 Name: ________________________________

Reasoning & Proof

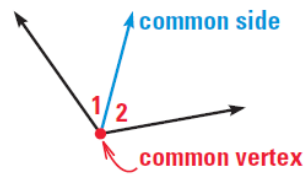
2.2 – Angle Pair Relationships

OBJECTIVES:

- Identify different types of angle relationships formed by intersecting lines and perpendicular lines
- Apply angle pair relationships to find measures of complementary angles, supplementary angles, linear pairs, and vertical angles

❖ Adjacent Angles

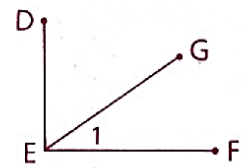
➢ Two angles with a common side, a common vertex, and no common interior points.



❖ Perpendicular

➢ Lines, rays, or segments that intersect at right angles are <u>perpendicular</u> (⊥).

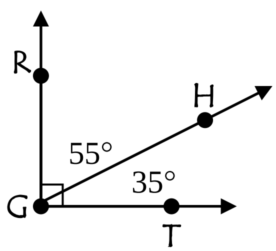
- Given: $\overline{DE} \perp \overline{EF}$
- Conclusion: $\angle DEF$ is a right angle



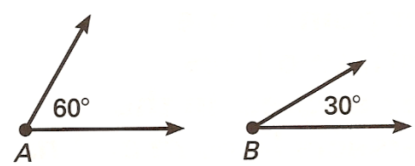
❖ Complementary Angles

➢ Two or more angles whose sum is 90° (or a right angle)

- Each angle is called the *complement* of the other.



∠RGH & ∠HGT ARE COMPLEMENTARY

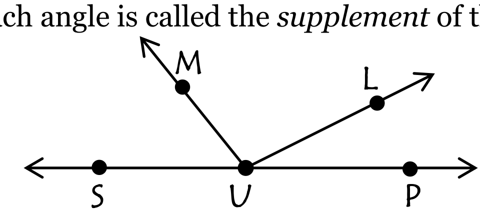


∠A & ∠B ARE COMPLEMENTARY

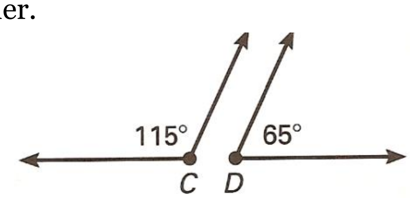
❖ Supplementary Angles

➢ Two or more angles whose sum is 180° (or a straight angle)

- Each angle is called the *supplement* of the other.



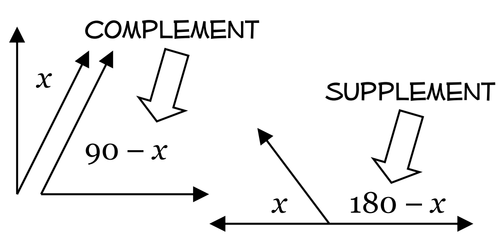
∠SUM, ∠MUL, & ∠LUP ARE SUPPLEMENTARY



∠C & ∠D ARE SUPPLEMENTARY

➢ The Alge-Geometric Connection

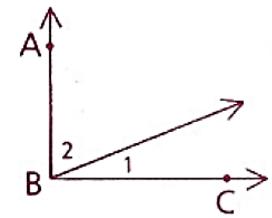
- x = the measure of the angle
- $90 - x$ = the measure of the complement
- $180 - x$ = the measure of the supplement of the angle



EXAMPLES:

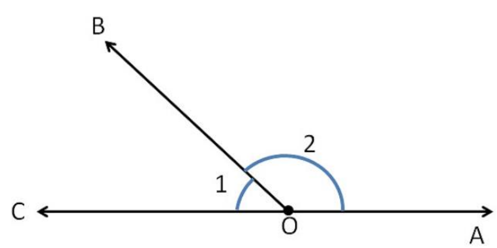
1. Find the measure of the complement and supplement of a $43°$ angle.

2. Given: $\overrightarrow{BA} \perp \overrightarrow{BC}$, $m\angle 2$ is three greater than twice $m\angle 1$.
   Find the measure of each angle.



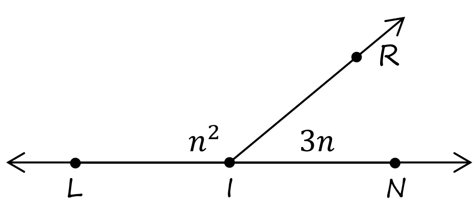
3. The measure of the supplement of an angle is 60 less than 3 times the measure of the complement of the angle. Find the measure of the complement.

❖ Linear Pairs
  ➢ A pair of adjacent angles whose non-common sides are on the same line.
    ▪ *We are allowed to assume linear pairs from the diagram.*
  ➢ **Linear Pair Postulate** ~ If two angles form a linear pair, then they are supplementary.
    ▪ $\angle 1$ & $\angle 2$ form a linear pair
    ▪ $m\angle 1 + m\angle 2 = 180°$

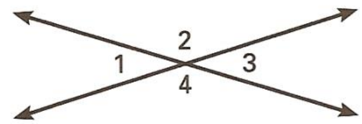


EXAMPLE:

4. Set up and solve a quadratic equation to find the value of *x*. Then find $m\angle LIR$ and $m\angle RIN$.

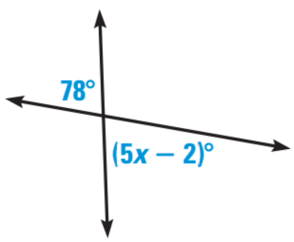


- Vertical Angles
  - Whenever two lines intersect, two pairs of vertical angles are formed.
    - *We are allowed to assume vertical angles from the diagram.*
  - Their sides form two pairs of opposite rays
    - <u>Opposite rays</u>—two collinear rays that have a common endpoint & extend in different directions
  - **Vertical angles are congruent.**
    - $\angle 1$ & $\angle 3$ and $\angle 2$ & $\angle 4$ are vertical angles
    - $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$

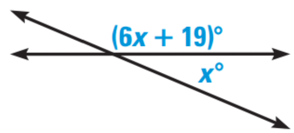


<u>EXAMPLES:</u> Use vertical angle or linear pair relationships to set up and solve an equation to find the value of x.

5.



6.



~ ~

2.4 – Beginning Proofs

OBJECTIVES:

- Structure statements and reasons to form a logical argument
- Interpret geometric diagrams

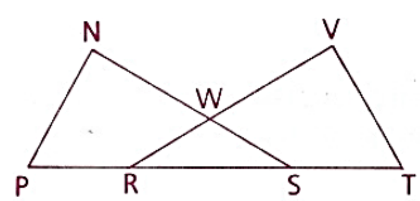
- *Assumptions from Diagrams*
  - ***YOU SHOULD ASSUME:***
    - Straight lines & angles
    - Linear Pairs
    - Collinearity & betweeness of points
    - Vertical angles

YOU SHOULD NEVER ASSUME:
- Right angles
- Congruent segments
- Congruent angles
- Relatives sizes of segments & angles

Example:

1. Which of the following can we assume from the diagram? Select ALL that apply.

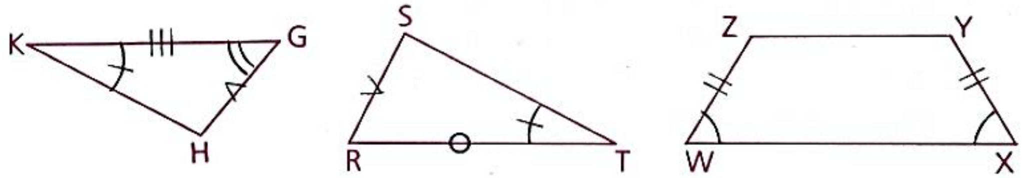
   a. $\angle NWR$ & $\angle VWT$ are vertical angles
   b. $\angle V$ is a right angle
   c. R is between P and S
   d. $\angle P \cong \angle T$
   e. $\angle RWV$ is a straight angle
   f. $\angle PRV$ & $\angle SRW$ form a linear pair
   g. $\overline{NP} \cong \overline{VT}$



OFTEN, WE USE IDENTICAL TICK MARKS TO INDICATE CONGRUENT SEGMENTS AND ARC MARKS TO INDICATE CONGRUENT ANGLES.

Example:

2. Write a congruence statement for all pairs of congruent segments and angles in each diagram.



❖ Writing Two-Column Proofs

- Proof – A convincing argument that shows why a statement is true
  - The proof begins with the given information & ends with the statement you are trying to prove.
  - Two-Column Proof:

Statements	Reasons
• Specific – applies only to this proof	• General – can apply to any proof

❖ Procedure for Drawing Conclusions

1. Memorize theorems, definitions, & postulates.
2. Look for key words & symbols in the given information.
3. Think of all the theorems, definitions, & postulates that involve those keys.
4. Decide which theorem, definition, or postulate allows you to draw a conclusion.
5. Draw a conclusion, & give a reason to justify the conclusion. Be certain that you have not used the *reverse* of the correct reason.
   - The "If…" part of the reason matches the given information, and the "then…" part matches the conclusion being justified.

TODAY WE WILL LEARN HOW TO USE THE FOLLOWING REASONS IN PROOFS:

Definition of congruent angles *Angles w/the same measure are congruent.*	definition of right angle *An angle with a measure of 90° is a right angle.*
Assumed from diagram. *Straight angles, linear pairs, vertical angles*	Right angles are congruent.
Straight angles are congruent.	**Angle Addition Postulate**

Substitution

- Substitution Property
  - Think of it as "replacement."

Example #3

Given: $m\angle 1 + m\angle 2 = 90°$,

$\angle 1 \cong \angle 3$

Prove: $m\angle 3 + m\angle 2 = 90°$

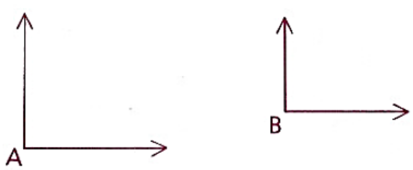
Statements	Reasons

<u>Theorem</u> – A mathematical statement that can be proved.

IF TWO ANGLES ARE RIGHT ANGLES, THEN THEY ARE CONGRUENT.

Given: $\angle A$ is a right $\angle$.
$\angle B$ is a right $\angle$.

Prove: $\angle A \cong \angle B$



Proof:

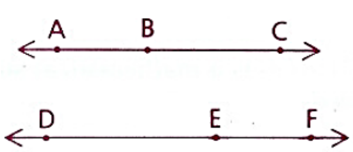
Statements	Reasons
1 $\angle A$ is a right angle.	1 Given
2 $m\angle A = 90$	2 If an angle is a right angle, then its measure is 90.
3 $\angle B$ is a right angle.	3 Given
4 $m\angle B = 90$	4 Same as 2
5 $\angle A \cong \angle B$	5 If two angles have the same measure, then they are congruent.

Steps 2 & 4 can be omitted in future proofs.

IF TWO ANGLES ARE STRAIGHT ANGLES, THEN THEY ARE CONGRUENT.

Given: $\angle ABC$ is a straight angle.
$\angle DEF$ is a straight angle.

Prove: $\angle ABC \cong \angle DEF$



Proof:

Statements	Reasons
1 $\angle ABC$ is a straight angle.	1 Given
2 $m\angle ABC = 180$	2 If an angle is a straight angle, then its measure is 180.
3 $\angle DEF$ is a straight angle.	3 Given
4 $m\angle DEF = 180$	4 Same as 2
5 $\angle ABC \cong \angle DEF$	5 If two angles have the same measure, then they are congruent.

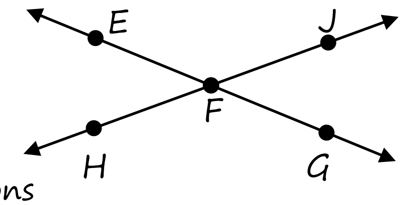
Steps 2 & 4 can be omitted in future proofs.

<u>NOW THAT WE HAVE "PROVEN" THESE THEOREMS, WE CAN USE THEM IN PROOFS!</u>

Example #4

Given: Diagram as shown

Prove: $\angle EFG \cong \angle HFJ$

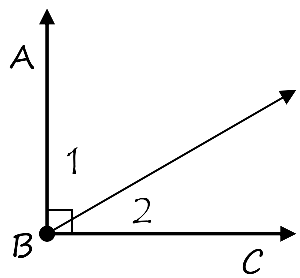


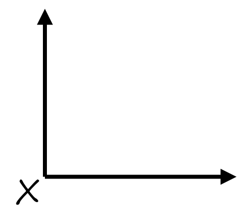
Statements	Reasons

Example #5

Given: $m\angle 1 = 50°$
$m\angle 2 = 40°$
$\angle X$ is a right angle

Prove: $\angle ABC \cong \angle X$





Statements	Reasons

2.5 – Bisectors, Perpendicularity, & Vertical Angles

OBJECTIVES:

- Prove conjectures involving midpoints, bisectors, perpendicularity, and vertical angles
- Structure statements and reasons to form a logical argument

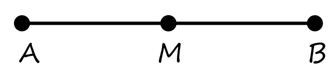
TODAY WE WILL LEARN HOW TO USE THE FOLLOWING REASONS IN PROOFS:

Vertical angles are congruent.	**TRANSITIVE PROPERTY** *If angles (or segments) are congruent to the same (or congruent) angle (or segment), then they are congruent to each other.*
Definition of bisects (or trisects) *If a ray bisects an angle, then it divides the angle into two congruent angles.*	**Definition of perpendicular (⊥)** *If two lines are perpendicular, then they intersect and form right angles.*
Definition of midpoint *If a point is a midpoint of a segment, then it divides the segment into two congruent segments.*	

❖ Midpoints & Bisectors of Segments

➢ A point (or segment, ray, or line) that divides a segment into two congruent segments <u>bisects</u> the segment.

- The bisection point is called the <u>midpoint</u> of the segment.
- Only segments have midpoints.

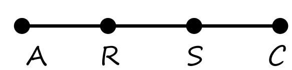


Statements	Reasons
1. M is the midpoint of $\overline{AB}$	1. Given
2.	2. Definition of midpoint

❖ Trisection Points & Trisecting a Segment

➢ Two points (or segments, rays, or lines) that divide a segment into three congruent segments <u>trisect</u> the segment.

- The two points at which the segment is divided are called the <u>trisection points</u> of the segment.
- Only segments have trisection points.

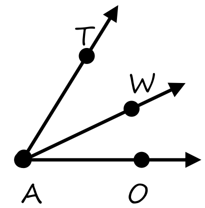


Statements	Reasons
1. R and S are trisection points of $\overline{AC}$	1. Given
2.	2. Definition of trisects

❖ Angle Bisectors

➢ A ray that divides an angle into two congruent angles bisects the angle.

- The dividing ray is called the bisector of the angle.

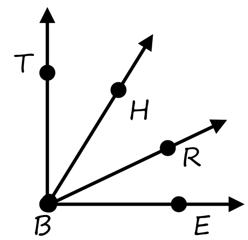


Statements	*Reasons*
1. $\overrightarrow{AW}$ bisects $\angle TAO$	1. Given
2.	2. Definition of bisects

❖ Angle Trisectors

➢ Two rays that divide an angle into three congruent angles trisect the angle.

- The two dividing rays are called the trisectors of the angle.



Statements	*Reasons*
1. $\overrightarrow{BH}$ and $\overrightarrow{BR}$ trisect $\angle TBE$	1. Given
2.	2. Definition of trisects

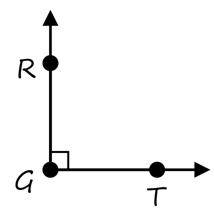
❖ Perpendicular Lines, Rays, & Segments

➢ Perpendicularity, right angles, & 90° measurements all go together.

➢ Lines, rays, or segments that intersect at right angles are perpendicular ($\perp$).

- A pair of perpendicular lines forms four right angles.

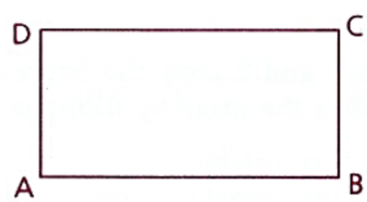
➢ Do not assume perpendicularity from a diagram!



Statements	*Reasons*
1. $\overrightarrow{GR} \perp \overrightarrow{GT}$	1. Given
2.	2. Definition of perpendicular

Examples ~

1. Given: $\overline{AB} \perp \overline{BC}$
   $\overline{DC} \perp \overline{AD}$
   Prove: $\angle B \cong \angle D$



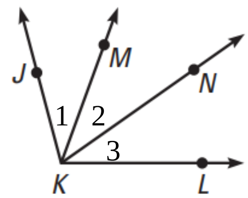
Statements	*Reasons*

- Transitive Properties of Congruence
  - If angles (or segments) are congruent to the same angle (or segment), they are congruent to each other.
  - If angles (or segments) are congruent to congruent angles (or segments), they are congruent to each other.

Examples ~

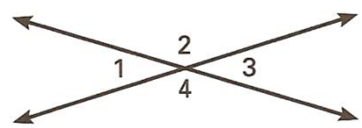
2. Given: $\overrightarrow{KM}$ bisects $\angle JKN$
   $\overrightarrow{KN}$ bisects $\angle MKL$

   Prove: $\angle 1 \cong \angle 3$



Statements	Reasons

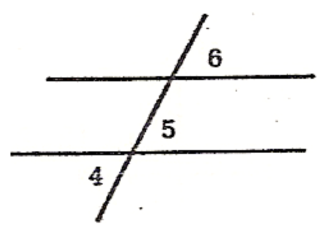
- Vertical Angles
  - Whenever two lines intersect, two pairs of vertical angles are formed.
    - *We are allowed to assume vertical angles from the diagram.*
  - **Vertical Angles Theorem:** Vertical angles are congruent.
    - $\angle 1$ & $\angle 3$ and $\angle 2$ & $\angle 4$ are vertical angles
    - $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$



Examples ~

3. Given: $\angle 4 \cong \angle 6$

   Prove: $\angle 5 \cong \angle 6$



Statements	Reasons

2.6 – Complementary & Supplementary Angles

OBJECTIVES:

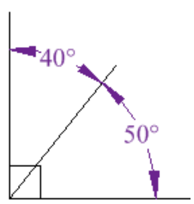
- Prove conjectures involving complementary and supplementary angles
- Structure statements and reasons to form a logical argument

TODAY WE WILL LEARN HOW TO USE THE FOLLOWING REASONS IN PROOFS:

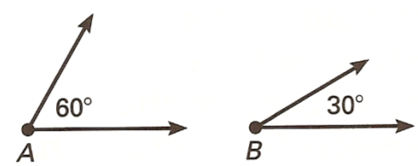
LINEAR PAIR POSTULATE *If two angles form a linear pair, then they are supplementary.*	**Definition of complementary angles** *If the sum of two angles is a right angle, then they are complementary.*
Congruent Supplements Theorem *If angles are supplementary to the same angle (or congruent angles), then they are congruent.*	**Congruent Complements Theorem** *If angles are complementary to the same angle (or congruent angles), then they are congruent.*

❖ Complementary angles—two angles whose sum is 90° (a right angle)

➢ Each of the two angles is called the complement of the other.

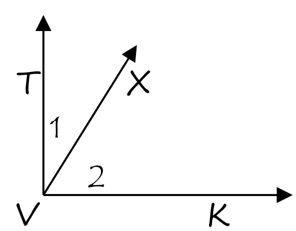


NO NEED FOR ANGLES TO BE ADJACENT TO BE COMPLEMENTARY.



Example ~

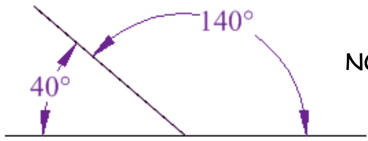
1. Given: $\angle TVK$ is a right $\angle$.
   Prove: $\angle 1$ is complementary to $\angle 2$.



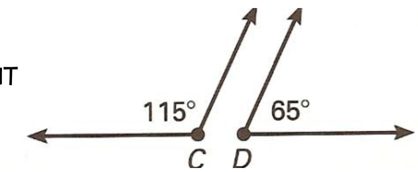
Statements	Reasons

Pattern for proving complementary angles – it follows from the definition of complementary angles: (1) Right angle; (2) Angle Addition Postulate; (3) Complementary

- <u>Supplementary angles</u>—two angles whose sum is 180° (a straight angle)
  - Each of the two angles is called the <u>supplement</u> of the other.



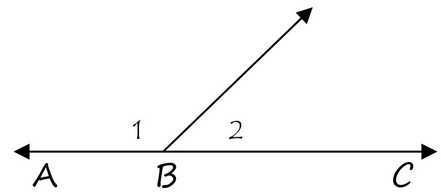
NO NEED FOR ANGLES TO BE ADJACENT TO BE SUPPLEMENTARY.



- **Linear Pair Postulate** ~ If two angles form a linear pair, then they are supplementary.

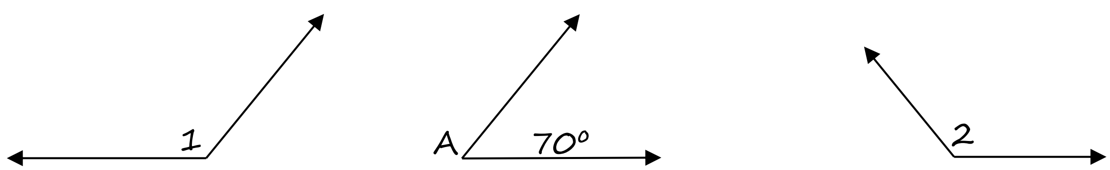
Example ~

2. Given: Diagram as shown
   Prove: ∠1 is supplementary to ∠2.



Statements	Reasons

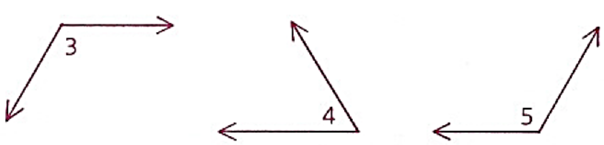
- Congruent Complements & Supplements
  - In the diagram below, ∠1 is supplementary to ∠*A*, and ∠2 is also supplementary to ∠*A*.

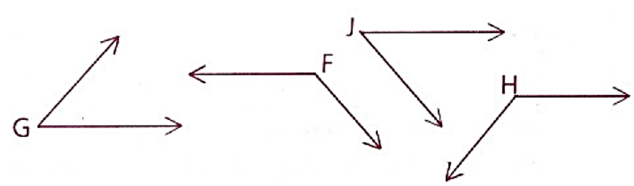


- How large is ∠1? How large is ∠2? How does ∠1 compare with ∠2?

CONGRUENT SUPPLEMENTS THEOREM

IF ANGLES ARE SUPPLEMENTARY TO THE <u>SAME ANGLE</u>, THEN THEY ARE CONGRUENT.	IF ANGLES ARE SUPPLEMENTARY TO <u>CONGRUENT ANGLES</u>, THEN THEY ARE CONGRUENT.
Given: ∠3 is supp. to ∠4. ∠5 is supp. to ∠4. Prove: ∠3 ≅ ∠5	Given: ∠F is supp. to ∠G. ∠H is supp. to ∠J. ∠G ≅ ∠J Conclusion: ∠F ≅ ∠H



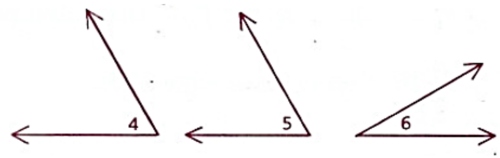


Congruent Complements Theorem

These work in a similar fashion.

- If angles are complementary to the same angle, then they are congruent.
- If angles are complementary to congruent angles, then they are congruent.

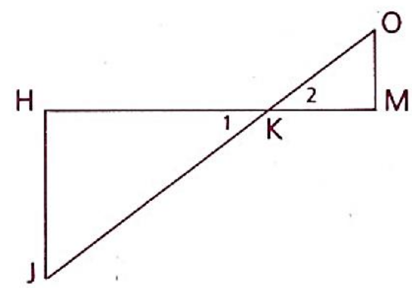
Given: $\angle 4$ is comp. to $\angle 6$.
$\angle 5$ is comp. to $\angle 6$.
Prove: $\angle 4 \cong \angle 5$



Example ~

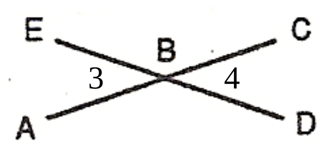
3. Given: $\angle O$ is comp. to $\angle 2$
   $\angle J$ is comp. to $\angle 1$

   Prove: $\angle O \cong \angle J$



Statements	Reasons

4. ***Proof of the Vertical Angles Theorem***
   Given: Diagram as shown
   Prove: $\angle 3 \cong \angle 4$
   Do not use vertical angles.



Statements	Reasons

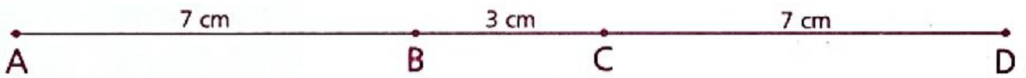
2.7 – PROPERTIES OF SEGMENTS & ANGLES

OBJECTIVES:

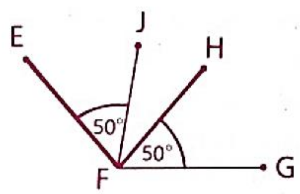
- Prove conjectures involving the addition and subtraction of segments and angles
- Structure statements and reasons to form a logical argument

INVESTIGATION:

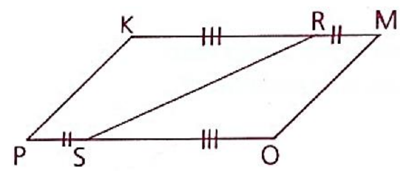
If $\overline{AB} \cong \overline{CD}$, does $\overline{AC} \cong \overline{BD}$? EXPLAIN YOUR REASONING.



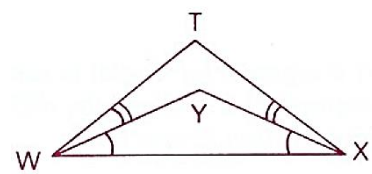
Is $\angle EFH \cong \angle JFG$?
EXPLAIN YOUR REASONING.



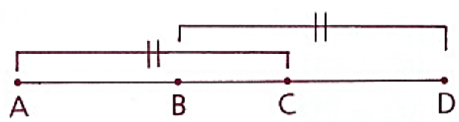
Is $\overline{KM} \cong \overline{PO}$?
EXPLAIN YOUR REASONING.



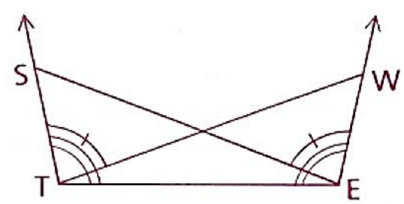
Is $\angle TWX \cong \angle TXW$?
EXPLAIN YOUR REASONING.



If $\overline{AC} \cong \overline{BD}$, does $\overline{AB} \cong \overline{CD}$?
EXPLAIN YOUR REASONING.



If $\angle STE \cong \angle WET$ & $\angle STW \cong \angle WES$…
Is $\angle SET \cong \angle WTE$?
EXPLAIN YOUR REASONING.



TODAY WE WILL LEARN HOW TO USE THE FOLLOWING REASONS IN PROOFS:

- ***SEGMENT ADDITION PROPERTY***
  - If a segment is added to two congruent segments, the sums are congruent.
  - If congruent segments are added to congruent segments, the sums are congruent.
- ***ANGLE ADDITION PROPERTY***
  - If an angle is added to two congruent angles, the sums are congruent.
  - If congruent angles are added to congruent angles, the sums are congruent.
- ***SEGMENT SUBTRACTION PROPERTY***
  - If a segment is subtracted from two congruent segments, the differences are congruent.
  - If congruent segments are subtracted from congruent segments, the differences are congruent.
- ***ANGLE SUBTRACTION PROPERTY***
  - If an angle is subtracted from two congruent angles, the differences are congruent.
  - If congruent angles are subtracted from congruent angles, the differences are congruent.

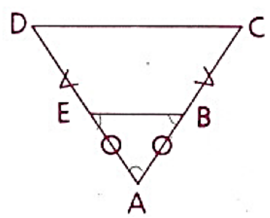
Go back to the Investigation and identify which property is represented in each problem.

- **REFLEXIVE PROPERTY**: *Any segment or angle is congruent to itself.*
  - Whenever a segment or an angle is shared by two figures, we can say that the segment or angle is congruent to itself.

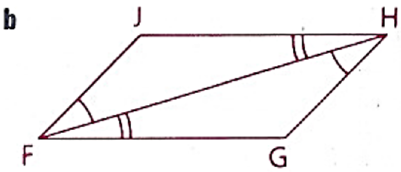
examples:

1. Name the angles or segments that are congruent by the Addition Property.

a

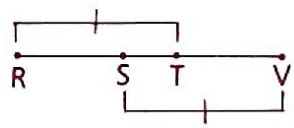


b



2. Name the angles or segments that are congruent by the Subtraction Property.

a



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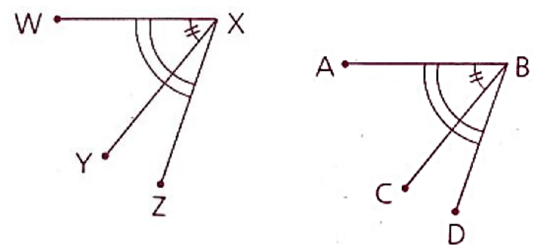
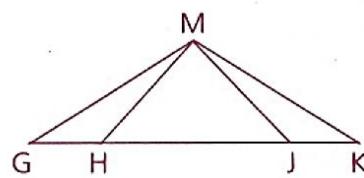


3. Given: $\overline{GJ} \cong \overline{HK}$

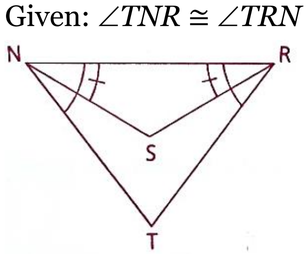
Prove: $\overline{GH} \cong \overline{JK}$



Statements	Reasons

4. What can you conclude from the given information? Which property would you use?

a. Given: $\angle TNR \cong \angle TRN$ & $\angle NRS \cong \angle RNS$



b. Given: $\overline{AB} \cong \overline{CD}$