZDECIMAL or ZINTEGER yield a square, friendly window on some graphers.

The graph of f is V-shaped and the graph of g is an upward opening parabola. They have two points of intersection. Thus the equation $|x| = x^2 + x - 2$ has two solutions. These solutions are the x-coordinates (one positive and one negative) of the two points of intersection.

- Trace along either graph to approximate the positive solution, that is, to estimate the x-coordinate of the point of intersection in the first quadrant.

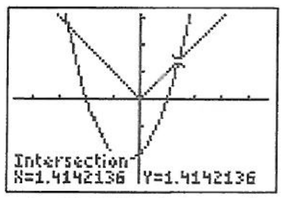
To get a better approximation for the positive solution we can *zoom in* by picking smaller and smaller windows that contain the point of intersection in the first quadrant. Three common ways to zoom in are to

1. change the WINDOW settings manually;
2. use ZOOMBOX, which lets you use the cursor to select the opposite corners of a "box" to define a new window; and

3. use ZOOMIN, which magnifies the graph around the cursor location by a factor that you can set.

Most graphers have an INTERSECTION feature that can be used to automate the process of solving equations graphically without adjusting the viewing window dimensions.

- Graph the equations of Figure 1.14 in a friendly window (Figure 1.15). Then use the INTERSECTION feature to locate the point of intersection in the first quadrant, as shown in Figure 1.17.



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

Figure 1.17

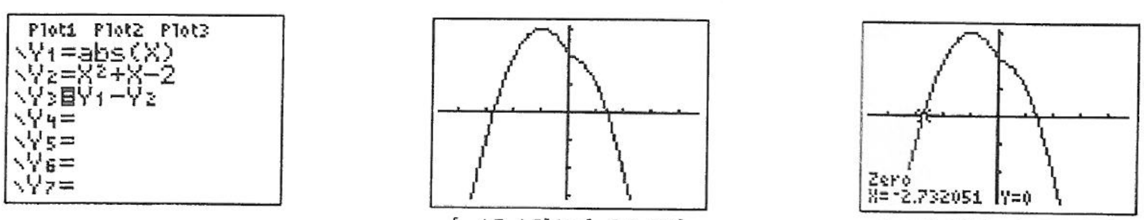
Solving the equation $|x| = x^2 + x - 2$ algebraically reveals that the positive solution is $x = \sqrt{2} = 1.4142\ldots$, which confirms the graphical solution.

Solving by Finding *x*-Intercepts

To solve an equation of the form $f(x) = g(x)$, we can solve $f(x) - g(x) = 0$. Then the problem becomes one of finding where the functions $y = f - g$ and $y = 0$ intersect, or simply the x-intercepts of $y = f - g$. For example, to solve the equation $|x| = x^2 + x - 2$, we can find the x-intercepts of

$$y = h(x) = f(x) - g(x) = |x| - (x^2 + x - 2).$$

- Load the Y= edit screen, as shown in Figure 1.18a, selecting only $y_3 = y_1 - y_2 = \text{abs}\,(x) - (x^2 + x - 2)$ to be graphed.
- Graph y_3 in a friendly window, as shown in Figure 1.18b.
- The x-intercepts are also the "zeros" of the equation. Use the ZERO feature (CYGOM if necessary) to locate the negative x-intercept, as shown in Figure 1.18c.



$[-4.7, 4.7]$ by $[-3.1, 3.1]$ $[-4.7, 4.7]$ by $[-3.1, 3.1]$

(a) (b) (c)

Figure 1.18 Finding a solution of an equation by finding an x-intercept of the difference function.

CALCULATE Menu

To display the CALCULATE menu, press [2nd] [CALC]. Use the items on this menu to analyze the current graph functions.

CALCULATE	
1: value	Calculates a function **Y** value for a given **X**.
2: zero	Finds a zero (x-intercept) of a function.
3: minimum	Finds a minimum of a function.
4: maximum	Finds a maximum of a function.
5: intersect	Finds an intersection of two functions.
6: dy/dx	Finds a numeric derivative of a function.
7: ∫f(x)dx	Finds a numeric integral of a function.

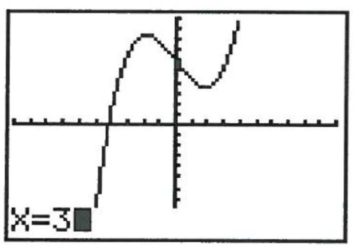
value

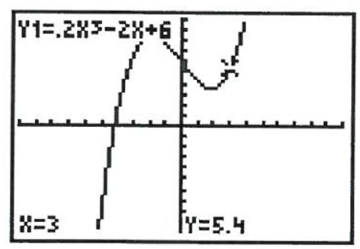
value evaluates one or more currently selected functions for a specified value of **X**.

Note: When a value is displayed for **X**, press [CLEAR] to clear the value. When no value is displayed, press [CLEAR] to cancel the **value** operation.

To evaluate a selected function at **X**, follow these steps.

1. Select **1:value** from the CALCULATE menu. The graph is displayed with **X=** in the bottom-left corner.
2. Enter a real value, which can be an expression, for **X** between **Xmin** and **Xmax**.
3. Press [ENTER].





The cursor is on the first selected function in the Y= editor at the **X** value you entered, and the coordinates are displayed, even if **CoordOff** format is selected.

To move the cursor from function to function at the entered **X** value, press [▲] or [▼]. To restore the free-moving cursor, press [◄] or [►].

zero

zero finds a zero (x-intercept or root) of a function using **solve(**. Functions can have more than one x-intercept value; **zero** finds the zero closest to your guess.

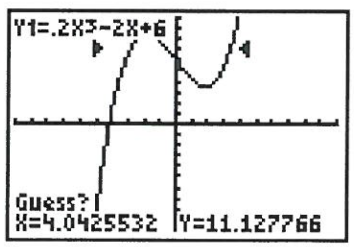
The time **zero** spends to find the correct zero value depends on the accuracy of the values you specify for the left and right bounds and the accuracy of your guess.

To find a zero of a function, follow these steps.

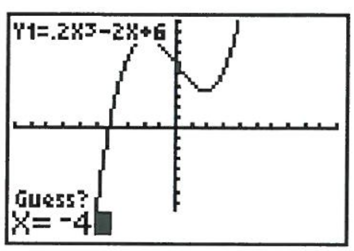
1. Select **2:zero** from the CALCULATE menu. The current graph is displayed with **Left Bound?** in the bottom-left corner.

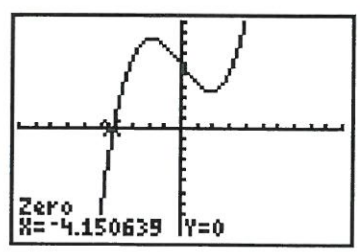
2. Press [▲] or [▼] to move the cursor onto the function for which you want to find a zero.

3. Press [◄] or [►] (or enter a value) to select the x-value for the left bound of the interval, and then press [ENTER]. A ► indicator on the graph screen shows the left bound. **Right Bound?** is displayed in the bottom-left corner. Press [◄] or [►] (or enter a value) to select the x-value for the right bound, and then press [ENTER]. A ◄ indicator on the graph screen shows the right bound. **Guess?** is then displayed in the bottom-left corner.



4. Press [◄] or [►] (or enter a value) to select a point near the zero of the function, between the bounds, and then press [ENTER].





The cursor is on the solution and the coordinates are displayed, even if **CoordOff** format is selected. To move to the same x-value for other selected functions, press [▲] or [▼]. To restore the free-moving cursor, press [◄] or [►].

minimum, maximum

minimum and **maximum** find a minimum or maximum of a function within a specified interval to a tolerance of 1E-5.

To find a minimum or maximum, follow these steps.

1. Select **3:minimum** or **4:maximum** from the CALCULATE menu. The current graph is displayed.
2. Select the function and set left bound, right bound, and guess as described for **zero**

The cursor is on the solution, and the coordinates are displayed, even if you have selected **CoordOff** format; **Minimum** or **Maximum** is displayed in the bottom-left corner.

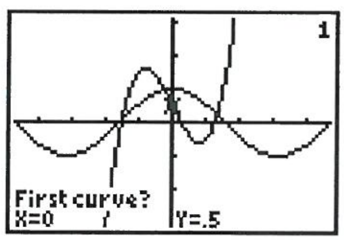
To move to the same x-value for other selected functions, press [▲] or [▼]. To restore the free-moving cursor, press [◄] or [►].

intersect

intersect finds the coordinates of a point at which two or more functions intersect using **solve(**. The intersection must appear on the display to use **intersect**.

To find an intersection, follow these steps.

1. Select **5:intersect** from the CALCULATE menu. The current graph is displayed with **First curve?** in the bottom-left corner.



2. Press [▼] or [▲], if necessary, to move the cursor to the first function, and then press [ENTER]. **Second curve?** is displayed in the bottom-left corner.
3. Press [▼] or [▲], if necessary, to move the cursor to the second function, and then press [ENTER].
4. Press [►] or [◄] to move the cursor to the point that is your guess as to location of the intersection, and then press [ENTER].

The cursor is on the solution and the coordinates are displayed, even if **CoordOff** format is selected. **Intersection** is displayed in the bottom-left corner. To restore the free-moving cursor, press [◄], [▲], [►], or [▼].

Studying Graph Behavior

As you have seen, TRACE allows you to move from pixel to pixel on a graph with the coordinates of the points displayed to illustrate the numerical behavior of the function. For example, you can see whether the y-coordinate increases, decreases, or remains constant as x increases. ZOOMIN permits a "close-up" examination of the *local behavior* of graphs.

Three other grapher features are useful for investigating graph behavior

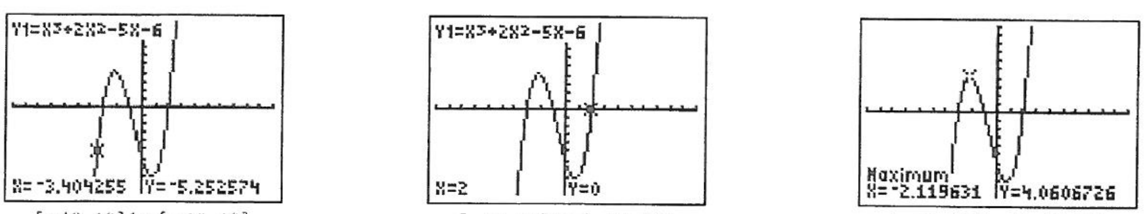
1. VALUE evaluates a function for a given domain value, which often avoids the need for a friendly window.
2. MINIMUM finds a local minimum value of a function and the associated domain value.
3. MAXIMUM finds a local maximum value of a function and the associated domain value.

Example _ Investigating Graph Behavior

Graph $f(x) = x^3 + 2x^2 - 5x - 6$ and study its behavior.

Solution Do the following on your grapher.

- Graph $y = f(x)$ in the standard window. Trace over until $x \approx -3.4$, as shown in Figure 1.19a. Then trace from left to right. Observe whether the function values (y-coordinates) increase, decrease, or remain constant as x increases.
- The graph appears to show that $f(2) = 0$, but TRACE fails to give a y value for $x = 2$. Use VALUE to find $f(2)$, as shown in Figure 1.19b.
- To determine precisely the intervals on which f is increasing or decreasing, locate the domain values at which the local maximums and minimums occur. Use MAXIMUM and MINIMUM to find these values. Figure 1.19c shows the result of using the MAXIMUM feature.



[−10, 10] by [−10, 10] (a) | [−10, 10] by [−10, 10] (b) | [−10, 10] by [−10, 10] (c)

Figure 1.19 Exploring a graph with (a) TRACE, (b) VALUE, and (c) MAXIMUM.

Curve Fitting and Statistics

A grapher can help you organize, process, and analyze data, as well as compute and plot models for paired data. The procedures for data analysis and curve fitting vary a great deal from grapher to grapher

Example Plotting and Fitting Data

Plot the national debt data given in Table 1.3, find a model for the data, and then overlay a graph of the model on the scatter plot.

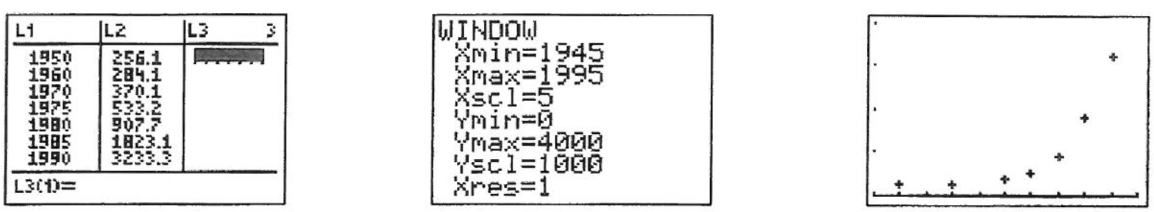
Table 1.3 U.S. Public Debt, 1950–1990

Year	Debt (Billions of dollars)
1950	256.1
1960	284.1
1970	370.1
1975	533.2
1980	907.7
1985	1,823.1
1990	3,233.3

Source: The World Almanac and Book of Facts (1995, Mahwah, N.J.: Funk & Wagnalls), p. 109.

Solution Follow these steps:

- Enter the data shown in Table 1.3 into the statistical memory of your grapher, as shown in Figure 1.27a.
- Set an appropriate window for the data, letting x be the year and y be the amount of the debt, as shown in Figure 1.27b.
- Make a scatter plot of the data, as shown in Figure 1.27c.



[1945, 1995] by [0, 4000]

(a) (b) (c)

Figure 1.27 The steps involved in making a scatter plot on a grapher.

Most graphers have several regression options. Typically, linear, quadratic, exponential, logarithmic, and power functions are available as regression models.

Editing a List Element

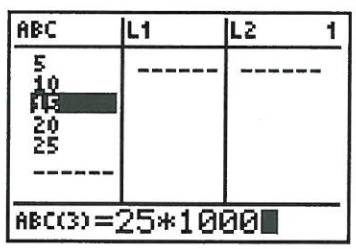
To edit a list element, follow these steps.

1. Move the rectangular cursor onto the element you want to edit.
2. Press [ENTER] to move the cursor to the entry line.

   Note: If you want to replace the current value, you can enter a new value without first pressing [ENTER]. When you enter the first character, the current value is cleared automatically.

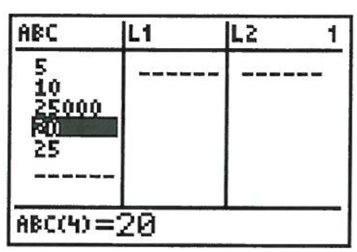
3. Edit the element in the entry line.
   - Press one or more keys to enter the new value. When you enter the first character, the current value is cleared automatically.
   - Press [▶] to move the cursor to the character before which you want to insert, press [2nd] [INS], and then enter one or more characters.
   - Press [▶] to move the cursor to a character you want to delete, and then press [DEL] to delete the character.

   To cancel any editing and restore the original element at the rectangular cursor, press [CLEAR] [ENTER].



Note: You can enter expressions and variables for elements.

4. Press [ENTER], [▲], or [▼] to update the list. If you entered an expression, it is evaluated. If you entered only a variable, the stored value is displayed as a list element.



When you edit a list element in the stat list editor, the list is updated in memory immediately.

Steps for Plotting Statistical Data in Lists

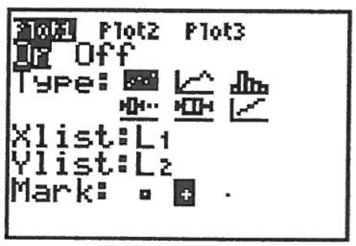
You can plot statistical data that is stored in lists. The six types of plots available are scatter plot, xyLine, histogram, modified box plot, regular box plot, and normal probability plot. You can define up to three plots.

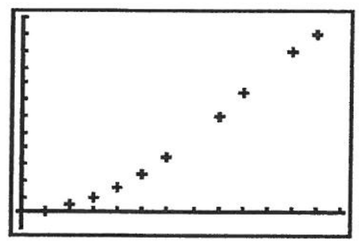
To plot statistical data in lists, follow these steps.

1. Store the stat data in one or more lists.
2. Select or deselect Y= functions as appropriate.
3. Define the stat plot.
4. Turn on the plots you want to display.
5. Define the viewing window.
6. Display and explore the graph.

(Scatter)

Scatter plots plot the data points from **Xlist** and **Ylist** as coordinate pairs, showing each point as a box (□), cross (+), or dot (•). **Xlist** and **Ylist** must be the same length. You can use the same list for **Xlist** and **Ylist**.

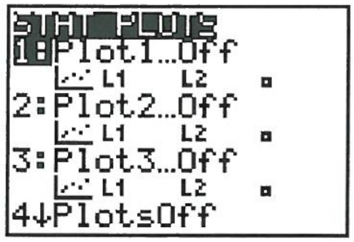




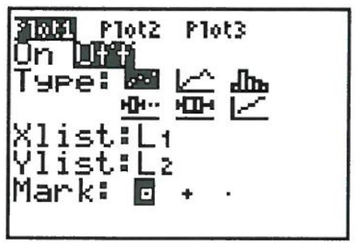
Defining the Plots

To define a plot, follow these steps.

1. Press [2nd] [STAT PLOT]. The STAT PLOTS menu is displayed with the current plot definitions.



2. Select the plot you want to use. The stat plot editor is displayed for the plot you selected.



3. Press [ENTER] to select **On** if you want to plot the statistical data immediately. The definition is stored whether you select **On** or **Off**.

4. Select the type of plot. Each type prompts for the options checked in this table.

Plot Type	XList	YList	Mark	Freq	Data List	Data Axis
Scatter	☑	☑	☑	☐	☐	☐
xyLine	☑	☑	☑	☐	☐	☐
Histogram	☑	☐	☐	☑	☐	☐
ModBoxplot	☑	☐	☑	☑	☐	☐
Boxplot	☑	☐	☐	☑	☐	☐
NormProbPlot	☐	☐	☑	☐	☑	☑

5. Enter list names or select options for the plot type.

   - **Xlist** (list name containing independent data)
   - **Ylist** (list name containing dependent data)
   - **Mark** (□ or + or •)
   - **Freq** (frequency list for **Xlist** elements; default is **1**)
   - **Data List** (list name for **NormProbPlot**)
   - **Data Axis** (axis on which to plot **Data List**)

Table 1.5 Official Census Population (in millions of persons), 1900–1990

Year	Florida	Pennsylvania
1900	0.5	6.3
1910	0.8	7.7
1920	1.0	8.7
1930	1.5	9.6
1940	1.9	9.9
1950	2.7	10.5
1960	5.0	11.3
1970	6.8	11.8
1980	9.7	11.9
1990	12.9	11.9

Source: U.S. Census Bureau as reported in *The World Almanac and Book of Facts* (1995, Mahwah, N.J.: Funk & Wagnalls), p. 377.