Name: ______________________________

Parallel & Perpendicular Lines

3.1 • Lines in the Coordinate Plane

OBJECTIVES:

- Identify the slope and x- and y-intercepts of linear equations
- Graph and write linear equations

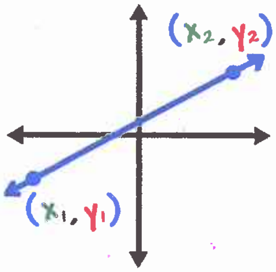
❖ Slope

➢ The slope, m, of a line is the ratio of the vertical change (rise) to the horizontal change (run).

$$m = \frac{\text{rise}}{\text{run}}$$

If you're given two points (x_1, y_1) and (x_2, y_2)

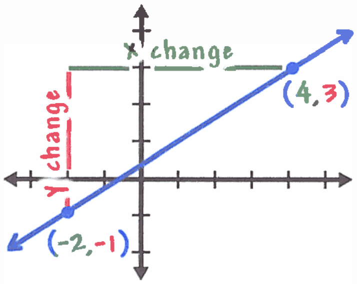
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



EXAMPLE: (-2,-1) and (4,3)

Look at $\frac{\text{rise}}{\text{run}}$ as $\frac{\text{the change in the y's}}{\text{the change in the x's}}$

$$= \frac{3-(-1)}{4-(-2)} = \frac{4}{6} = \frac{2}{3}$$

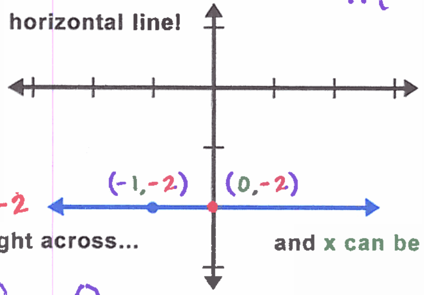


❖ Horizontal & Vertical Lines

Horizontal Lines: $y = \#$

$m=0$

It's a horizontal line!



$y = -2$

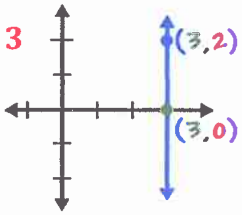
straight across...

and x can be anything!

$$m = \frac{-2-(-2)}{0-(-1)} = \frac{0}{1} = 0$$

Vertical Lines: $x = \#$

$x = 3$



It's a vertical line!
What's the slope?

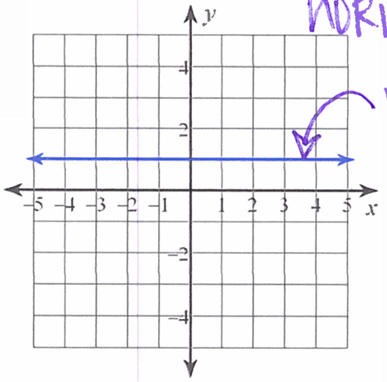
$$m = \frac{2-0}{3-3} = \frac{2}{0}$$

The slope of a vertical line is undefined.

This is the official word.

EXAMPLES: Find the slope and equation for each line shown.

1.

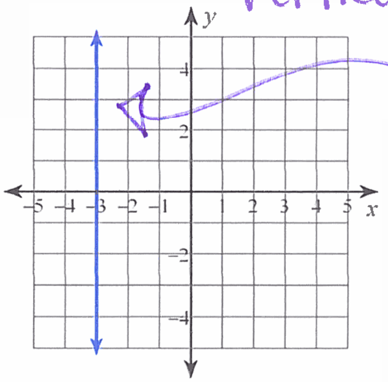


horizontal line

$y=1$

$m=0$

2.



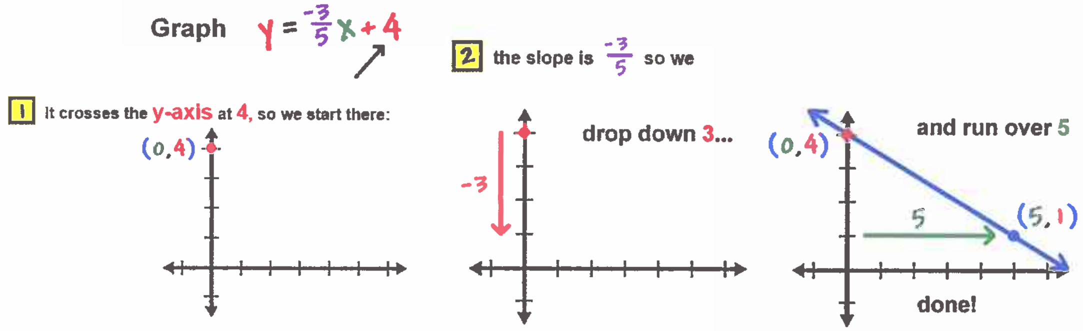
vertical line

$X=-3$

slope is undefined

- Slope-Intercept Form
  - $y = mx + b$
    - The coefficient of x, which is m, is the slope of the line.
    - The constant term, b, is the y-coordinate of the y-intercept.
- Graphing a Function in Slope-Intercept Form
  - Plot the y-intercept: $(0, b)$
  - From there, use the slope, m, to determine a second point
    - If the slope is positive, go up the number of units in the numerator
    - If the slope is negative, go down the number of units in the numerator
    - Always go right the number of units in the denominator

If you're going to go off the grid, then do the complete opposite.

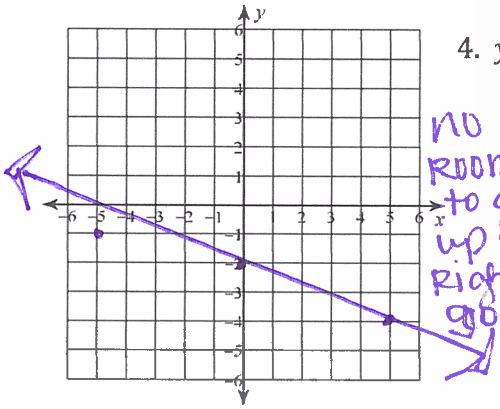


EXAMPLES: GRAPHING A FUNCTION IN SLOPE-INTERCEPT FORM

3. $y = -\frac{2}{5}x - 2$

down 2
Right 5

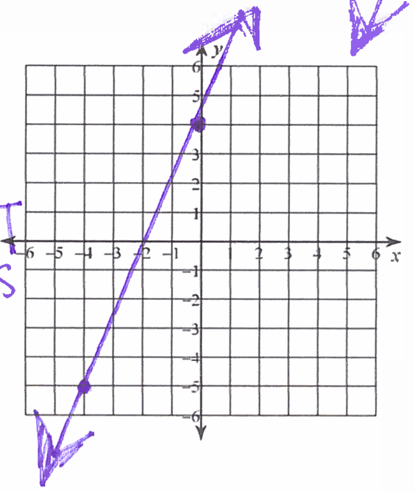
START on y-axis



4. $y = \frac{9}{2}x + 4$

no room to go up 9 & Right 4
go down 9 & left 4

START y-axis



take note

Key Concept Forms of Linear Equations

Definition

The **slope-intercept form** of an equation of a nonvertical line is $y = mx + b$, where m is the slope and b is the y-intercept.

The **point-slope form** of an equation of a nonvertical line is $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) is a point on the line.

Symbols

$$y = \underset{\underset{\text{slope}}{\uparrow}}{m}x + \underset{\underset{y\text{-intercept}}{\uparrow}}{b}$$

$$y - \underset{\underset{y\text{-coordinate}}{\uparrow}}{y_1} = \underset{\underset{\text{slope}}{\uparrow}}{m}(x - \underset{\underset{x\text{-coordinate}}{\uparrow}}{x_1})$$

- ❖ Writing Linear Functions

$Y - Y_1 = m(X - X_1)$

$m = -2$ $(4, -3)$

$\uparrow x_1$ $\uparrow y_1$

Substitute ① $Y-(-3) = -2(X-4)$

simplify ② $Y+3 = -2X+8$ ← be careful here!

$-3 \quad -3$

Solve for y ③ $Y = -2X+5$ done

You could substitute the point & the slope into the slope-intercept equation of a line and then solve for *b*:

$y = mx + b$ ① Substitute

$-3 = -2(4) + b$ ② simplify

$-3 = -8 + b$ ③ Solve for b

$5 = b$

Then only plug in the slope & y-intercept (*b*): ④ write the function

$y = -2x + 5$

EXAMPLES: WRITING LINEAR FUNCTIONS

Write the slope-intercept form of the line described.

5. The line that passes through (−1, 4) with slope −3

$x_1 \; y_1 \qquad m$

$y - y_1 = m(x - x_1)$

$y - 4 = -3(x+1)$

$y - 4 = -3x - 3$

$y = -3x + 1$

$y = mx + b$

$4 = -3(-1) + b$

$4 = 3 + b$

$1 = b$

$y = -3x + 1$

6. The line that passes through the points (3, −5) & (−4, −1)

NEED SLOPE!

$m = \frac{-1-(-5)}{-4-3} = \frac{4}{-7}$

Function:

$y + 5 = -\frac{4}{7}(x-3)$

$y + 5 = -\frac{4}{7}x + \frac{12}{7}$

$y = -\frac{4}{7}x - \frac{23}{7}$

- ❖ Standard Form
  - ➢ A linear function can be written in standard form as $Ax + By = C$ where A, B, and C are real numbers and A and B cannot both be zero.

- ❖ Graphing a Function in Standard Form
  - ➢ Find and plot the vertical and horizontal intercepts of the function.

Graph $x + 2y = 4$

x	y
0	
	0

To find the x-intercept: Let y = 0 & solve for x.

x  $= 4$

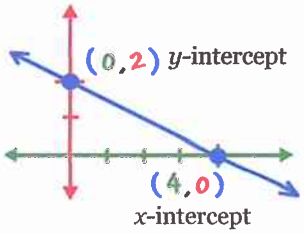
$x = 4$

To find the y-intercept: Let x = 0 & solve for y.

 $+ 2y = 4$

$y = 2$

This is also ***b*** *in slope-intercept form.*



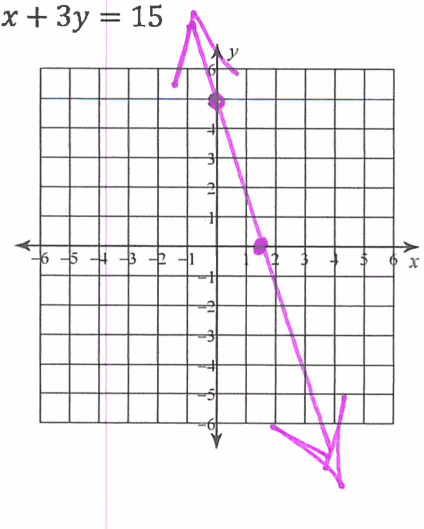
EXAMPLES: GRAPHING A LINEAR FUNCTION IN STANDARD FORM

Find the vertical and horizontal intercepts of the function. Then graph the function.

7. $10x + 3y = 15$

$10x = 15$
$x = 1.5$

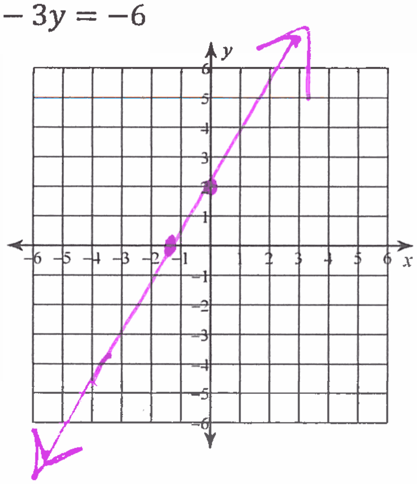
$3y = 15$
$y = 5$



8. $5x - 3y = -6$

$5x = -6$
$x = -1.2$

$-3y = -6$
$y = 2$



3.2 • Parallel & Perpendicular Lines

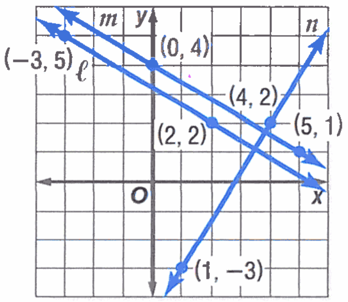
OBJECTIVES:

- Prove the slope criteria for parallel and perpendicular lines and use them to sole geometric problems
- Write equations to describe lines parallel or perpendicular to a given lines

PARALLEL AND PERPENDICULAR LINES

Examine the graphs of lines ℓ, m, and n. Lines ℓ and m are parallel, and n is perpendicular to ℓ and m. Let's investigate the slopes of these lines.

slope of ℓ	slope of m	slope of n
$m = \dfrac{2-5}{2-(-3)}$	$m = \dfrac{1-4}{5-0}$	$m = \dfrac{2-(-3)}{4-1}$
$= -\dfrac{3}{5}$	$= -\dfrac{3}{5}$	$= \dfrac{5}{3}$



❖ Slopes of Parallel Lines $\ell \parallel m$

- ➢ If two nonvertical lines are parallel, then their slopes are the SAME!
- ➢ Any two vertical or horizontal lines are parallel.

❖ Slopes of Perpendicular Lines $\ell \perp n$ & $m \perp n$

- ➢ The slopes of perpendicular lines are opposite Reciprocals
- ➢ Any horizontal line and vertical line are perpendicular.

ex. $-\frac{3}{5}$ & $\frac{5}{3}$ opp. signs
flip to find Recip.

EXAMPLES: DETERMINE LINEAR RELATIONSHIPS

$m = \frac{y_2 - y_1}{x_2 - x_1}$

1. Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel, perpendicular, or neither: $\underbrace{A(-8,-7), B(4,-4)}, \underbrace{C(-2,-5), D(1,7)}$

$m = \frac{-4+7}{4+8} = \frac{3}{12} = \frac{1}{4}$

$m = \frac{7+5}{1+2} = \frac{12}{3} = 4$

neither
not the
same or
Opp. Recip.

EXAMPLES: DETERMINE LINEAR RELATIONSHIPS

$y = mx + b$

slope ↑ y-intercept ↖

Find the slope of each line, then determine if the lines are parallel, perpendicular, or neither.

2. $2x + 3y = 6$ → $3y = -2x + 6$

$y = -\frac{2}{3}x + 2$

$m = \boxed{-\frac{2}{3}}$

$6x - 4y = 24$

↓

$-4y = -6x + 24$

$y = \frac{3}{2}x - 6$

$m = \boxed{\frac{3}{2}}$ ⊥ Opp. Recips

3. $2x + 5y = -1$ → $5y = -2x - 1$

$y = -\frac{2}{5}x - \frac{1}{5}$

$m = \boxed{-\frac{2}{5}}$

$10y = -4x - 20$

↓

$y = -\frac{4}{10}x - 2$

$y = \boxed{-\frac{2}{5}}x - 2$

// same slopes!

EXAMPLES: WRITE LINEAR FUNCTIONS

$y - y_1 = m(x - x_1)$

Write the slope-intercept form of the line described.

4. The line that passes through $(-8, 6)$ and is parallel to the line $\underbrace{4x + y = -5}$

① Identify slope of given line.

$y = \boxed{-4}x - 5$

② ↓ $m_{\parallel} = -4$

③ $y - 6 = -4(x + 8)$

$y - 6 = -4x - 32$

$y = -4x - 26$

5. The line that passes through $(-3, -4)$ and is $\overline{\text{perpendicular}}$ to the line $\underbrace{y = 2x - 3}$

① $m = 2$

② ↓ $m_{\perp} = -\frac{1}{2}$

③ $y + 4 = -\frac{1}{2}(x + 3)$

$y + 4 = -\frac{1}{2}x - \frac{3}{2}$

$y = -\frac{1}{2}x - \frac{11}{2}$

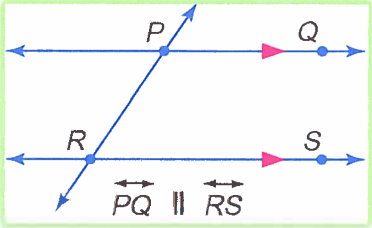
3.3 • Angles Formed by Parallel Lines

OBJECTIVES:

- Identify the pairs of angles formed by a transversal cutting parallel lines
- Use postulates and theorems involving parallel lines to find angle measures

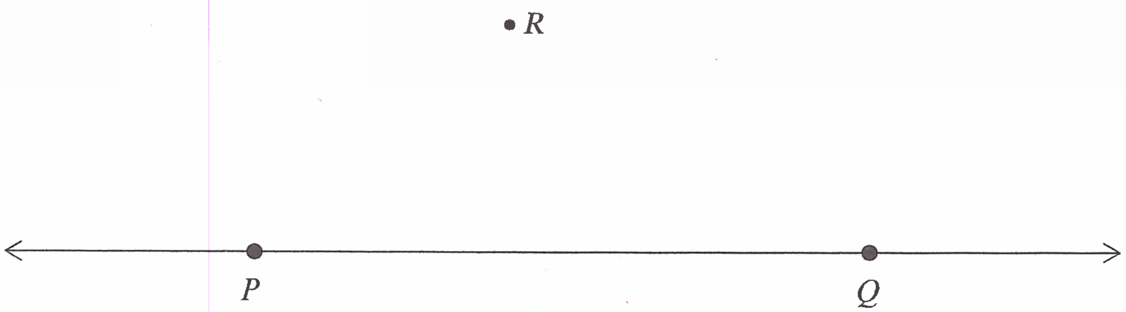
❖ Relationships Between Lines & Planes

- ➢ Coplanar lines that do not intersect are called parallel lines.
  - ▪ $\overleftrightarrow{PQ} \parallel \overleftrightarrow{RS}$
  - ▪ Arrows are used in diagram to identify parallel lines.



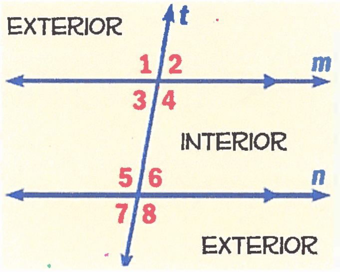
❖ Constructing Parallel Lines

- ➢ Constructing a line parallel to a given line that passes through a given point
  - ▪ Construction Link



❖ Angles Formed When Parallel Lines Are Cut by a Transversal

There are interior angles and exterior angles and corresponding angles.



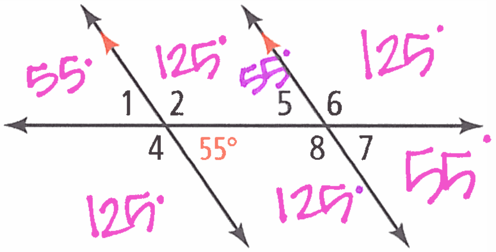
Corresponding Angles
One is on the inside; the other is on the outside; both are on the same side and they are not adjacent.

❖ Angle Pairs Formed by Transversals

- ➢ *Identify from the diagram (above)*

CORRESPONDING ANGLES	ALTERNATE EXTERIOR ANGLES	ALTERNATE INTERIOR ANGLES	SAME-SIDE INTERIOR ANGLES	SAME-SIDE EXTERIOR ANGLES
∠1 & ∠5 ∠2 & ∠6 ∠3 & ∠7 ∠4 & ∠8	∠1 & ∠8 ∠2 & ∠7	∠3 & ∠6 ∠4 & ∠5	∠3 & ∠5 ∠4 & ∠6	∠1 & ∠7 ∠2 & ∠8

EXAMPLE 1: FINDING ANGLE MEASURES



Above, you constructed **CORRESPONDING ANGLES** which you know to be congruent since you copied ∠RPQ. In the diagram (at right), the corresponding angles would be represented by the given 55° angle and ∠7.

Use your knowledge of vertical angles and linear pairs, to find the remaining angle measurements.

m∠1 = 55°
vert. ∠s are ≅
m∠5 = 55°

m∠2 = m∠4 = 125°
linear pair: ∠1 & ∠2
supplementary

m∠6 = m∠8 = 125

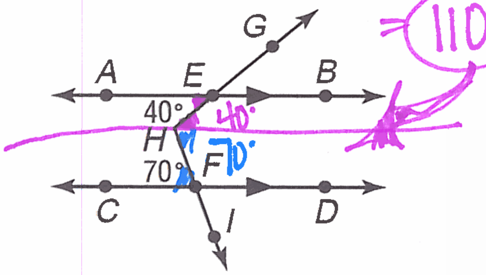
- Relationships Between Parallel Lines & Angles
  - *If two parallel lines are cut by a transversal, then any pair of the angles formed are either congruent or supplementary.*

CONGRUENT ANGLE PAIRS	SUPPLEMENTARY ANGLE PAIRS
corresponding alternate interior & exterior	same-side interior & exterior

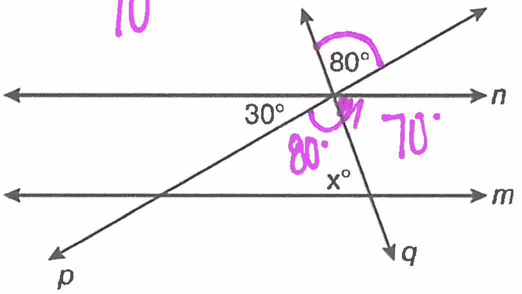
EXAMPLES: FINDING ANGLE MEASURES

2. Find m∠GHI.

40+70
= 110°

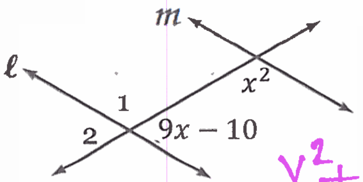


3. Given: m ∥ n
Find x. = 70°



4. Given: ℓ ∥ m
Find x (that makes sense).

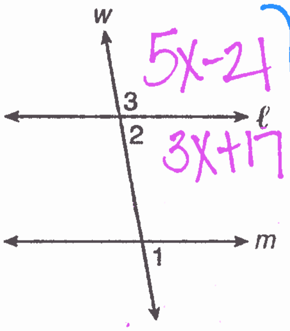
Same-side int.
supp.



x=10

$x^2+9x-10=180°$
$x^2+9x-190=0$
$(x+19)(x-10)=0$
x = −19 OR x = 10

5. Given: ℓ ∥ m, m∠2 = 3x + 17 & m∠3 = 5x − 21
Find m∠1.



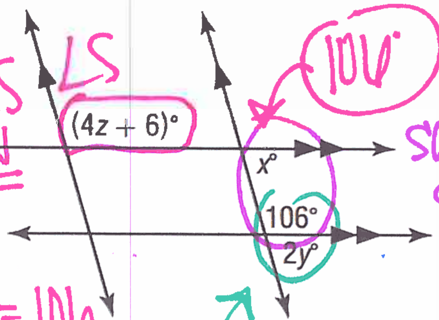
∠2 & ∠3 form a linear pair
↓
Supp.

8x − 4 = 180
8x = 184
x = 23

∠1 ≅ ∠2
corrs. ∠s
m∠1 = 86°

Find the value of the variable(s) in each figure.

6.



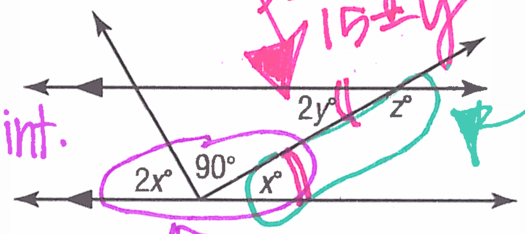
CORRS ∠s are ≅
↓
$4z+6=106$
$4z=100$
$z=25$

106°

same-side int.
Supp.
↓
$x=74°$

linear pair
↓
Supp
↓
$2y+106=180$
$y=37$

7.



Alt. int. ∠s are ≅

$x=2y$
↓
$30=2y$
$15=y$

Same-side supp.
$x+z=180$
↓
$30+z=180$
$z=150$

supp.
↓
$3x+90=180$
$3x=90$
$x=30$

3.4 • Proofs With Parallel Lines

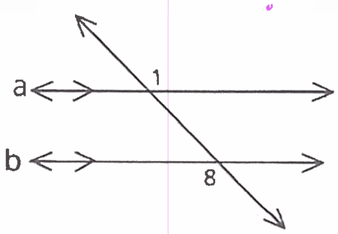
OBJECTIVES:

- Use postulates and theorems involving parallel lines to find angle measures
- Prove conjectures involving parallel lines
- Structure statements and reasons to form a logical argument

❖ Postulates & Theorems Involving Parallel Lines

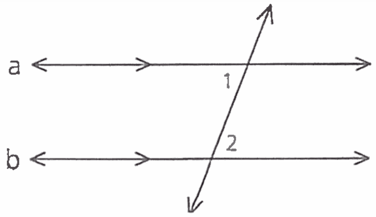
➢ If two parallel lines are cut by a transversal, then any pair of the angles formed are either congruent or supplementary.

Alternate Exterior Angles



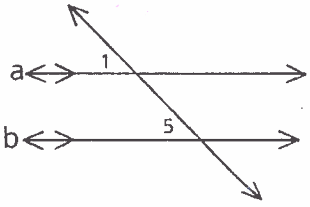
Congruent: ∠1 ≅ ∠8

Alternate Interior Angles



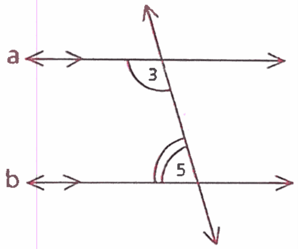
Congruent: ∠1 ≅ ∠2

Corresponding Angles



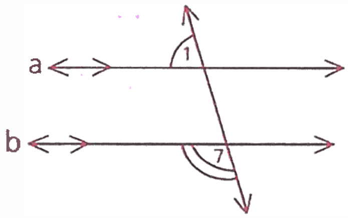
Congruent: ∠1 ≅ ∠5

Same-Side Interior Angles



Supplementary: $m\angle 3 + m\angle 5 = 180°$

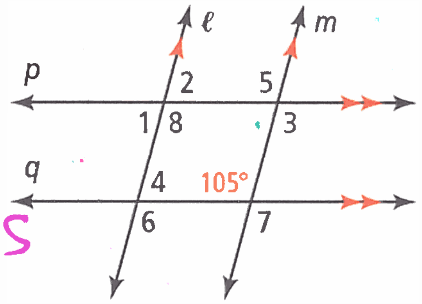
Same-Side Exterior Angles



Supplementary: $m\angle 1 + m\angle 7 = 180°$

EXAMPLES: FIND ANGLE MEASURES

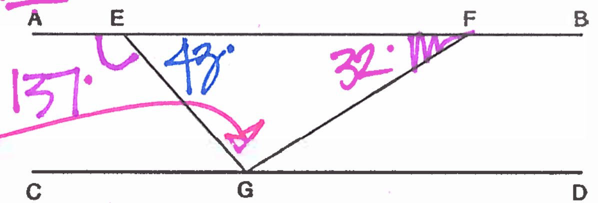
1. There are two sets of parallel lines. Each parallel line also acts as a transversal. Find the measure of ∠1. Justify your answer.



m∠4 = 75° same-side int. ∠s are supp.

m∠1 = 75 b/c ∠1 ≅ ∠4, alt. int. ∠s are ≅

2. In the diagram below, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$. If m∠EFG = 32° and m∠AEG = 137°, what is m∠EGF?



Δ sum = 180

therefore m∠EGF = 105°

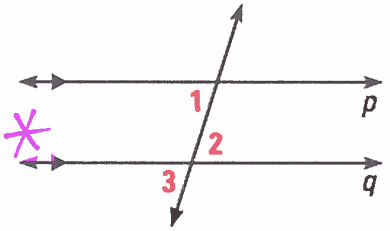
PROVE THE ALTERNATE INTERIOR ANGLES THEOREM:

If two parallel lines are cut by a transversal, each pair of alternate interior angles are congruent.

Given: p ∥ q

Prove: ∠1 ≅ ∠2

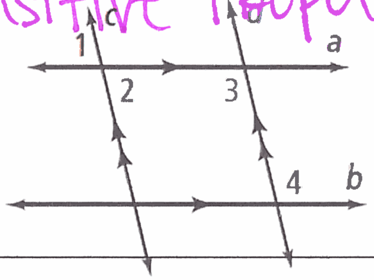
* can't use alt. int. ∠s *



Statements	Reasons
1. p ∥ q	1. Given
2. ∠1 ≅ ∠3	2. Corrs. ∠s are ≅
3. ∠2 & ∠3 are vert. ∠s	3. Assumed from diagram
4. ∠2 ≅ ∠3	4. Vert. ∠s are ≅
5. ∠1 ≅ ∠2	5. Transitive Property

3. Given: a ∥ b & c ∥ d

Prove: ∠1 is supp. to ∠4



Statements	Reasons
1. a ∥ b & c ∥ d	1. Given
2. ∠3 ≅ ∠4	2. Alt. Int. ∠s Theorem
3. ∠2 is supp to ∠3	3. Same-Side Int. ∠s theorem
4. ∠2 is supp to ∠4	4. Substitution

3.5 • Proving Lines Parallel

OBJECTIVES:

- Use postulates and theorems to prove that two lines are parallel
- Structure statements and reasons to form a logical argument

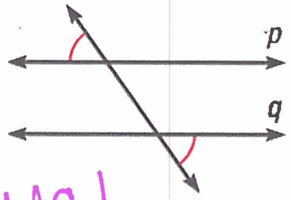
❖ Parallel Line Converse Theorems

- Converse of the Corresponding Angles Postulate
  - If two lines and a transversal form corresponding angles that are congruent, then the two lines are parallel.
- Converse of the Alternate Interior Angles Theorem
  - If two lines and a transversal form alternate interior angles that are congruent, then the two lines are parallel.
- Converse of the Alternate Exterior Angles Theorem
  - If two lines and a transversal form alternate exterior angles that are congruent, then the two lines are parallel.
- Converse of the Same-Side Interior Angles Theorem
  - If two lines and a transversal form same-side interior angles that are supplementary, then the two lines are parallel.
- Converse of the Same-Side Exterior Angles Theorem
  - If two lines and a transversal form same-side exterior angles that are supplementary, then the two lines are parallel.

EXAMPLES: RECOGNIZE PARALLEL LINES

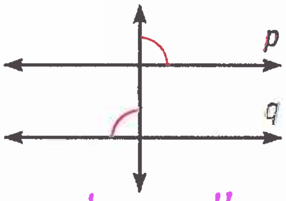
Can you prove that lines *p* and *q* are parallel? If so, identify the theorem or postulate used.

1.



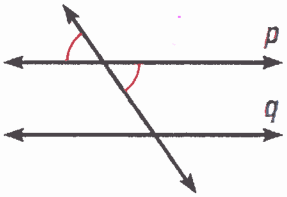
yes!
Conv. of Alt. Ext. ∠s theorem

2.



NOT //

3.



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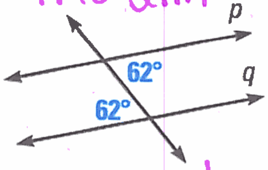
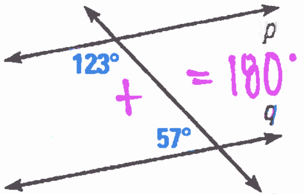
4.



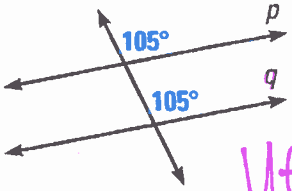
yes!
Conv. of Alt. Int. ∠s Theorem

5.



yes!
Conv. of Same-Side Int. ∠s Theorem

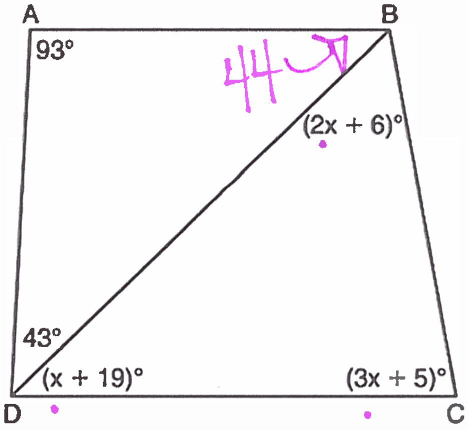
6.



yes!
Converse of ~~San~~ Corrs. ∠s Theorem

Δ sum $= 180°$

7. The diagram shows quadrilateral $ABCD$ with diagonal $\overline{BD}$. Is $\overline{AB} \parallel \overline{DC}$? Show all work and explain your reasoning.



$\Delta BCD: 6x + 30 = 180$

$6x = 150$

$x = 25$

If $x = 25$, then $m\angle BDC = 44$

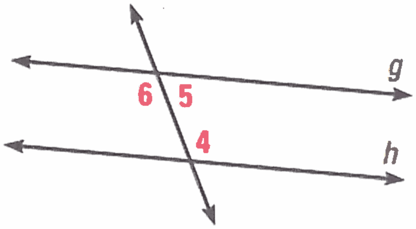
$\angle BDC \cong \angle ABD$, therefore $\overline{AB} \parallel \overline{DC}$ by Conv. Alt. Int. ∠s Theorem

8. PROVE THE CONVERSE OF THE SAME-SIDE INTERIOR ANGLES THEOREM:

If two lines and a transversal form same-side interior angles that are supplementary, then the two lines are parallel.

Given: $\angle 4$ is supp. to $\angle 5$

Prove: $g \parallel h$

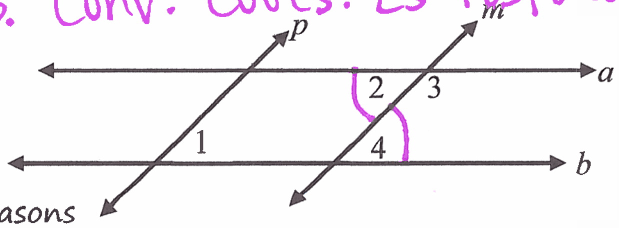


Statements	Reasons
1. $\angle 4$ is supp to $\angle 5$	1. Given
2. $\angle 5$ & $\angle 6$ form a linear pair	2. Assumed from diagram
3. $\angle 5$ is supp to $\angle 6$	3. Linear Pair Postulate
4. $\angle 4 \cong \angle 6$	4. ≅ Supps. Theorem
5. $g \parallel h$	5. Conv. Corrs. ∠s Postulate

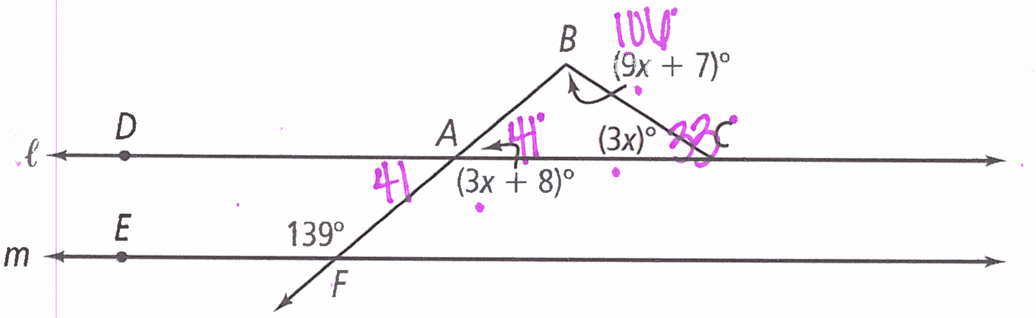
9. Given: $a \parallel b$

$\angle 1 \cong \angle 2$

Prove: $m \parallel p$



Statements	Reasons
1. $a \parallel b$	1. & 2. } Given
2. $\angle 1 \cong \angle 2$	
3. $\angle 2 \cong \angle 4$	3. Alt. Int. ∠s Theorem
4. $\angle 1 \cong \angle 4$	4. Transitive Prop
5. $m \parallel p$	5. Conv. Of Corrs. ∠s Post



10. Given that the sum of the measures of the angles in a triangle is 180°, solve for x.

$15X+15=180$

$15X=165$

$X=11$

11. Find the measure of each angle in △*ABC*.

m∠BAC = 41°   m∠B = 104°   m∠C = 33°

12. Explain why ℓ ∥ *m*.

∠BAC ≅ ∠DAF vert. ∠s are ≅

∠DAF is supp. to ∠EFA: 139+41=180°

therefore ℓ∥m b/c of the Converse of the Same-Side Int. ∠s Theorem