 Name: ______________________________

The Tools of Geometry

1.1 – THE BASICS OF GEOMETRY

OBJECTIVES:

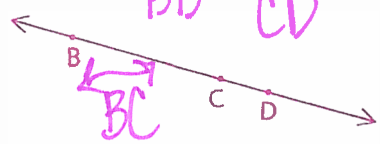
- Identify and name points, lines, planes, rays, and line segments
- Use symbolic notation to describe points, lines, planes, rays, and line segments
- Describe possible intersections of lines and planes

❖ Undefined Terms

In geometry, the terms point, line, and plane are undefined terms.

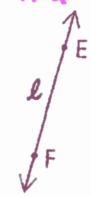
- A point is a specific location. It has no dimension and is represented by a dot.
  - We use capital letters to name points.
- A line is made up of points and is straight. It has no thickness and it continues forever in both directions.
  - We can name a line in terms of any two points on it; or with single scripted letter.
  - Collinear points are points that lie on the same line.

$\overleftrightarrow{BD}$ $\overleftrightarrow{CD}$



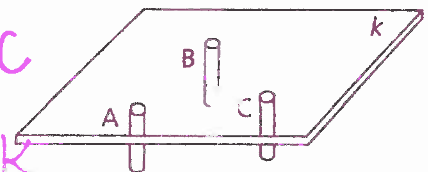
$\overleftrightarrow{BC}$

Line ℓ



- A plane is a flat surface that has no thickness and extends forever.
  - Four Ways to Determine a Plane
    - Three noncollinear points
    - A line and a point not on the line
    - Two intersecting lines
    - Two parallel lines

ABC OR Plane K



  - If a line intersects a plane (not containing it) then the intersection is exactly one point (the foot).
  - If two planes intersect, their intersection is exactly one line.

❖ Defined Terms

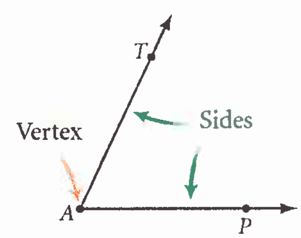
- Line Segment (aka Segment)
  - Segments are made up of points and are straight. A segment has a definite beginning and end.
  - We name segments in terms of its two endpoints.

$\overline{RS}$

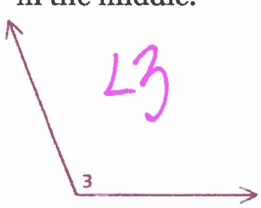
- Ray
  - Rays are made up of points and are straight. A ray begins at an endpoint and then extends infinitely far in only one direction.
  - A ray is name with the endpoint first and any other point on the ray is given second.

$\overrightarrow{CD}$ or $\overrightarrow{CE}$

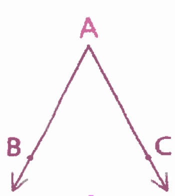
- Angle
  - An angle is formed by two rays (the sides) that share a common endpoint (the vertex), provided that the two rays are non collinear.
    - We can never name an angle in a way that could result in confusion. The safest way to name an angle is with three letters, with the vertex in the middle.



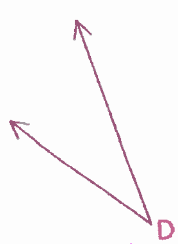
Sides $\overrightarrow{AT}$ & $\overrightarrow{AP}$



$\angle 3$

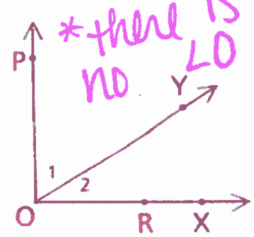


$\angle A$ or $\angle BAC$



$\angle D$

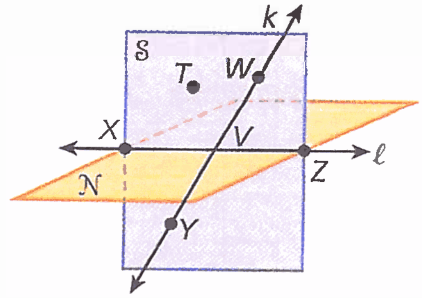
*there is no $\angle O$



$\angle 2 = \angle YOR$ or $\angle YOX$

FIRST THINK; THEN, PAIR & SHARE

Use the figure to name each of the following. Remember to use proper symbolic notation.



1. Three coplanar and non-collinear points

   T, W, V or X, W, Z or ...

2. Three other (different) names for line k

   $\overleftrightarrow{YW}$ $\overleftrightarrow{VY}$ $\overleftrightarrow{WV}$

3. A ray that lies on plane S (but not plane N)

   $\overrightarrow{WY}$ or $\overrightarrow{WV}$ or $\overrightarrow{VY}$

4. Plane S

   TWV or XWZ or

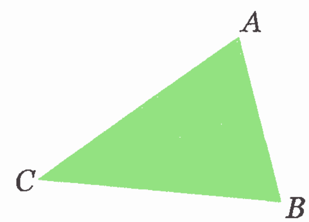
❖ Unions & Intersections

➢ The triangle is the union ($\cup$) of three segments

- $\triangle ABC = \overline{AB} \cup \overline{BC} \cup \overline{AC}$
- Union: *What do they form?*

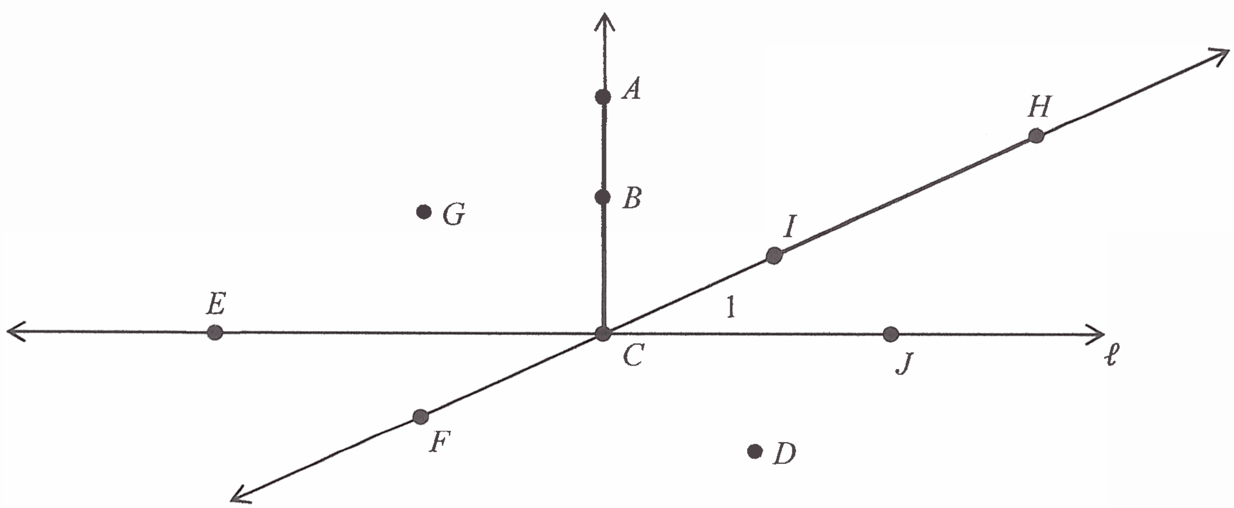
➢ The intersection ($\cap$) of any two sides is a vertex of the triangle.

- $\overline{AB} \cap \overline{BC} = B$
- Intersection: *What do they have in common?*



FIRST THINK; THEN, PAIR & SHARE

Use the diagram below to answer the questions that follow.



5. $\overrightarrow{CH} \cup \overrightarrow{CE}$ $\angle ECH$

6. $\overleftrightarrow{EJ} \cap \overleftrightarrow{HI}$ C

7. $\overrightarrow{IF} \cap \overrightarrow{CH}$ $\overline{CI}$

8. $\overrightarrow{IF} \cup \overrightarrow{CH}$ $\overleftrightarrow{FH}$

9. $\overrightarrow{JC} \cap \overrightarrow{CE}$ $\overrightarrow{CE}$

10. $\overrightarrow{EC} \cap \overrightarrow{JE}$ EJ

11. Prove three different names for line ℓ

    $\overleftrightarrow{EC}$ $\overleftrightarrow{EJ}$ $\overleftrightarrow{CJ}$

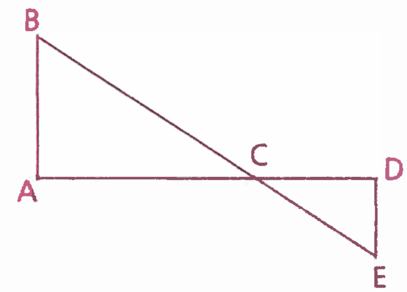
12. Prove three other names for $\angle 1$

    $\angle HCJ$, $\angle ICJ$, $\angle JCH$

- Assumptions from a Diagram
  - You SHOULD Assume
    - Straight lines and angles
    - Collinearity and betweeness of points
    - Vertical angles formed by 2 intersecting lines
  - You SHOULD NOT Assume
    - Right angles
    - Congruent segments and angles
    - Relative sizes of segments and angles

What SHOULD we assume from the given diagram?

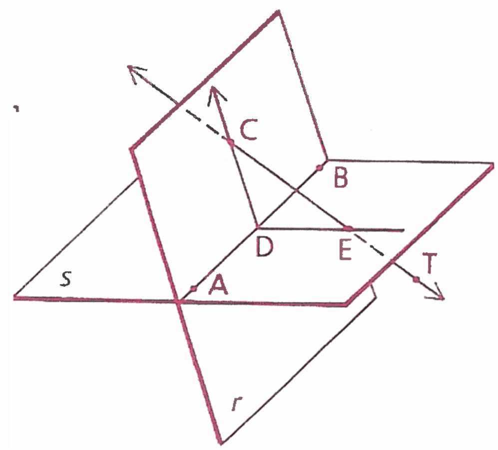
13. $\angle ADE$ is a right angle NO
14. C is between B and E yes
15. $\angle BCA$ and $\angle ECD$ are vertical angles yes
16. $\angle BAC \cong \angle EDA$ NO
17. $m\angle BCE = 180°$ yes



FIRST THINK; THEN, PAIR & SHARE

Use the diagram to answer the questions below.

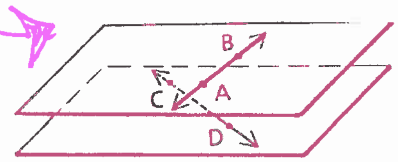
18. $r \cap s$ $\overleftrightarrow{AB}$
19. $\overleftrightarrow{AB} \cap s$ $\overleftrightarrow{AB}$
20. What plane do points A, B, and C determine? r
21. What plane do lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{ED}$ determine? S
22. Name the foot of $\overleftrightarrow{TC}$ in plane r. C
23. Name the foot of $\overleftrightarrow{TC}$ in plane s. E



True or False? For every false statement, rewrite it so that it's true. (Change that which is underlined.)

24. __T__ If a plane intersects two parallel planes, the lines of intersection are <u>parallel</u>.

25. __F__ Two lines must either intersect or be ~~parallel~~. Skew



26. __T__ A three-legged stool will not rock, even if the legs are of different lengths because only <u>three</u> points are necessary to determine a plane.

27. __T__ If a line intersects a plane, not containing it, then the intersection is <u>exactly one point</u>.

28. __F__ If two planes intersect, their intersection is ~~a point~~. Line

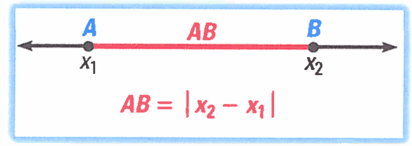
1.2 – SEGMENTS & THEIR MEASURES

OBJECTIVES:

- Apply the Segment Addition Postulate
- Use length and midpoint of a segment
- Bisect and trisect a segment

❖ Segments & Their Measures

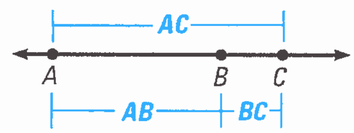
➢ The distance between points A and B, written as AB, is the absolute value of the difference between the coordinates of A and B.



- AB is also called the length of $\overline{AB}$.

❖ The Segment Addition Postulate

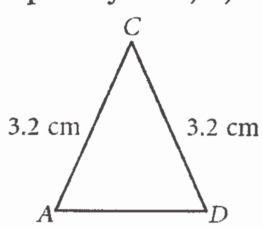
➢ If B is between A and C, then $AB + BC = AC$.



❖ Congruent

➢ Congruent segments are segments that have the same measure or length.

- The symbol for congruence: $\cong$ (use between congruent figures)
- Use the equals symbol, =, between equal numbers

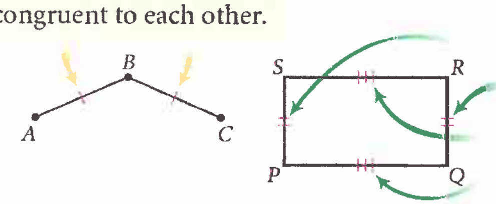


$AC = DC$ This notation shows that the measure of $\overline{AC}$ is equal to the measure of $\overline{DC}$.

$\overline{AC} \cong \overline{DC}$ This notation states that the two segments are congruent.

- Tick marks (identical markings) are used in a figure to show congruent segments.

These single marks mean these two segments are congruent to each other.

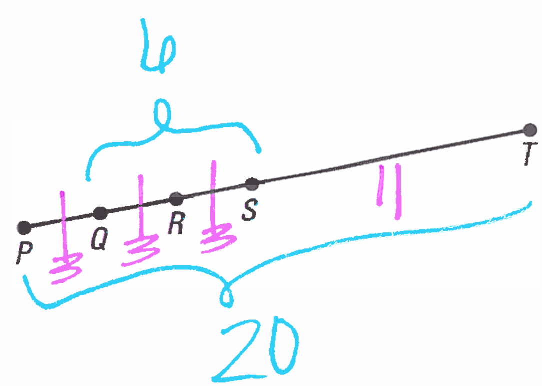


These double marks mean that $\overline{SP} \cong \overline{RQ}$, and these triple marks mean that $\overline{PQ} \cong \overline{SR}$.

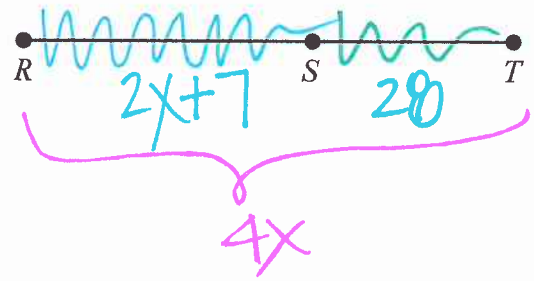
FIRST THINK; THEN, PAIR & SHARE

1. Points P, Q, R, S, and T are collinear. $PT = 20$, $QS = 6$, and $\overline{PQ} \cong \overline{QR} \cong \overline{RS}$. Find each length.

a. QR	3	e. RP	6
b. RS	3	f. RT	14
c. PQ	3	g. SP	9
d. ST	11	h. QT	17



2. S is between R and T. $RS = 2x + 7$, $ST = 28$, and $RT = 4x$. Use the Segment Addition Postulate to set up and solve an equation to find the value of x. Then find RT.

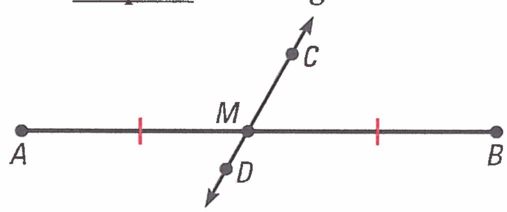


$RT = RS + ST$
$4x = 2x + 7 + 28$
$4x = 2x + 35$
$2x = 35 \rightarrow x = 17.5$ & $RT = 70$

❖ Bisecting a Segment

➢ A <u>segment bisector</u> is a line, segment, ray, or point that divides a segment into two congruent segments.

➢ The point where a bisector intersects a segment is the <u>midpoint</u> of the segment.



Given → ***M* is the midpoint of $\overline{AB}$**
Conclusion: $\overline{AM} \cong \overline{MB}$

Given = **$\overleftrightarrow{CD}$ is a bisector of $\overline{AB}$.**
Conclusion: $\overline{AM} \cong \overline{MB}$

<u>FIRST THINK; THEN, PAIR & SHARE</u>

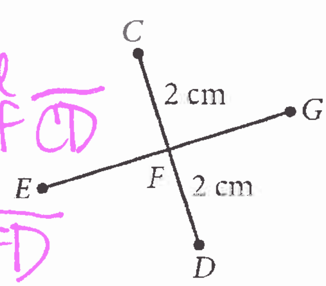
3. Can a line or a ray have a midpoint? Explain why or why not.

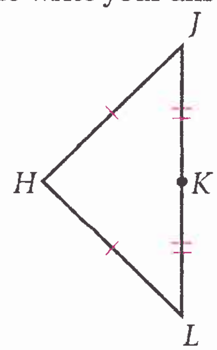
NO, a line or a ray extend forever in @ least one direction

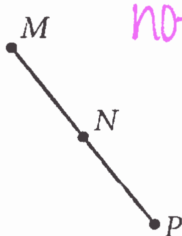
4. In the diagrams below, name each midpoint and the segment it bisects. Name all the congruent segments. Use the congruence symbol to write your answers.

F is the midpt. of $\overline{CD}$

$\overline{CF} \cong \overline{FD}$







not enough info

K is the midpt. of $\overline{JL}$
$\overline{JK} \cong \overline{KL}$

❖ Trisecting a Segment

➢ Two points (or segments, rays, or lines) that divide a segment into three congruent segments <u>trisect</u> the segment.

- The two points at which the segment is divided are called the <u>trisection points</u> of the segment.

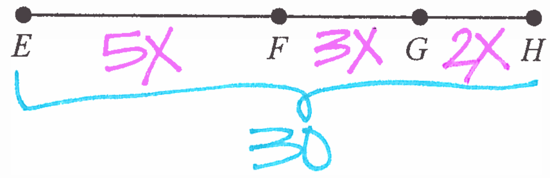
  - In Example 1, we were told that $\overline{PQ} \cong \overline{QR} \cong \overline{RS}$. Can we conclude that Q and R are trisection points of $\overline{PS}$?

YES!

FIRST THINK; THEN, PAIR & SHARE

include an x w/ each!

5. $\overline{EH}$ is divided by F and G in the ratio 5:3:2 from left to right. If $EH = 30$, find FG and name the midpoint of $\overline{EH}$.



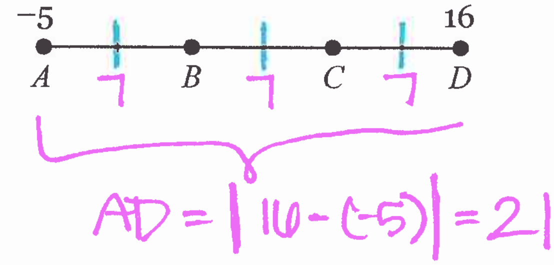
$EF + FG + GH = EH$

$5X + 3X + 2X = 30$

$10X = 30$

$X = 10$, $FG = 9$, F is the midpt. of $\overline{EH}$

6. B and C trisect $\overline{AD}$. Find the coordinates of B and C. Then find AC.

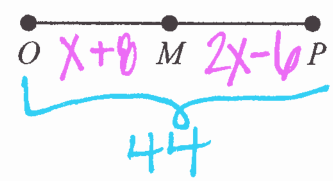


$\overline{AB} \cong \overline{BC} \cong \overline{CD}$

$AC = 14$, $B = 2$, $C = 9$

$AD = |16 - (-5)| = 21$

7. Points O, M, and P are collinear. $OM = x + 8$, $MP = 2x - 6$, & $OP = 44$. Is M the midpoint of $\overline{OP}$? Explain.



$OM + MP = OP$

$X + 8 + 2X - 6 = 44$

$3X + 2 = 44$

$3X = 42$

$X = 14$

therefore $\overset{14+8}{OM = 22}$ & $\overset{2(14)-6}{MP = 22}$

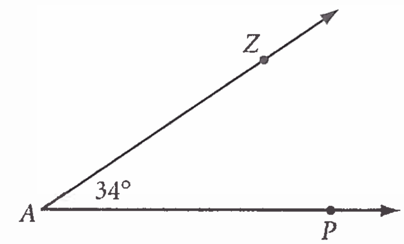
$\overline{OM} \cong \overline{MP}$

M is the midpoint of $\overline{OP}$ because it divides $\overline{OP}$ into 2 $\cong$ segments

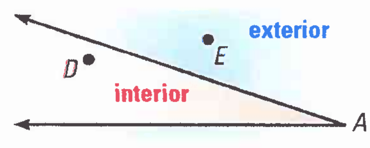
1.3 – ANGLES & THEIR MEASURES

OBJECTIVES:
- Classify angles by their measures
- Apply the Angle Addition Postulate
- Bisect or trisect an angle

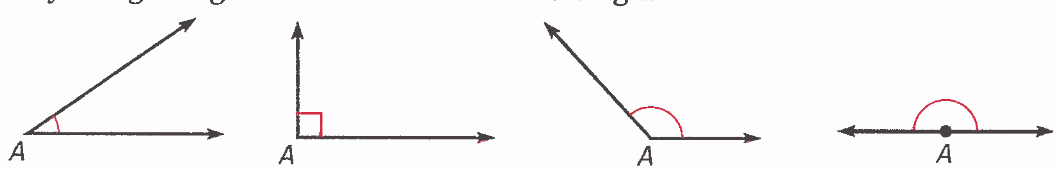
❖ Angles & Their Measures
- ➢ Recall that an <u>angle</u> consists of two different rays that have a common endpoint. The rays are the <u>sides</u> of the angle. The common endpoint is the <u>vertex</u> of the angle.
  - ▪ To show the measure of an angle, use an m before the angle symbol.
    - $m\angle ZAP = 34°$



- ➢ A point is in the <u>interior</u> of an angle if it is between points that lie on each side of the angle.
- ➢ A point is in the <u>exterior</u> of an angle if it is not on the angle or in its interior.



❖ Classifying Angles by Their Measures
- ➢ Angles are classified as acute, right, obtuse, and straight, according to their measures.
  - ▪ Recall that we cannot assume right angles or relative sizes of angles.
  - ▪ Only straight angles can be assumed from a diagram.

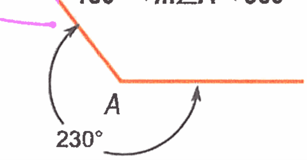


	Acute angle	Right angle	Obtuse angle	Straight angle
RESTRICTIONS ⇨	$0° < m\angle A < 90°$	$m\angle A = 90°$	$90° < m\angle A < 180°$	$m\angle A = 180°$

⇧

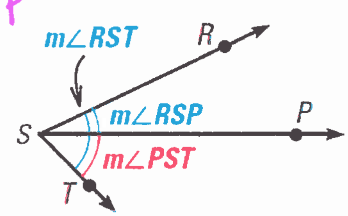
RESTRICTIONS ⇨ Reflex Angle $180° < m\angle A < 360°$

- ➢ A complete rotation around a point is 360°
- ➢ A reflex angle: $180° < m\angle A < 360°$

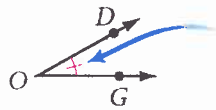


❖ Angle Terminology
- ➢ Two angles are <u>adjacent angles</u> if they share a common vertex and side, but have no common interior points.
  - ▪ Which angles are adjacent angles? ∠RSP & ∠TSP
  - ▪ What is the common side? SP

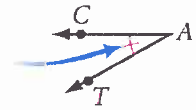
❖ The Angle Addition Postulate
- ➢ If P is in the interior of $\angle RST$, then $m\angle RSP + m\angle PST = m\angle RST$.



- Congruent
  - Two angles are <u>congruent angles</u> if and only if they have the same measure.
    - Use identical markings (arc marks) to show that two angles in a figure are congruent.

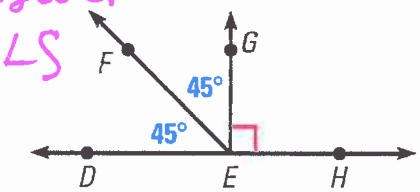


These markings mean that $\angle DOG \cong \angle CAT$ and $m\angle DOG = m\angle CAT$.



FIRST THINK; THEN, PAIR & SHARE

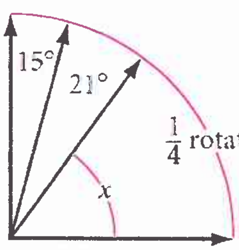
Use the diagram to answer the questions below. Explain your reasoning.



1. Is $\angle DEF \cong \angle FEG$? yes, same measure
2. Is $\angle DEG \cong \angle HEG$? yes, both are Rt ∠s
3. Are $\angle DEF$ and $\angle FEH$ adjacent? yes
4. Are $\angle GED$ and $\angle DEF$ adjacent? no

Use the Angle Addition Postulate to find the angle measures represented by each letter.

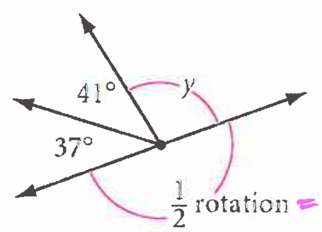
5.



= 90°

x = 54°

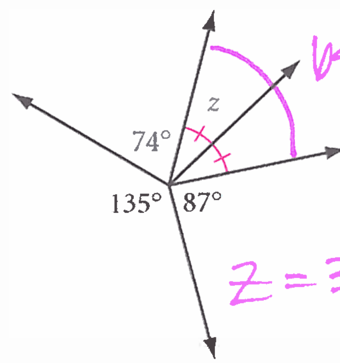
6.



= 180°

y = 102°

7.



64°

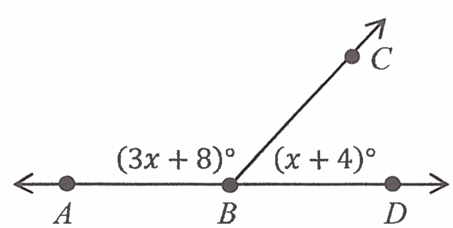
z = 32°

8. Given: $\angle J$ is an acute angle
   a. What are the restrictions on the $m\angle J$? 0° < m∠J < 90°
   b. What are the restrictions on x if $m\angle J = 3x - 21$?

$0 < 3x - 21 < 90$
$21 < 3x < 111$
$7 < x < 37$

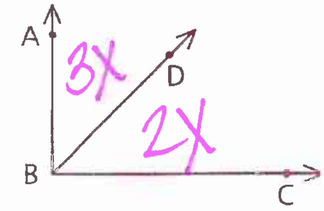
9. Use the Angle Addition Postulate to find the value of x. Then find $m\angle ABC$.

   Remember: We CAN assume straight angles.



m∠ABC + m∠CBD = m∠ABD
3x + 8 + x + 4 = 180°
4x + 12 = 180°
4x = 168
x = 42 & m∠ABC = 134°

10. $\angle ABC$ is a right angle. The ratio of the $m\angle ABD$ to $m\angle DBC$ is 3 to 2. Use the Angle Addition Postulate to find the value of x. Then find $m\angle ABD$.



$m\angle ABD + m\angle DBC = m\angle ABC$

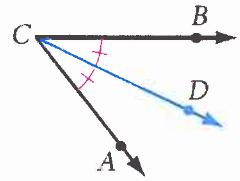
$3X + 2X = 90$

$5X = 90$

$X = 18$ & $m\angle ABD = 54°$

❖ Bisecting an Angle

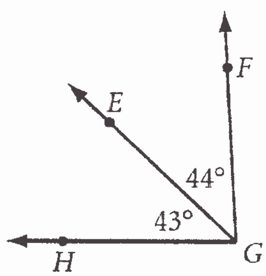
➢ A ray is the <u>angle bisector</u> if it contains the vertex and divides the angle into two congruent angles.



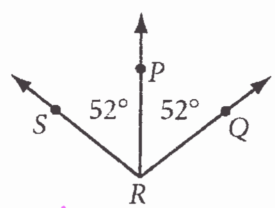
- $\overrightarrow{CD}$ bisects $\angle ACB$, therefore $\angle ACD \cong \angle BCD$
- $m\angle ACD = m\angle BCD$

FIRST THINK; THEN, PAIR & SHARE

11. In the diagrams below, name each angle bisector and the angle it bisects. Name all the congruent angles in the figure. Use the congruence symbol and name the angles so there is no confusion about which angle you mean.

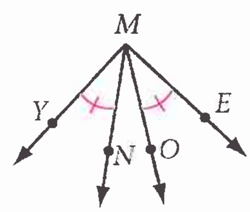


no angle bisector



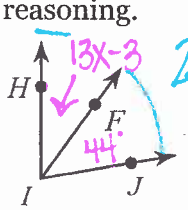
$\overrightarrow{RP}$ bisects $\angle SRQ$

$\angle SRP \cong \angle QRP$



no angle bisector

12. If $m\angle FIJ = 44°$, $m\angle HIF = 13x - 3$, and $m\angle HIJ = 25x + 5$, is $\overrightarrow{IF}$ an angle bisector? Explain your reasoning.



25X+5

$m\angle FIJ + m\angle HIF = m\angle HIJ$

$44 + 13X - 3 = 25X + 5$

$13X + 41 = 25X + 5$

$36 = 12X$

$3 = X \rightarrow m\angle HIF = 36°$

$\overrightarrow{IF}$ is NOT an $\angle$ bisector because it doesn't divide $\angle HIJ$ into 2 $\cong$ $\angle$s

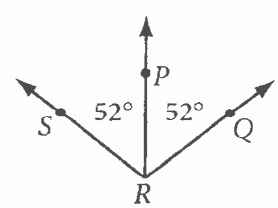
$m\angle HIF \neq m\angle FIJ$

❖ Trisecting an Angle

➢ Two rays that divide an angle into three congruent angles <u>trisect</u> the angle.

- The two dividing rays are called <u>trisectors</u> of the angle.

13. If $\angle SRQ$ were trisected, what would be the measure of each of the three angles?



$m\angle SRQ = 104$

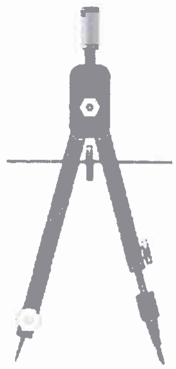
therefore the measure of all 3 $\angle$s would be $\approx 34.7°$

1.4 – BASIC CONSTRUCTIONS

OBJECTIVES:

Use a compass, a straight edge, and a construction to…

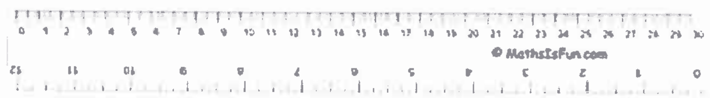
- Copy or duplicate a line segment and an angle
- Bisect a line segment and an angle



"Construction" in Geometry means to draw shapes, angles or lines accurately.

These constructions use only compass, straightedge (i.e. ruler) and a pencil.

This is the "pure" form of geometric construction: no numbers involved!



- Construction Videos
  - Go to my website: schultzjen.weebly.com
    - Click on Honors Geometry (under Backer's Schedule)
    - Scroll down the Lesson 1.4
      - The videos are located here.

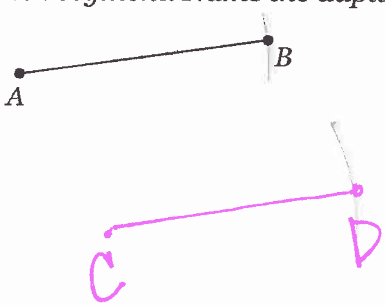
LESSON 1.4 ~ Basic Constructions

- Copying Line Segments
- Bisecting a Line Segment
- Constructing an Angle
- Bisecting an Angle

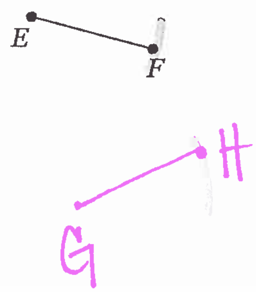
Copying a Line Segment

Copy each segment. Name the duplicate using the next two letters in the alphabet.

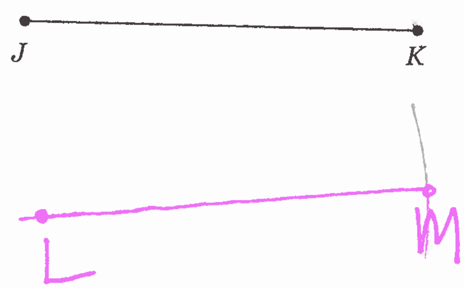
1.



2.



3.

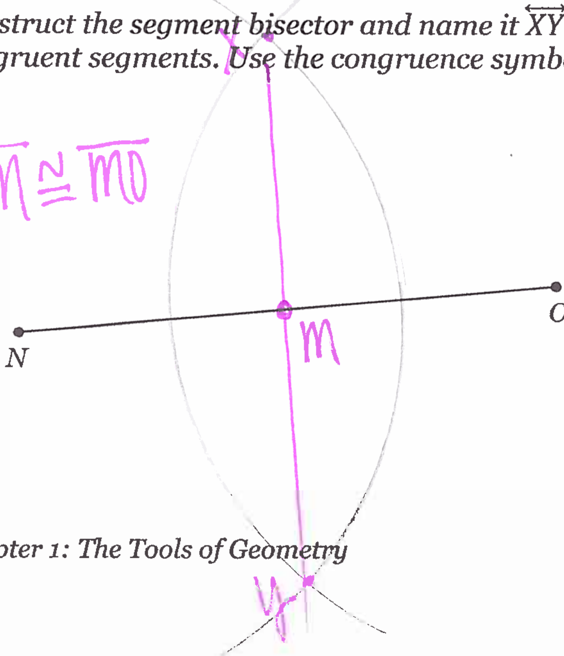


Constructing the Perpendicular Bisector of a Line Segment & Locating its Midpoint

Construct the segment bisector and name it $\overleftrightarrow{XY}$. Locate the midpoint and label it M. Name all the congruent segments. Use the congruence symbol to write your answers.

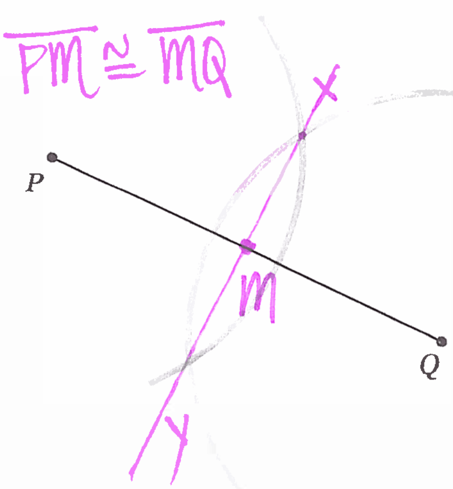
4.

$\overline{NM} \cong \overline{MO}$



5.

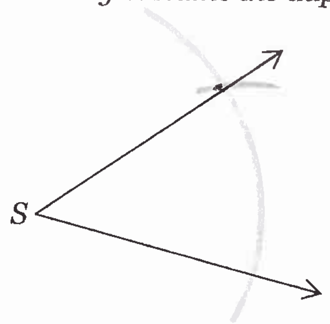
$\overline{PM} \cong \overline{MQ}$

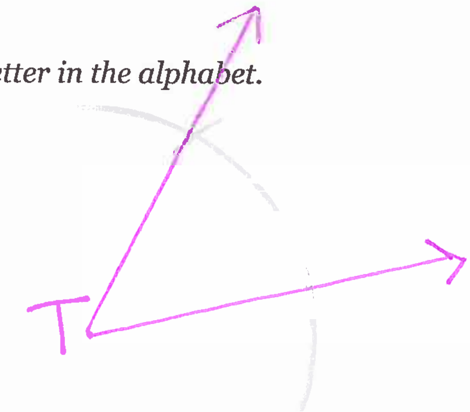


Copying/Duplicating an Angle

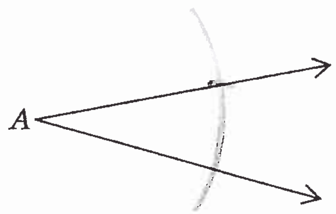
Copy each angle. Name the duplicate using the letter in the alphabet.

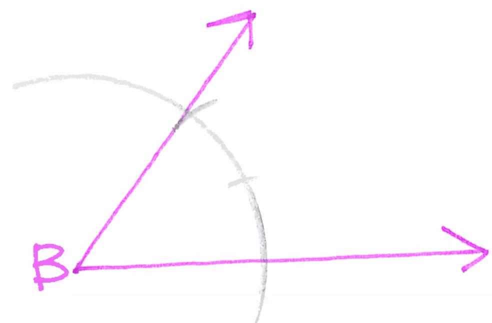
6.





7. Construct an angle that is twice the measure of ∠*A*. Then explain how you performed the construction.

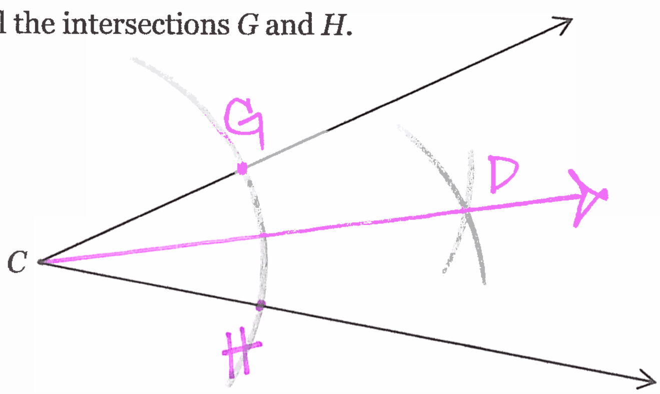




Bisecting an Angle

Construct the angle bisector and name it $\overrightarrow{CD}$. Name all the congruent angles in the figure. Use the congruence symbol and name the angles so there is no confusion about which angle you mean.

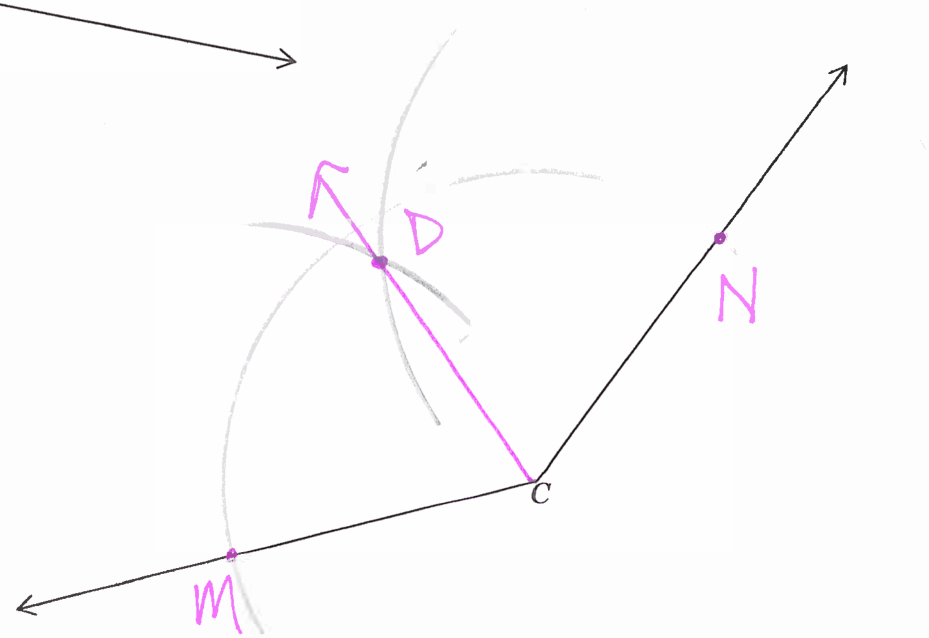
8. Label the intersections *G* and *H*.



$\angle GCD \cong \angle HCD$

9. Label the intersections *M* and *N*.

$\angle MCD \cong \angle NCD$



1.5.D1 – THE COORDINATE PLANE

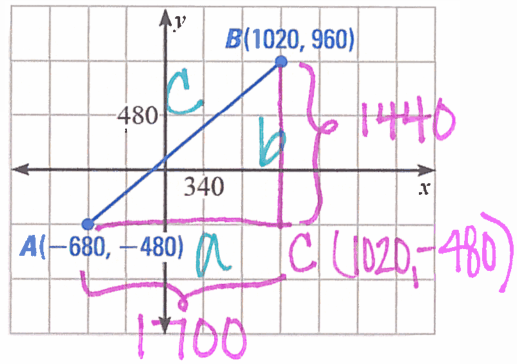
OBJECTIVES:

- Determine the distance between two points
- Use the Pythagorean Theorem to derive the distance formula
- Determine the length, midpoint, and slope of a line segment on a coordinate plane

❖ Deriving the Distance Formula

➢ On the map, the city blocks are 340 feet apart east-west and 480 feet apart north-south.

- Start at A and walk east and then turn north and walk to B. Label the point at which you turn C.
- Use the Pythagorean Theorem to determine the distance if a diagonal street existed between the two points. (Show all work.)



$AC = |1020 - (-680)| = 1700 = a$

$BC = |960 - (-480)| = 1440 = b$

$a^2 + b^2 = c^2$

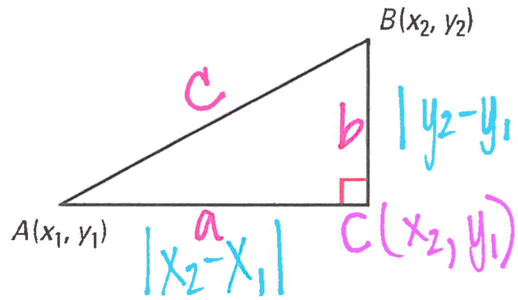
$1700^2 + 1440^2 = (AB)^2$

$4963600 = (AB)^2$

$2228 \approx AB$ feet

➢ Suppose we generalize the situation using the following coordinates for A and B: $A(x_1, y_1)$ and $B(x_2, y_2)$.

- What are the coordinates of C? (x_2, y_1)
- Write an expression for each distance.
  - $AC = |x_2 - x_1| = a$
  - $BC = |y_2 - y_1| = b$
- Use the Pythagorean Theorem to determine an expression for the distance AB



$a^2 + b^2 = c^2$

$|x_2 - x_1|^2 + |y_2 - y_1|^2 = (AB)^2$

$\sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2} = AB$

Congratulations! You used the Pythagorean Theorem to calculate the distance between two points on the coordinate plane. Your method can be written as the *Distance Formula*.

- The Distance Formula
  - If (x_1, y_1) and (x_2, y_2) are two points on the coordinate plane, then the distance d between (x_1, y_1) and (x_2, y_2) is...

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- The Midpoint Formula
  - If (x_1, y_1) and (x_2, y_2) are two points on the coordinate plane, then the midpoint (x_m, y_m) of the line segment that joins these two points is...

$$(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

- The Slope Formula
  - If (x_1, y_1) and (x_2, y_2) are two points on the coordinate plane, then the slope m of the line segment that joins these two points is...

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

FIRST THINK; THEN, PAIR & SHARE

1. Find the length, midpoint, and slope of the lines segment with endpoints $M(-15, 8)$ and $N(-1, 12)$. If necessary, express the length as a radical in simplest form; express midpoints and slope as an improper fraction in simplest form.

$$\text{length} = \sqrt{(-15-(-1))^2 + (8-12)^2} = \sqrt{(-14)^2 + (-4)^2} = \sqrt{212}$$

$$\text{midpoint} = \left(\frac{\overset{-16}{-15+(-1)}}{2}, \frac{\overset{20}{8+12}}{2}\right) = (-8, 10)$$

$$\text{slope} = \frac{8-12}{-15-(-1)} = \frac{-4}{-14} = \frac{2}{7}$$

2. One endpoint of segment is $(\underset{x_1}{12}, -8)$. The midpoint is $(3, 18)$. Find the coordinates of the other endpoint.

$$(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

X-COORD

$$3 = \frac{12 + x_2}{2}$$

$$6 = 12 + x_2 \rightarrow x_2 = -6$$

Y-COORD

$$18 = \frac{-8 + y_2}{2}$$

$$36 = -8 + y_2$$

$$44 = y_2$$

Solution: $(-6, 44)$

3. Line segment $\overline{QR}$ has a length of 12 units; $Q(\underset{x_2}{10}, \underset{y_2}{10})$ and $R(\underset{x_1}{x}, \underset{y_1}{-2})$. Find the missing coordinate x.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$(12)^2 = \left(\sqrt{(x-10)^2 + (\underbrace{-2-10}_{-12})^2}\right)^2$$

$$144 = (x-10)^2 + 144$$

$$\sqrt{0} = \sqrt{(x-10)^2}$$

$$0 = x - 10$$

$$10 = x$$

1.5.D2 – THE COORDINATE PLANE

OBJECTIVES:

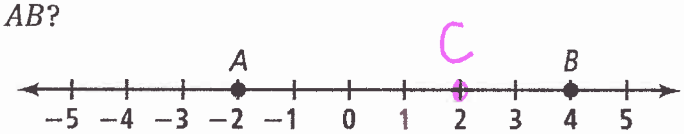
- Determine the length, midpoint, and slope of a line segment on a coordinate plane
- Find the point on a directed line segment between two given points that partitions the segment in a given ratio

You can find a point on a line segment such that the point's distance from one endpoint of the segment and the length of the entire line segment have a specified ratio.

- Partitioning a Directed Line Segment
  - A directed line segment is a segment between two points A and B with a specified direction, from A to B or from B to A.
  - To partition a directed line segment is to divide it into two segments with a given ratio.
    - When dividing a line segment into a given ratio, you need to first determine the number of parts.
      - A line segment divided into a 3:2 ratio has 5 equal parts.
      - A line segment divided into a $\frac{3}{5}$ ratio has 8 equal parts.

Examples:

4. On a number line, A is at -2 and B is at 4. What is the location of C between A and B, such that AC is 2/3 the length of $\overline{AB}$?



a. Find the length of $\overline{AB}$. 6

b. What is 2/3 the length of $\overline{AB}$? $\frac{2}{3}\cdot 6 = \frac{12}{3} = 4$

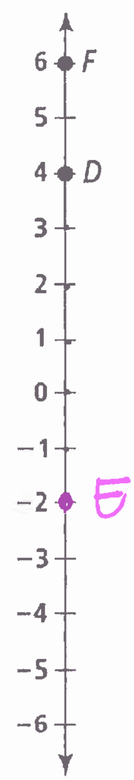
c. What is the location of point C that is 2/3 the length of $\overline{AB}$ from A to B?

$-2+4=2$

C is @ 2

d. Find the ratio $\frac{AC}{AB}$. Explain how this checks your solution.

$\frac{AC}{AB} = \frac{4}{6} = \frac{2}{3}$



5. On the vertical number line at the right, find point E such that $DE = 3FD$. Explain your work.

FD = 2

DE = 3·FD

DE = 3·2

DE = 6

E is @ -2

direction/ORDER

6. B is at 7 and T is at -3. Find S, so the directed line segment $\overline{BT}$ is partitioned in a 3:2 ratio.

$BT = 10$

$10 \cdot \frac{3}{5} = \frac{30}{5} = 6$ units from B

S is @ 1

❖ Finding the Coordinates of a Point in a Directed Line Segment

Find the point P along the directed line segment from point $A(-8, -7)$ to point $B(8, 5)$ that divides the segment in the ratio 3 to 1. → fraction $= \frac{3}{4}$

direction = up & Right

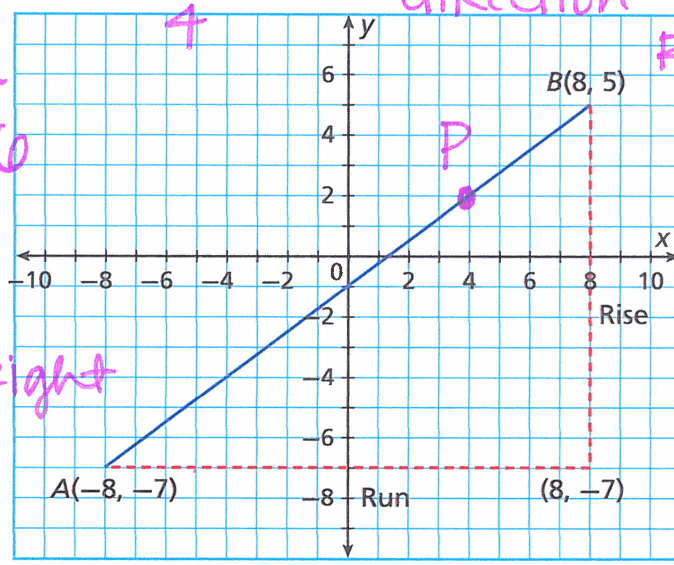
➢ Find the rise and run of the directed line segment.

- Rise $= |y_2 - y_1| = |5-(-7)| = 12$
- Run $= |x_2 - x_1| = |8-(-8)| = 16$

➢ Point P is $\frac{3}{4}$ of the way between points A and B, so find $\frac{3}{4}$ of both the rise and the run: $12 \cdot \frac{3}{4} = \frac{36}{4} = 9$ up

$16 \cdot \frac{3}{4} = 12$ Right

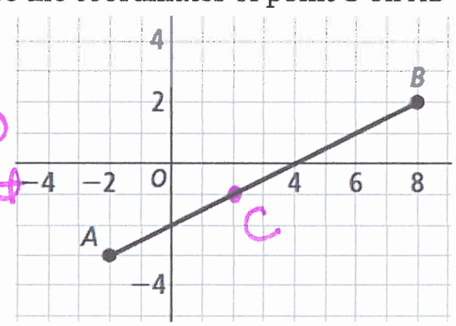
$A(-8, -7)$
$+12 \quad +9$
$P(4, 2)$



❖ Point P is ___9___ units up and ___12___ units right from the point A.

Its coordinates are ___(4, 2)___.

7. Points $A(-2, -3)$ and $B(8, 2)$ are the endpoints of $\overline{AB}$. What are the coordinates of point C on $\overline{AB}$ such at AC is 2/5 the length of $\overline{AB}$?



direction = up & Right

Rise $= |-3-2| = 5$ Run $= |-2-8| = 10$

$5 \cdot 2/5 = 2$ up $\quad 10 \cdot 2/5 = 4$ Right

$A(-2, -3) \rightarrow C(2, -1)$
$+4 \quad +2$

8. Find the point Q along the directed line segment from point $R(-2, 4)$ to point $S(18, -6)$ that divides the segment in the ratio 3 to 7. fraction $= 3/10$

direction = down & Right

Rise $= |4-(-6)| = 10$ Run $= |-2-18| = 20$

$10 \cdot \frac{3}{10} = 3$ down $\quad 20 \cdot \frac{3}{10} = 6$ Right

$R(-2, 4) \rightarrow Q(4, 1)$
$+6 \quad -3$

1.6 – TRANSFORMATIONS: PART 1

OBJECTIVES:

- Translate, rotate, and reflect geometric figures on the coordinate plane
- Without graphing, determine the coordinates of translated, rotated, and/or reflected figures

❖ Transformations

➢ A transformation is the mapping, or movement, of all the points of a figure in a place according to a common operation.

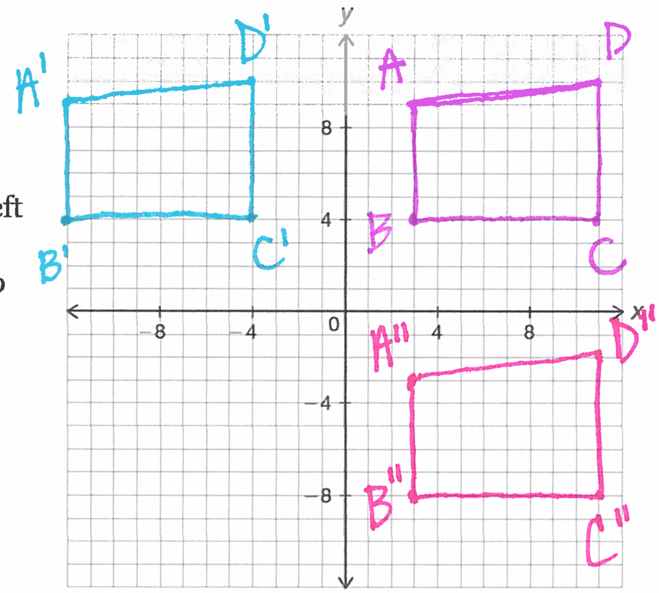
- A rigid motion is a transformation of points in space.
- The original figure is called the pre-image.
- The new figure created is called the image.

❖ Translating Geometric Figures on the Coordinate Plane

➢ A translation is a rigid motion that "slides" each point of a figure the same distance and direction.

- Sliding a figure left or right is a horizontal translation, and sliding it up or down is a vertical translation.

1. Graph trapezoid *ABCD* by plotting the points $A(3,9)$, $B(3,4)$, $C(11,4)$, and $D(11,10)$.
2. Translate trapezoid *ABCD* on the coordinate plane. Graph the image and record the vertex coordinates in the table.
   a. Translate trapezoid *ABCD* 15 units to the left to form trapezoid *A'B'C'D'*.
   b. Translate trapezoid *ABCD* 12 units down to form trapezoid *A"B"C"D"*.



COORDINATES OF *ABCD*	COORDINATES OF *A'B'C'D'*	COORDINATES OF *A"B"C"D"*
$A(3,9)$	(-12, 9)	(3, -3)
$B(3,4)$	(-12, 4)	(3, -8)
$C(11,4)$	(-4, 4)	(11, -8)
$D(11,10)$	(-4, 10)	(11, -2)

3. What patterns do you observe when translating left?

y-coords stayed the same, x-coords decreased by 15

4. What patterns do you observe when translating down?

x-coords stayed the same. y-coords decreased by 12

Summarize your observations about translations in the table.

Translation	Right *h* units	Left *h* units	Up *k* units	Down *k* units
Coordinate Rule	(x+h, y)	(x-h, y)	(x, y+k)	(x, y-k)

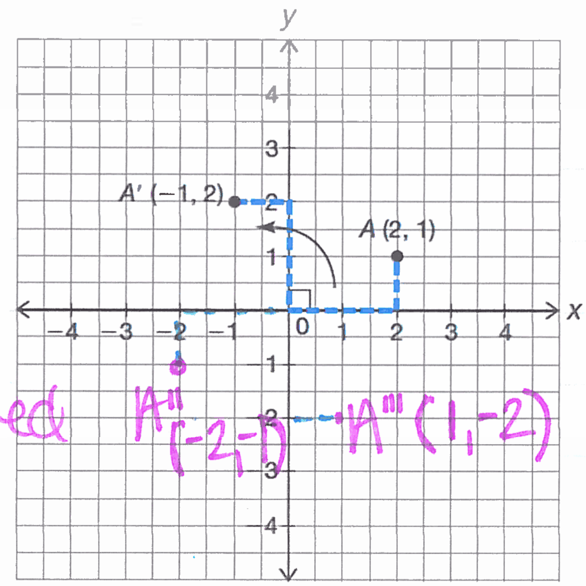
- Rotating Geometric Figures on the Coordinate Plane
  - A <u>rotation</u> is a rigid motion that turns a figure about a fixed point, called the point of rotation.
    - The figure is rotated in a given direction for a given angle, called the angle of rotation.
    - The angle of rotation is the measure of the amount the figure is rotated about the point of rotation.
    - The direction of rotation can either be clockwise or counterclockwise.

Let's rotate point A about the origin. The origin will be the point of rotation and you will rotate point A 90°, 180°, and 270°.

WORKED EXAMPLE:

Rotate point A 90° counterclockwise about the origin.

1. Draw a "hook" from the origin to point A, using the coordinates and horizontal and vertical line segments as shown.
2. Rotate the "hook" 90° counterclockwise as shown. Point A' is located at $(-1, 2)$.



What do you notice about the coordinates of A and A'?

A (2,1) → A' (-1,2)
the x & y-coords switched
the new x-coord has a different sign

- Rotate point A' about the origin 90° counterclockwise on the coordinate plane shown above. Label the point A''.
  - What do you notice about the coordinates of points A and A''? How are the two points related?

    A (2,1) → A'' (-2,-1)
    A double sign change

    - A has just been rotated 180° about the origin.

- Rotate point A'' about the origin 90° counterclockwise on the coordinate plane shown above. Label the point A'''.
  - What do you notice about the coordinates of points A and A'''? How are the two points related?

    A (2,1) → A''' (1,-2) the x & y-coords switched the new y-coord has a different sign

    - A has just been rotated 270° counterclockwise about the origin.

Summarize your observations about rotations in the table.

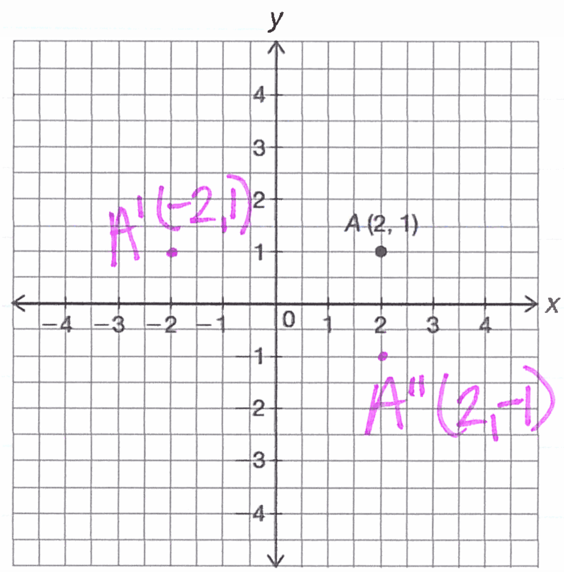
Rotation (x,y)	90° counterclockwise	180°	270° counterclockwise	90° clockwise
Coordinate Rule	$(-1\cdot y, x)$	$(-1\cdot x, -1\cdot y)$	$(y, -1\cdot x)$	$(y, -1\cdot x)$

- ❖ Reflecting Geometric Figures on the Coordinate Plane
  - ➢ A reflection is a rigid motion that reflects, or "flips," a figure over a given line called a line of reflection.
    - ▪ A line of reflection is a line over which a figure is reflected so that corresponding points are the same distance from the line.

GUIDED EXAMPLE:

Let's reflect point A over the y-axis.

1. Count the number of x units from point A to the y-axis.
2. Then, count the same number of x units on the opposite side of the y-axis to locate the reflection of point A. Label the point A'.



What do you notice about the coordinates of A and A'?

A(2,1) → A'(-2,1)
the x-coord has changed signs; the y-coord stayed the same

- ❖ Reflect point A over the x-axis on the coordinate plane shown above. Label the point A''.
  - ➢ What do you notice about the coordinates of points A and A''? How are the two points related?

A(2,1) → A"(2,-1) the x-coord is the same
the y-coord changed signs

Summarize your observations about reflections in the table.

Reflection	Over the x-axis	Over the y-axis
Coordinate Rule	$(x, -1\cdot y)$	$(-1\cdot x, y)$

EXAMPLES:

1. Use the translation $(x, y) \rightarrow (x-5, y+8)$.

   a. What is the image of $D(-1,5)$?

   D'(-4,13)

   b. What is the pre-image of $K'(7,-5)$?

   K(2,-13)

2. Refer to the graph of △ABC. What are the coordinates of the image after the given transformation?

   a. A, reflection over x-axis

   (-1,3)

   b. B, rotation 90° clockwise about the origin

   (1,-1)

   c. C, translation right 4 & down 5

   (9,-6)

   d. C, reflection over y-axis

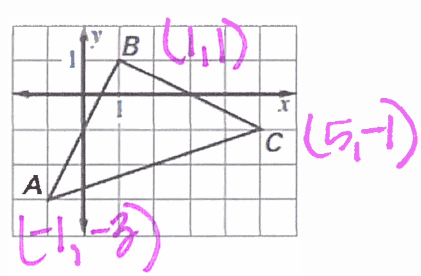
   (-5,-1)

   e. A, rotation 90° counter-clockwise about the origin

   (3,-1)

   f. B, translation left 7 & up 10

   (-6,11)



1.6 – TRANSFORMATIONS: PART 2

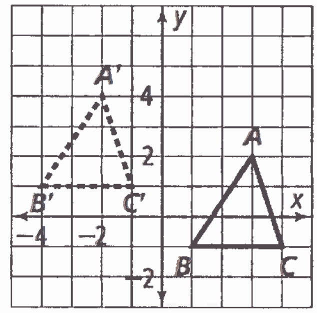
OBJECTIVES:

- Translate, rotate, and reflect geometric figures on the coordinate plane
- Without graphing, determine the coordinates of translated, rotated, and/or reflected figures
- Represent/draw the image of a composition of transformations
- Describe a sequence of transformations that will carry a given geometric figure onto itself (or another)

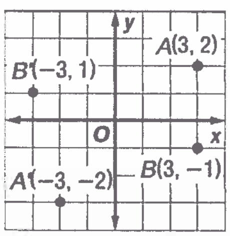
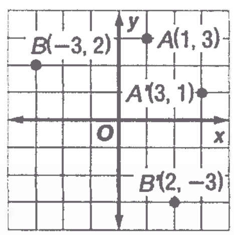
❖ Translations

The diagram at the right shows a translation in the coordinate plane. The preimage is $\triangle ABC$. The image is $\triangle A'B'C'$.

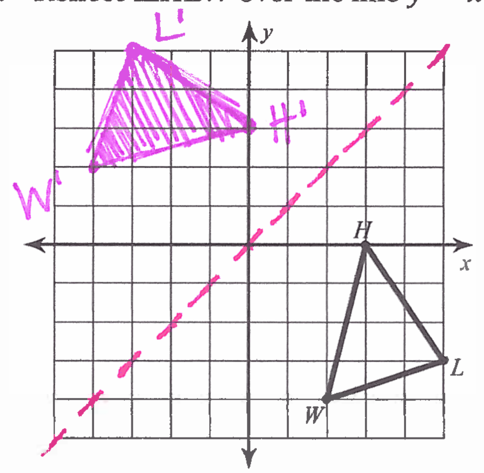
Each point of $\triangle ABC$ has moved 5 units left and 2 units up. Moving left is in the negative x direction, and moving up is in the positive y direction. So each (x, y) pair in $\triangle ABC$ is mapped to $(x - 5, y + 2)$ in $\triangle A'B'C'$.



❖ Other Reflections in the Coordinate Plane

Reflections in the Coordinate Plane	
origin	$y = x$
$(a, b) \quad (-a, -b)$	$(a, b) \quad (b, a)$
Multiply both coordinates by −1.	Interchange the x- and y-coordinates.
	

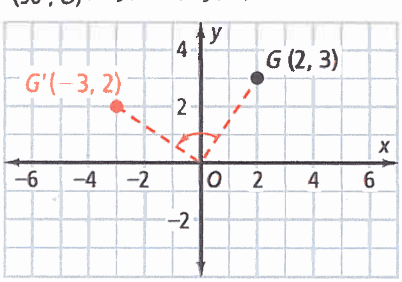
1. Reflect $\triangle HLW$ over the line $y = x$.



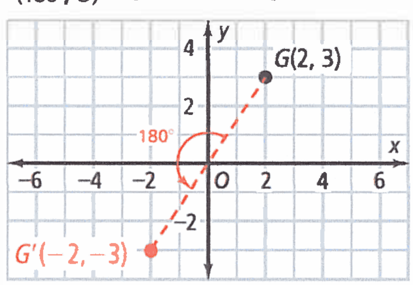
❖ Rotations

➢ For a point $P(x, y)$ in the coordinate plane, use the following rules to find the coordinates of the (counterclockwise) rotations 90°, 180°, & 270° about the origin O.

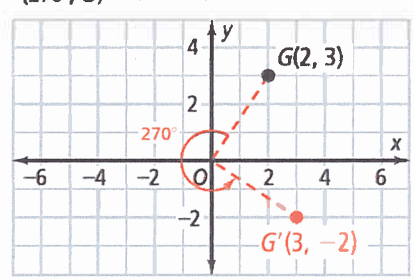
$r_{(90°,\, O)}(x, y) = (-y, x)$



$r_{(180°,\, O)}(x, y) = (-x, -y)$



$r_{(270°,\, O)}(x, y) = (y, -x)$

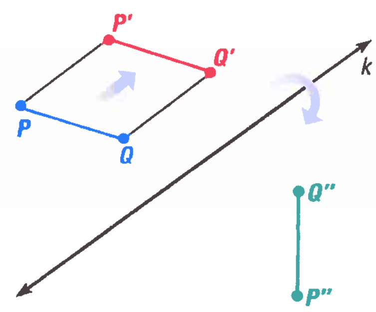


- Compositions of Reflections
  - A translation or rotation is a <u>composition</u> of two reflections.
  - A composition of reflections in...
    - ...two parallel lines is a translation.
    - ...two intersecting lines is a rotation.

- Glide Reflections
  - Fundamental Theorem of Isometries
    - In a plane, one of two congruent figures can be mapped onto the other by a composition of at most three reflections.

A translation, or glide, and a reflection can be performed one after the other to produce a transformation known as a *glide reflection*. A **glide reflection** is a transformation in which every point P is mapped onto a point P'' by the following steps:

1. A translation maps P onto P'.
2. A reflection in a line k parallel to the direction of the translation maps P' onto P''.



2. Use the information below to sketch the image of $\triangle ABC$ after a glide reflection.

   $A(-1,-3), B(-4,1), C(-6,-4)$

   Translation: $(x, y) \rightarrow (x + 10, y)$

   Reflection: in the x-axis

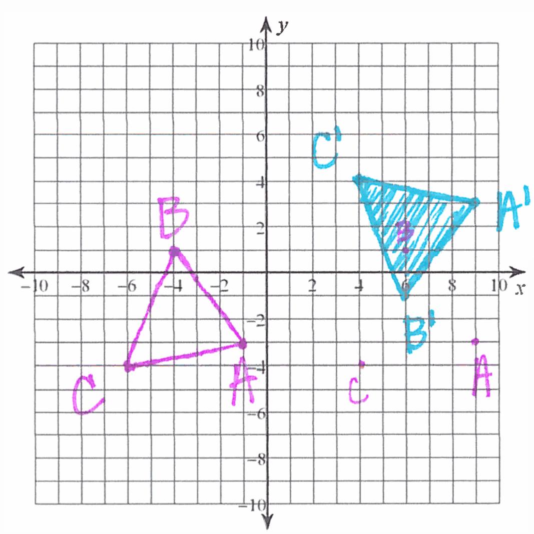
3. Sketch the image of $\overline{FG}$ after a composition using the given transformations in the order they appear. Then, perform the transformations in reverse order. Does the order affect the final image?

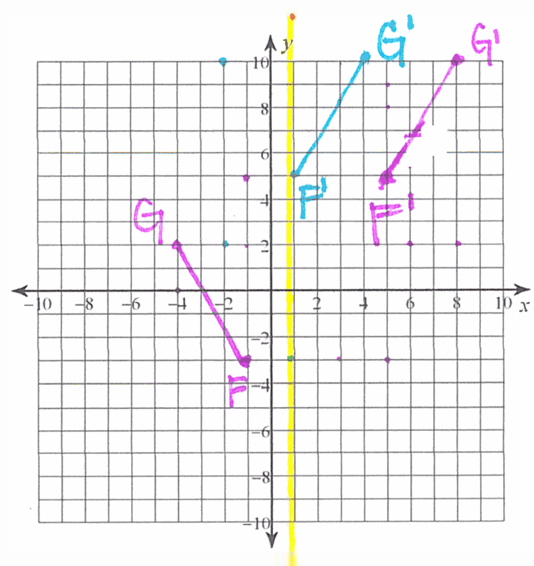
   yes!

   $F(-1,-3), G(-4,2)$

   Reflection: in the line $x = 1$

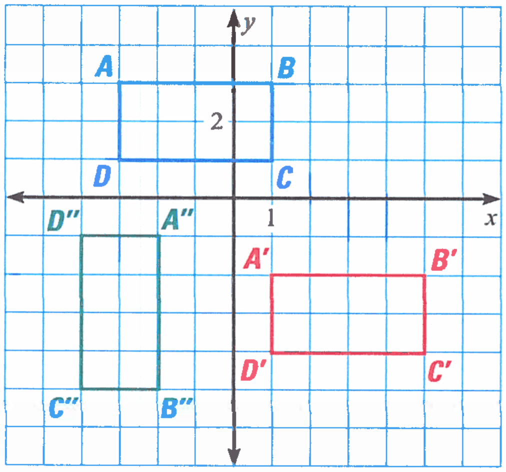
   Translation: $(x, y) \rightarrow (x + 2, y + 8)$





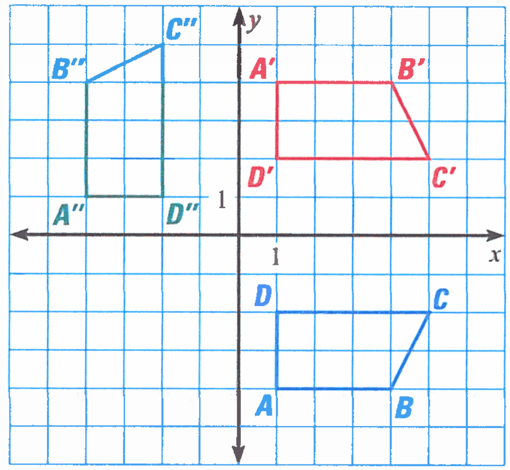
Describe the composition of the transformations.

4.



Right 4 & down 7
Rotate 90° clockwise

5.



Reflect over x-axis
Rotate 90° counterclockwise

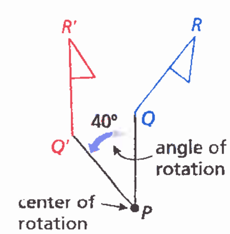
1.6 – TRANSFORMATIONS: PART 3

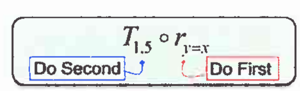
OBJECTIVES:

- Represent transformations in the plane
- Describe transformations as functions
- Develop definitions of translations, rotations, and reflections in terms of angles, circles, perpendicular lines, parallel lines, and line segments

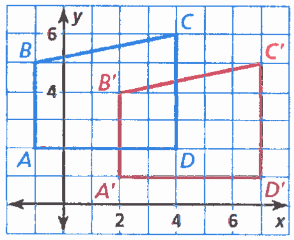
❖ Function Notation of Transformations

- ➢ Translations: $T_{a,b}$
  - ▪ $a > 0 =$ right, $a < 0 =$ left, $b > 0 =$ up, $b < 0 =$ down
- ➢ Rotations: $R_{C,\theta}$
  - ▪ C represents the center of rotation, with O representing the origin
  - ▪ θ the angle of rotation, positive indicates counterclockwise, while negative indicates clockwise
- ➢ Reflections: r_m
  - ▪ m represents the line of reflection
- ➢ Composition of Transformations: $\circ$ *(an open circle)*
  - ▪ Example: $T_{1,5} \circ r_{y=x}$
    - A translation $(x, y) \rightarrow (x + 1, y + 5)$ AFTER a reflection in the line $y = x$





1. Write a rule for the translation:

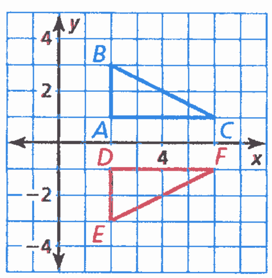


   a. Verbal rule:
   Right 3 & down 1
   b. Coordinate notation:
   $(x, y) \rightarrow (x+3, y-1)$
   c. Function notation:
   $T_{3,-1}$

TRANSLATIONS

2. Which of the following properties are true about a figure that has been translated? Select ALL that apply.
   a. Translations maps segments to parallel segments.
   b. The slopes of corresponding segments are opposite reciprocals.
   c. Segments connecting corresponding vertices of the image and pre-image are the same length.
   d. The perimeter of the pre-image is smaller than the perimeter of the image.
   e. Segments connecting corresponding vertices of the image and pre-image are parallel.
   f. The measures of corresponding angles of the image and pre-image are congruent.

3. Write a rule for the reflection:



   a. Verbal rule:
   reflect over x-axis
   b. Function notation:
   $r_{x\text{-axis}}$

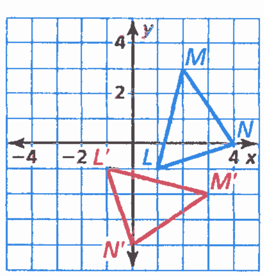
REFLECTIONS

4. Which of the following properties are true about a figure that has been reflected? Select ALL that apply.
   a. The slopes of corresponding segments are the same.
   b. Segments connecting corresponding vertices of the image and pre-image are the same length.
   c. The area of the pre-image is the same as the area of the image.
   d. Segments connecting corresponding vertices of the image and pre-image are parallel.
   e. The measures of the interior angles of an object may change under a reflection.
   f. Corresponding vertices are equidistant from the line of reflection.

5. Write a rule for the rotation:

$m_{MN} = \frac{-3}{2}$

$m_{M'N'} = \frac{2}{3}$



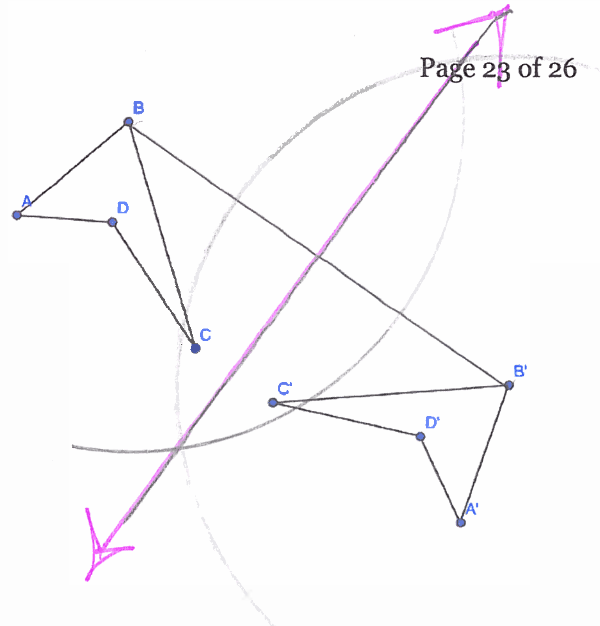
   a. Verbal rule:
   Rotate 90° clockwise
   b. Function notation:
   $R_{O,-90°}$

ROTATIONS

6. Which of the following properties are true about a figure that has been rotated 90°? Select ALL that apply.
   a. The slope of corresponding segments are opposite reciprocals.
   b. Segments connecting corresponding vertices of the image and pre-image are the same length.
   c. Lines that are parallel in the pre-image are not necessarily parallel in the image. N/A
   d. Segments connecting corresponding vertices of the image and pre-image are parallel.
   e. Corresponding segments are perpendicular.
   f. Corresponding vertices lie on the same circle. ?

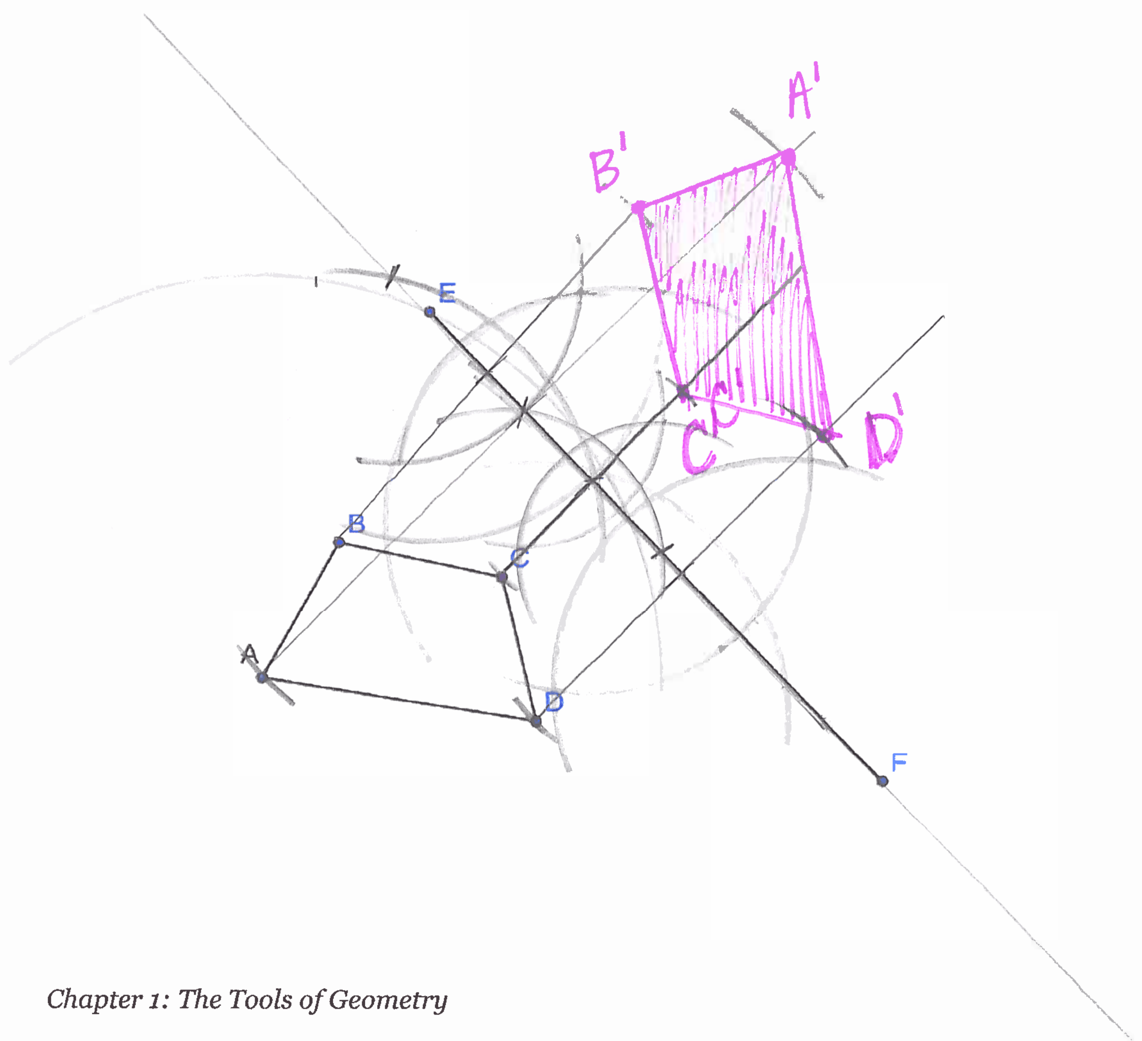
❖ Construct a Line of Reflection

- ➢ Connect any vertex to its image.
- ➢ Construct the perpendicular bisectors of the segment formed.



❖ Construct the Reflected Image

- ➢ Choose a starting vertex *A* and construct a perpendicular from the vertex *A* to the line of reflection.
- ➢ Measure the length from the vertex *A* to the intersection point and copy this length on the perpendicular bisector starting at the intersection point to find *A'*. (This is one vertex of the image.)
- ➢ Repeat this process for all other vertices.



❖ Find the Angle & Direction of Rotation

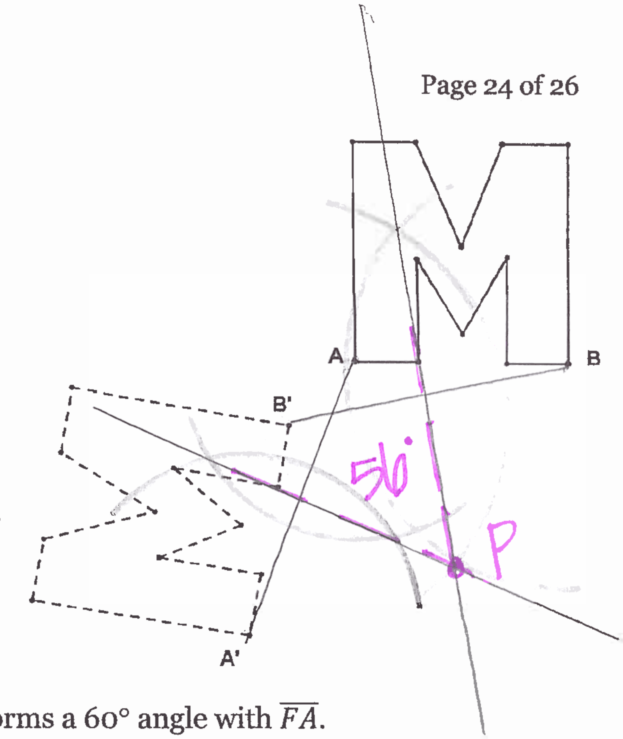
➤ Draw a segment connecting points A & A'

- Using a compass and a straightedge, find the perpendicular bisector of this segment.

➤ Draw a segment connecting points B & B'

- Using a compass and a straightedge, find the perpendicular bisector of this segment.

➤ The point of intersection of the two perpendicular bisectors is the center of rotation. Label this point P.

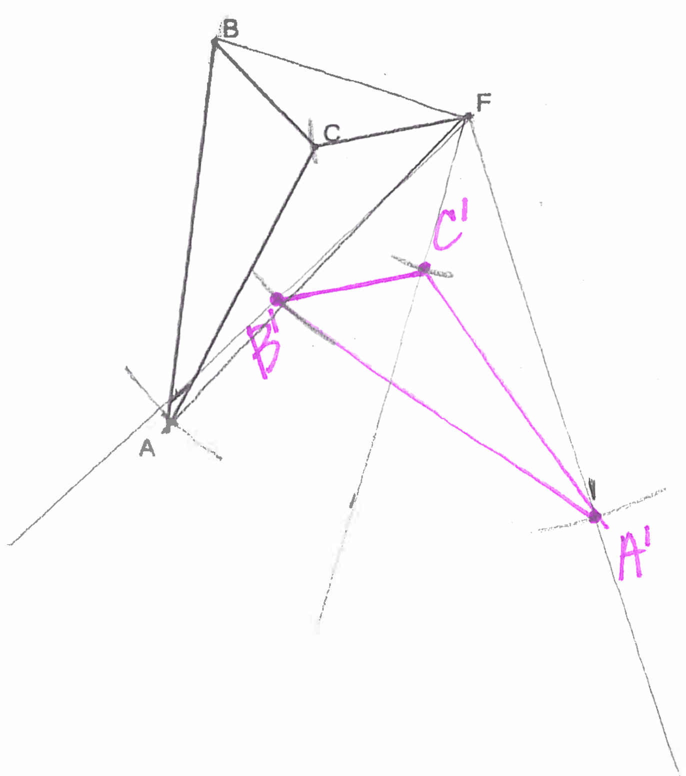


❖ Construct the Rotated Image

➤ Draw $\overline{FA}$. Then use a protractor to draw a ray that forms a 60° angle with $\overline{FA}$.

➤ Use a compass to mark point A' along the ray so that $FA' = FA$.

➤ Repeat the process for points B and C to locate points B' and C'.



1.7 – PERIMETER & AREA ON THE COORDINATE PLANE

OBJECTIVES:

- Use transformations to determine the perimeter and area of geometric figures
- Determine perimeters and areas of rectangles, triangles, and trapezoids on the coordinate plane
- Determine the perimeter and area of composite figures on the coordinate plane
- Use the boxing method to find the area of geometric figures

❖ Formulas

➢ Distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

➢ Perimeter & Area

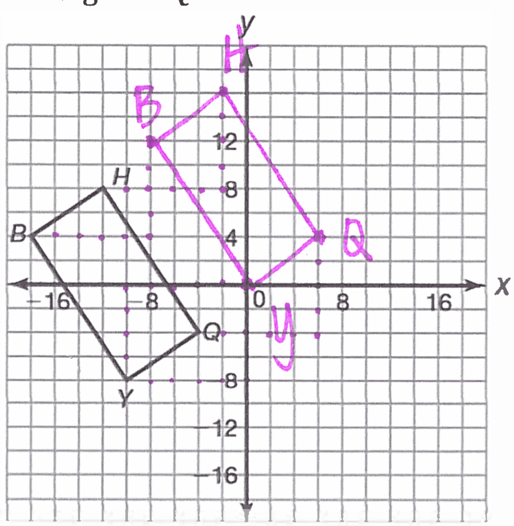
Square	Rectangle	Triangle	Trapezoid
$P = 4s$	$p = 2\ell + 2w$	$p =$ sum of the sides	$p =$ sum of the sides
$A = s^2$	$A = \ell w$ or $A = bh$	$A = \frac{1}{2}bh$	$A = \frac{1}{2}h(b_1 + b_2)$

❖ Using Transformations

➢ If a rigid motion is performed on a geometric figure, not only are the pre-image and the image congruent, but both the perimeter and area of the pre-image and image are equal.

Translate the rectangle such that one vertex of the image is located at the origin and label the vertices of the translated image. Calculate the perimeter and area of the image. Round your answer to the nearest hundredth, if necessary.

1. Rectangle *BHQY*



10 Right & 8 up

$QY = \sqrt{6^2 + 4^2} = \sqrt{40}$

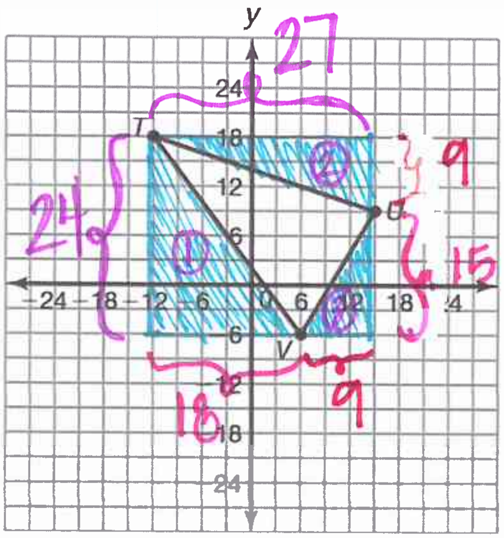
$BY = \sqrt{8^2 + 12^2} = \sqrt{208}$

Perimeter $= 2\sqrt{40} + 2\sqrt{208} \approx 41.49$ units

Area $= \sqrt{40} \cdot \sqrt{208} \approx 91.21$ units^2

- The Boxing Method
  - Sketch a rectangle so that each side of the rectangle passes through each vertex and all sides of the rectangle are either horizontal or vertical.
    - Calculate the area of the rectangle
  - Calculate the area of each right triangle
  - To find the area of the original geometric figure subtract the sum of the areas of the right triangles from the area of the rectangle

2. Use the boxing method to find the area of $\triangle TUV$.



Area of the rectangle: $24 \cdot 27 = 648$

Area of $\triangle 1$: $\frac{1}{2} \cdot 24 \cdot 18 = 216$

Area of $\triangle 2$: $\frac{1}{2} \cdot 27 \cdot 9 = 121.5$

Area of $\triangle 3$: $\frac{1}{2} \cdot 9 \cdot 15 = 67.5$

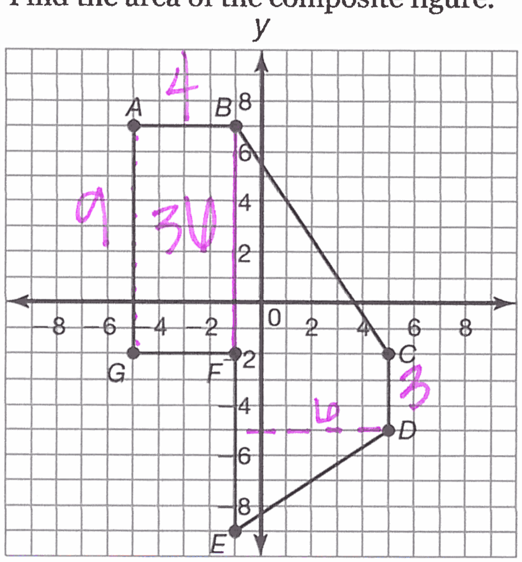
Δ Area sum $\rightarrow$ 405

Area of $\triangle TUV$: Rect − Δ sum

$648 - 405 = 243$ units^2

- Area of Composite Figures
  - A composite figure is a figure formed by combining different shapes.
  - The area of a composite figure can be calculated by drawing line segments on the figure to divide it into familiar shapes, determining the area of each shape, and finding the sum of these areas.

3. Find the area of the composite figure.



BE = 16

$\underbrace{\text{rect } ABFG}_{9 \cdot 4} + \underbrace{\text{trap. } BCDE}_{\frac{1}{2} \cdot 6(16+3)}$

$36 + 57$

93 units^2