Chapter 3 Notes Packet Name: ______________________________

Parallel & Perpendicular Lines

3.1 • Lines in the Coordinate Plane

OBJECTIVES:

- Identify the slope and x- and y-intercepts of linear equations
- Graph and write linear equations

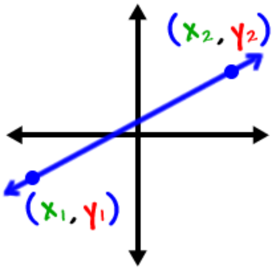
❖ Slope

➢ The slope, m, of a line is the ratio of the vertical change (rise) to the horizontal change (run).

$$m = \frac{\text{rise}}{\text{run}}$$

If you're given two points (x_1, y_1) and (x_2, y_2)

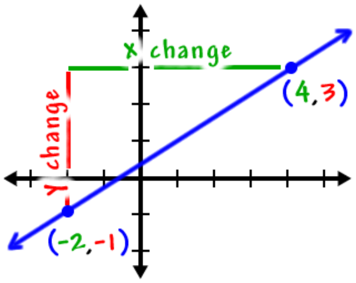
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



EXAMPLE: $(-2, -1)$ and $(4, 3)$

Look at $\frac{\text{rise}}{\text{run}}$ as $\frac{\text{the change in the y's}}{\text{the change in the x's}}$

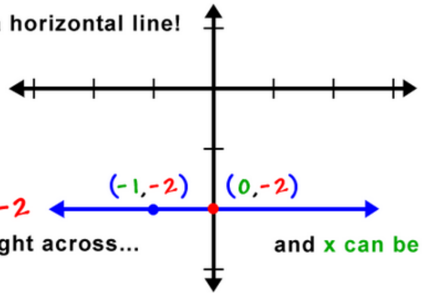
$$= \frac{3-(-1)}{4-(-2)} = \frac{4}{6} = \frac{2}{3}$$



❖ Horizontal & Vertical Lines

Horizontal Lines: $y = \#$

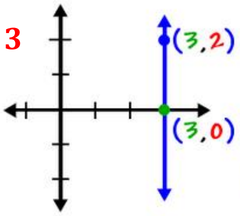
It's a horizontal line!



straight across... and x can be anything!

Vertical Lines: $x = \#$

$x = 3$



It's a vertical line!

What's the slope?

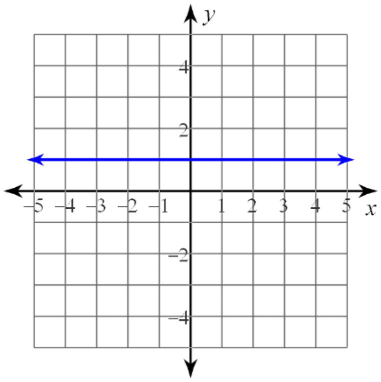
$$m = \frac{2-0}{3-3} = \frac{2}{0}$$

The slope of a vertical line is undefined.

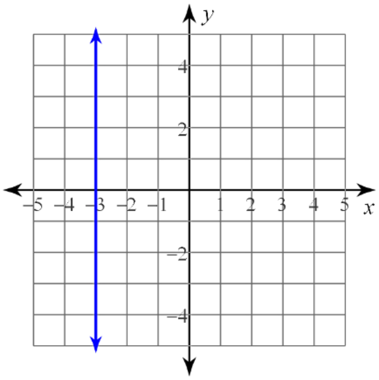
This is the official word.

EXAMPLES: Find the slope and equation for each line shown.

1.

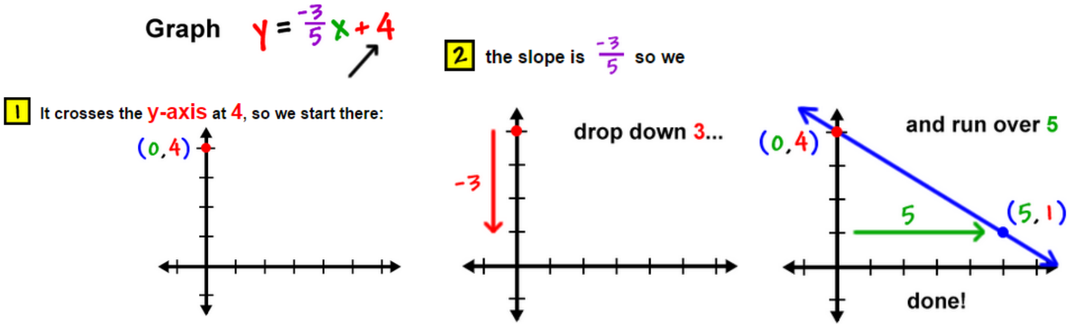


2.



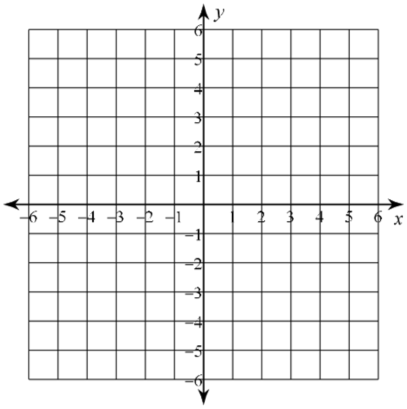
- Slope-Intercept Form
  - $\boldsymbol{y = mx + b}$
    - The coefficient of x, which is m, is the slope of the line.
    - The constant term, b, is the y-coordinate of the y-intercept.
- Graphing a Function in Slope-Intercept Form
  - Plot the y-intercept: $(0, b)$
  - From there, use the slope, m, to determine a second point
    - If the slope is positive, go up the number of units in the numerator
    - If the slope is negative, go down the number of units in the numerator
    - Always go right the number of units in the denominator

If you're going to go off the grid, then do the complete opposite.

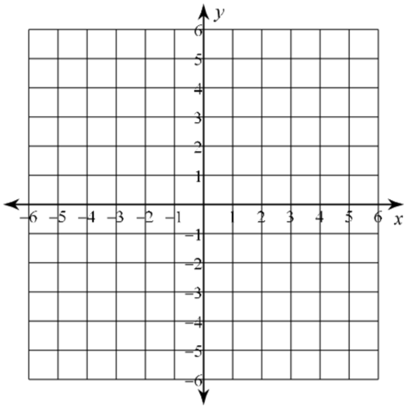


EXAMPLES: GRAPHING A FUNCTION IN SLOPE-INTERCEPT FORM

3. $y = -\frac{2}{5}x - 2$



4. $y = \frac{9}{2}x + 4$



take note

Key Concept Forms of Linear Equations

Definition

The **slope-intercept form** of an equation of a nonvertical line is $y = mx + b$, where m is the slope and b is the y-intercept.

The **point-slope form** of an equation of a nonvertical line is $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) is a point on the line.

Symbols

$y = mx + b$

↑ ↑

slope y-intercept

$y - y_1 = m(x - x_1)$

↑ ↑ ↑

y-coordinate slope x-coordinate

- ❖ Writing Linear Functions

$$y - y_1 = m(x - x_1)$$

$m = -2$ $\quad (4, -3)$ $\quad x_1 \nearrow$ $\nwarrow y_1$

$$y - (-3) = -2(x - 4)$$

$$y + 3 = -2x + 8 \leftarrow \text{be careful here!}$$

$$\underline{-3 \qquad\qquad -3}$$

$$y = -2x + 5 \quad \text{done}$$

You could substitute the point & the slope into the slope-intercept equation of a line and then solve for b:

$$y = mx + b$$
$$-3 = -2(4) + b$$
$$-3 = -8 + b$$
$$5 = b$$

Then only plug in the slope & y-intercept (b):

$$y = -2x + 5$$

EXAMPLES: WRITING LINEAR FUNCTIONS

Write the slope-intercept form of the line described.

5. The line that passes through (–1, 4) with slope –3

6. The line that passes through the points (3, –5) & (–4, –1)

- ❖ Standard Form
  - ➢ A linear function can be written in standard form as $Ax + By = C$ where A, B, and C are real numbers and A and B cannot both be zero.

- ❖ Graphing a Function in Standard Form
  - ➢ Find and plot the vertical and horizontal intercepts of the function.

Graph $x + 2y = 4$

x	y
0	
	0

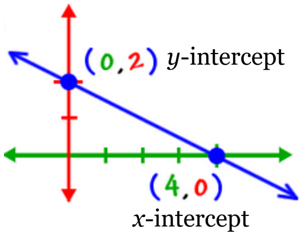
To find the x-intercept: Let y = 0 & solve for x.

$$x \ ☐ = 4$$
$$x = 4$$

To find the y-intercept: Let x = 0 & solve for y.

$$☐ + 2y = 4$$
$$y = 2$$

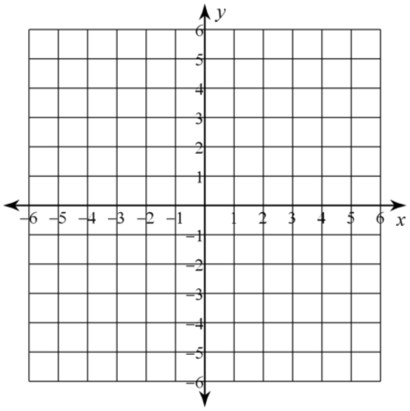
This is also ***b*** *in slope-intercept form.*



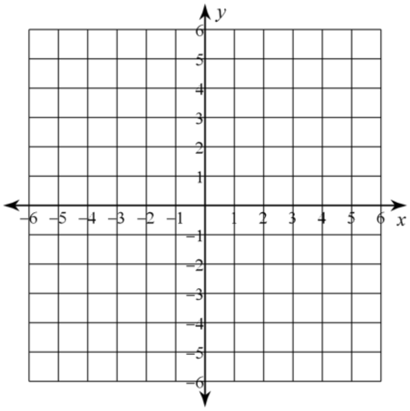
EXAMPLES: GRAPHING A LINEAR FUNCTION IN STANDARD FORM

Find the vertical and horizontal intercepts of the function. Then graph the function.

7. $10x + 3y = 15$



8. $5x - 3y = -6$



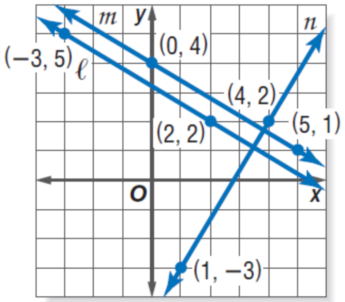
3.2 • Parallel & Perpendicular Lines

OBJECTIVES:

- Prove the slope criteria for parallel and perpendicular lines and use them to sole geometric problems
- Write equations to describe lines parallel or perpendicular to a given lines

PARALLEL AND PERPENDICULAR LINES

Examine the graphs of lines ℓ, m, and n. Lines ℓ and m are parallel, and n is perpendicular to ℓ and m. Let's investigate the slopes of these lines.



slope of ℓ

$$m = \frac{2 - 5}{2 - (-3)} = -\frac{3}{5}$$

slope of m

$$m = \frac{1 - 4}{5 - 0} = -\frac{3}{5}$$

slope of n

$$m = \frac{2 - (-3)}{4 - 1} = \frac{5}{3}$$

- ❖ Slopes of Parallel Lines
  - ➢ If two nonvertical lines are parallel, then their slopes are _______________.
  - ➢ Any two vertical or horizontal lines are parallel.

- ❖ Slopes of Perpendicular Lines
  - ➢ The slopes of perpendicular lines are _______________________________.
  - ➢ Any horizontal line and vertical line are perpendicular.

EXAMPLES: DETERMINE LINEAR RELATIONSHIPS

1. Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel, perpendicular, or neither:
   $A(-8,-7), B(4,-4), C(-2,-5), D(1,7)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

EXAMPLES: DETERMINE LINEAR RELATIONSHIPS

$$y = \underset{\text{slope}}{m}x + \underset{\text{y-intercept}}{b}$$

Find the slope of each line, then determine if the lines are parallel, perpendicular, or neither.

2. $2x + 3y = 6$
   $6x - 4y = 24$

3. $2x + 5y = -1$
   $10y = -4x - 20$

EXAMPLES: WRITE LINEAR FUNCTIONS

$$y - y_1 = m(x - x_1)$$

Write the slope-intercept form of the line described.

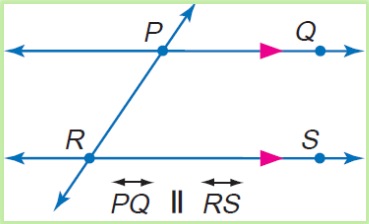
4. The line that passes through $(-8, 6)$ and is parallel to the line $4x + y = -5$

5. The line that passes through $(-3, -4)$ and is perpendicular to the line $y = 2x - 3$

3.3 • Angles Formed by Parallel Lines

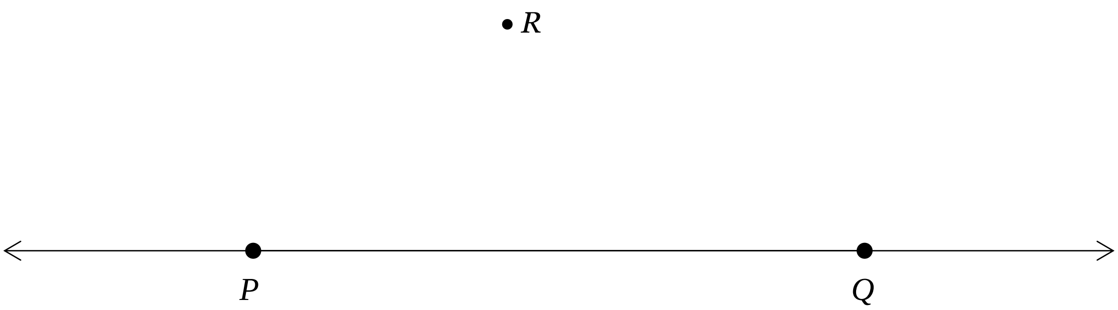
OBJECTIVES:

- Identify the pairs of angles formed by a transversal cutting parallel lines
- Use postulates and theorems involving parallel lines to find angle measures



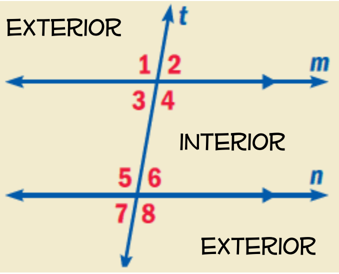
- Relationships Between Lines & Planes
  - Coplanar lines that do not intersect are called parallel lines.
    - $\overleftrightarrow{PQ} \parallel \overleftrightarrow{RS}$
    - Arrows are used in diagram to identify parallel lines.

- Constructing Parallel Lines
  - Constructing a line parallel to a given line that passes through a given point
    - [Construction Link]



- Angles Formed When Parallel Lines Are Cut by a Transversal

There are interior angles and exterior angles and corresponding angles.



<u>Corresponding Angles</u>
One is on the inside; the other is on the outside; both are on the same side and they are not adjacent.

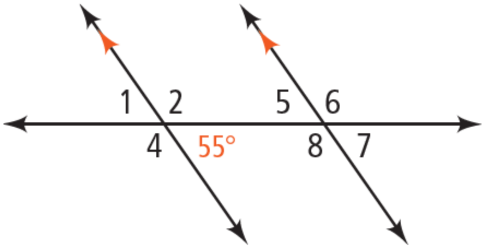
- Angle Pairs Formed by Transversals
  - *Identify from the diagram (above)*

Corresponding Angles	Alternate Exterior Angles	Alternate Interior Angles	Same-Side Interior Angles	Same-Side Exterior Angles

EXAMPLE 1: FINDING ANGLE MEASURES

Above, you constructed **CORRESPONDING ANGLES** which you know to be congruent since you copied $\angle RPQ$. In the diagram (at right), the corresponding angles would be represented by the given 55° angle and $\angle 7$.

Use your knowledge of vertical angles and linear pairs, to find the remaining angle measurements.



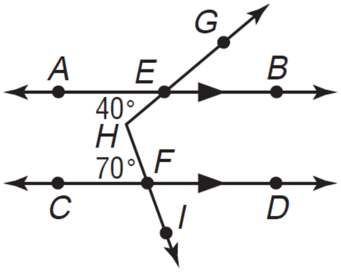
❖ Relationships Between Parallel Lines & Angles

- ➢ *If two parallel lines are cut by a transversal, then any pair of the angles formed are either congruent or supplementary.*

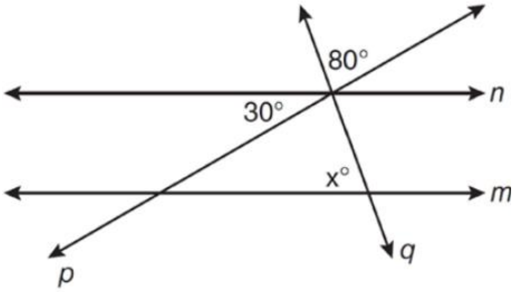
CONGRUENT ANGLE PAIRS	SUPPLEMENTARY ANGLE PAIRS

EXAMPLES: FINDING ANGLE MEASURES

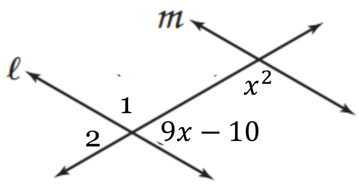
2. Find $m\angle GHI$.



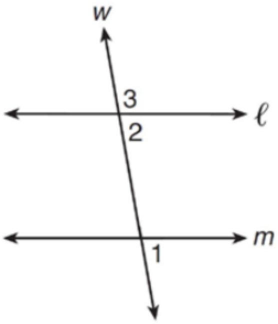
3. Given: $m \parallel n$
Find x.



4. Given: $\ell \parallel m$
Find x (that makes sense).

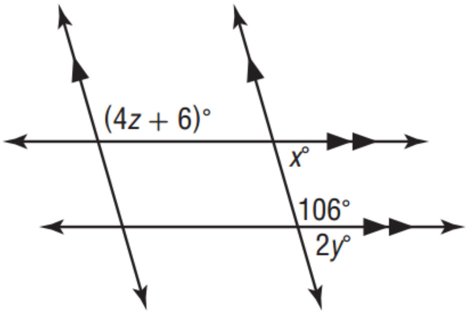


5. Given: $\ell \parallel m$, $m\angle 2 = 3x + 17$ & $m\angle 3 = 5x - 21$
Find $m\angle 1$.



Find the value of the variable(s) in each figure.

6.



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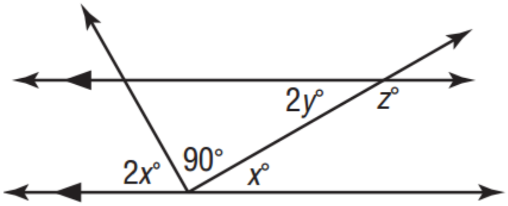


3.4 • Proofs with Parallel Lines

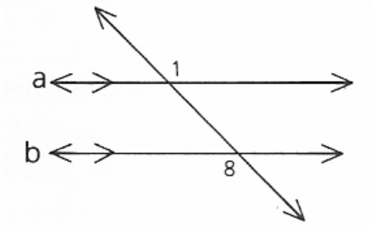
OBJECTIVES:

- Use postulates and theorems involving parallel lines to find angle measures
- Prove conjectures involving parallel lines
- Structure statements and reasons to form a logical argument

❖ Postulates & Theorems Involving Parallel Lines

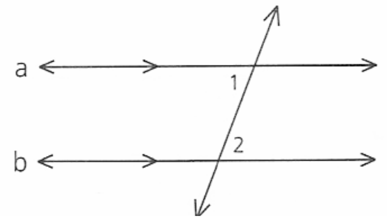
- ➢ If two parallel lines are cut by a transversal, then any pair of the angles formed are either congruent or supplementary.

ALTERNATE EXTERIOR ANGLES



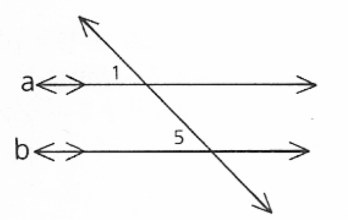
Congruent: ∠1 ≅ ∠8

ALTERNATE INTERIOR ANGLES



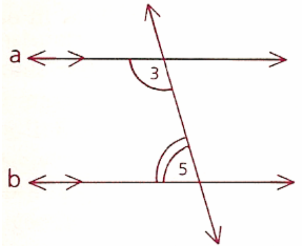
Congruent: ∠1 ≅ ∠2

CORRESPONDING ANGLES



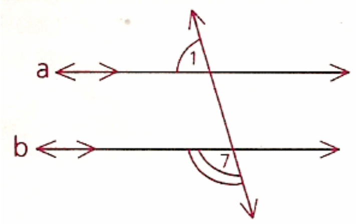
Congruent: ∠1 ≅ ∠5

SAME-SIDE INTERIOR ANGLES



Supplementary: $m\angle 3 + m\angle 5 = 180°$

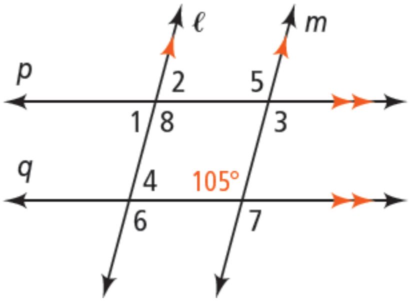
SAME-SIDE EXTERIOR ANGLES



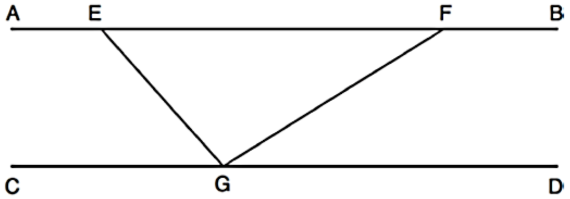
Supplementary: $m\angle 1 + m\angle 7 = 180°$

EXAMPLES: FIND ANGLE MEASURES

1. There are two sets of parallel lines. Each parallel line also acts as a transversal. Find the measure of $\angle 1$. Justify your answer.



2. In the diagram below, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$. If $m\angle EFG = 32°$ and $m\angle AEG = 137°$, what is $m\angle EGF$?

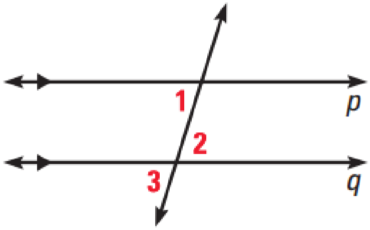


PROVE THE ALTERNATE INTERIOR ANGLES THEOREM:

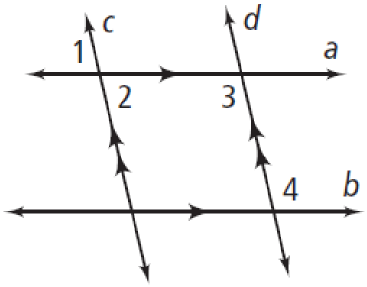
If two parallel lines are cut by a transversal, each pair of alternate interior angles are congruent.

Given: $p \parallel q$

Prove: $\angle 1 \cong \angle 2$



Statements	Reasons



3. Given: $a \parallel b$ & $c \parallel d$

Prove: $\angle 1$ is supp. to $\angle 4$

Statements	Reasons

3.5 • Proving Lines Parallel

OBJECTIVES:

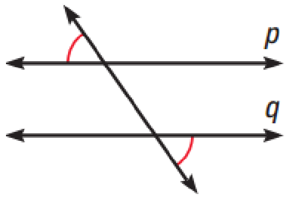
- Use postulates and theorems to prove that two lines are parallel
- Structure statements and reasons to form a logical argument

- Parallel Line Converse Theorems
  - **Converse of the Corresponding Angles Postulate**
    - If two lines and a transversal form corresponding angles that are congruent, then the two lines are parallel.
  - **Converse of the Alternate Interior Angles Theorem**
    - If two lines and a transversal form alternate interior angles that are congruent, then the two lines are parallel.
  - **Converse of the Alternate Exterior Angles Theorem**
    - If two lines and a transversal form alternate exterior angles that are congruent, then the two lines are parallel.
  - **Converse of the Same-Side Interior Angles Theorem**
    - If two lines and a transversal form same-side interior angles that are supplementary, then the two lines are parallel.
  - **Converse of the Same-Side Exterior Angles Theorem**
    - If two lines and a transversal form same-side exterior angles that are supplementary, then the two lines are parallel.

EXAMPLES: RECOGNIZE PARALLEL LINES

Can you prove that lines *p* and *q* are parallel? If so, identify the theorem or postulate used.

1.



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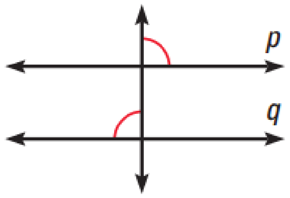

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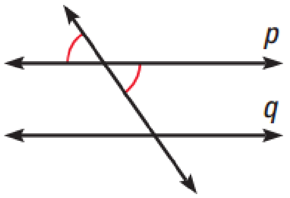

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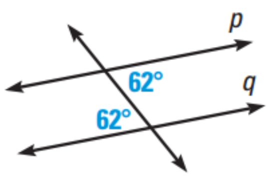

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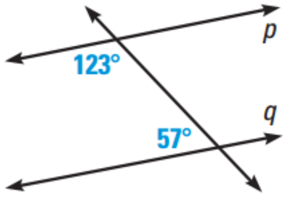

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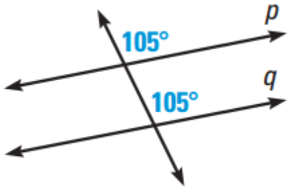
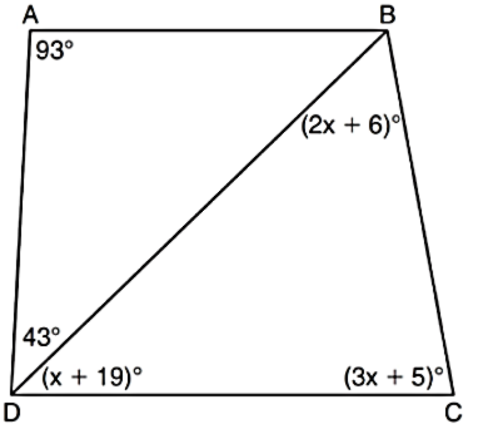


7. The diagram shows quadrilateral $ABCD$ with diagonal $\overline{BD}$. Is $\overline{AB} \parallel \overline{DC}$? Show all work and explain your reasoning.

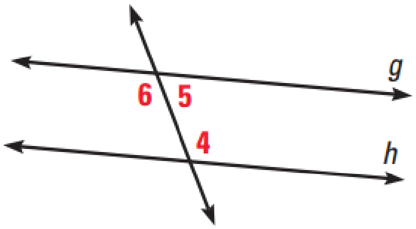


8. PROVE THE CONVERSE OF THE SAME-SIDE INTERIOR ANGLES THEOREM:

If two lines and a transversal form same-side interior angles that are supplementary, then the two lines are parallel.

Given: $\angle 4$ is supp. to $\angle 5$

Prove: $g \parallel h$

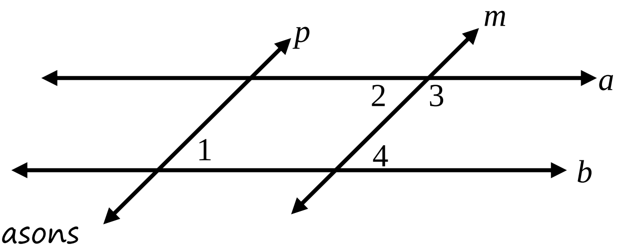


Statements	Reasons

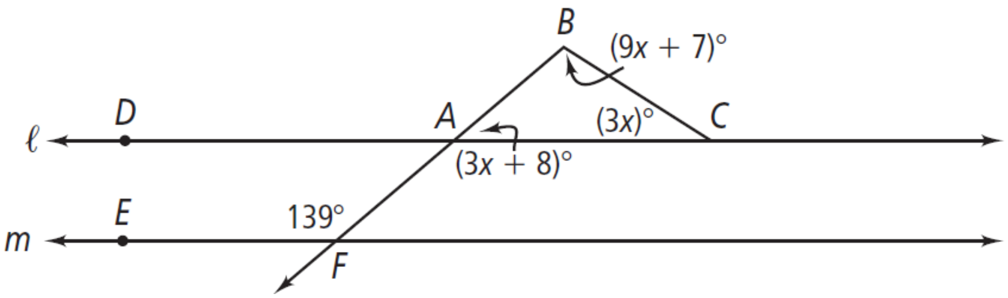
9. Given: $a \parallel b$

$\angle 1 \cong \angle 2$

Prove: $m \parallel p$



Statements	Reasons



10. Given that the sum of the measures of the angles in a triangle is 180°, solve for x.

11. Find the measure of each angle in $\triangle ABC$.

$m\angle BAC$ = _______ $m\angle B$ = _______ $m\angle C$ = _______

12. Explain why $\ell \parallel m$.