1.6 Graphical Transformations

Understanding how algebraic alterations change the shapes, sizes, positions, and orientations of graphs is helpful for understanding the connection between algebraic and graphical models of functions.

- Transformations
    - Functions that map real numbers to real numbers

All functions can be written in such a way:

$$a\,f(b(x+c))+d$$

- Reflections Across Axes (flips)
    - The following transformations result in reflections of the graph of $y = f(x)$:
        - $y = -f(x)$: a reflection across the x-axis
        - $y = f(-x)$: a reflection across the y-axis

- Size Changes
    - Let a be a positive real number. Then the following transformations result in VERTICAL size changes of the graph of $y = f(x)$
        - $y = af(x)$
            - A stretch by a factor of a if $a > 1$
            - A compression by a factor of a if $0 < a < 1$
    - Let b be a positive real number. Then the following transformations result in HORIZONTAL size changes of the graph of $y = f(x)$
        - $y = f\left(\frac{x}{b}\right)$
            - A stretch by a factor of b if $b > 1$
            - A compression by a factor of b if $0 < b < 1$

- Vertical & Horizontal Translations (shifts)
    - Let c be a positive real number. Then the following transformations result in HORIZONTAL translations of the graph of $y = f(x)$
        - $y = f(x - c)$ a shift right c units
        - $y = f(x + c)$ a shift left c units
    - Let d be a positive real number. Then the following transformations result in VERTICAL translations of the graph of $y = f(x)$
        - $y = f(x) + d$ a shift up d units
        - $y = f(x) - d$ a shift down d units