Name: ___________________________________

SIMILARITY

4.1 – Ratios & Proportions

OBJECTIVES:

- WRITE RATIOS; SOLVE APPLICATIONS INVOLVING RATIOS
- USE THE CROSS PRODUCT PROPERTY TO SOLVE PROPORTIONS

❖ Ratios

➢ If a and b are two quantities that are measured in the same units, then the ratio of a to b is $\frac{a}{b}$

▪ Ratios are usually expressed in simplest form.

EXAMPLE 1: The perimeter of a rectangle is 60 centimeters. The ratio of the width to the length is 3 : 2.

a. Find the length and the width.

b. Find the area of the rectangle.

❖ Proportions

➢ An equation that equates two ratios is a proportion.

▪ Cross Product Property – The product of the extremes equals the product of the means:

If $\frac{a}{b} = \frac{c}{d}$ then $ad = bc$

If $\frac{x+5}{20} = \frac{3}{10}$ then $(x+5)\cdot 10 = 20$

EXAMPLES: Use the Cross Product Property to solve each proportion. *If necessary, round to the nearest hundredth.*

2. $\frac{3x-8}{6} = \frac{2x}{10}$

3. $\frac{x+3}{3} = \frac{8}{x-2}$

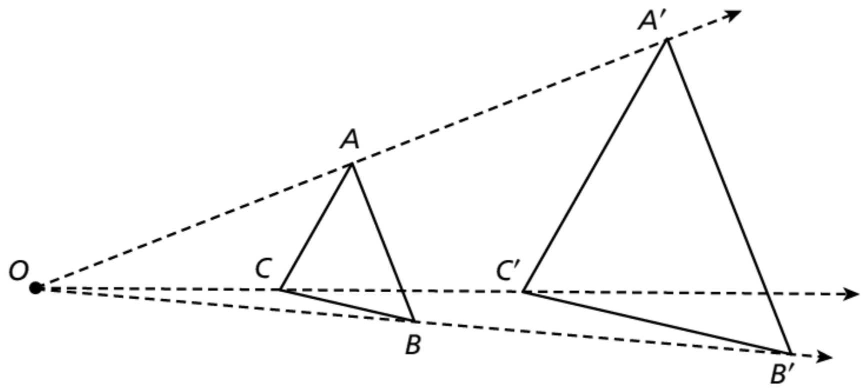
4.2 – Dilations

OBJECTIVES:

- Determine the scale factor of a dilation
- Given the pre-image, scale factor, and center of dilation, graph dilations
- Given the vertex coordinates, scale factor, and center of dilation, determine the coordinates of dilated figures
- Prove that figures are similar using transformations

❖ Dilations

➢ In the figure below $\triangle A'B'C'$ is the image of $\triangle ABC$ after a dilation with center O and scale factor __?__.



- Use a ruler to find the following lengths in millimeters:

  OA = ________________ OA' = ________________

  OB = ________________ OB' = ________________

  OC = ________________ OC' = ________________

  AB = ________________ $A'B'$ = ________________

  BC = ________________ $B'C'$ = ________________

  AC = ________________ $A'C'$ = ________________

- What do you notice about the lengths of the corresponding sides of the two triangles?

- Use a protractor and find the measures of the corresponding angles.

  $m\angle A$ = _______ $m\angle B$ = _______ $m\angle C$ = _______

  $m\angle A'$ = _______ $m\angle B'$ = _______ $m\angle C'$ = _______

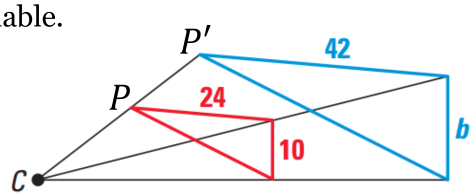
- What do you notice about the measures of the corresponding angles?

➢ The scale factor, k, for a dilation is the ratio of a linear measurement of the image to a corresponding measurement of the preimage.

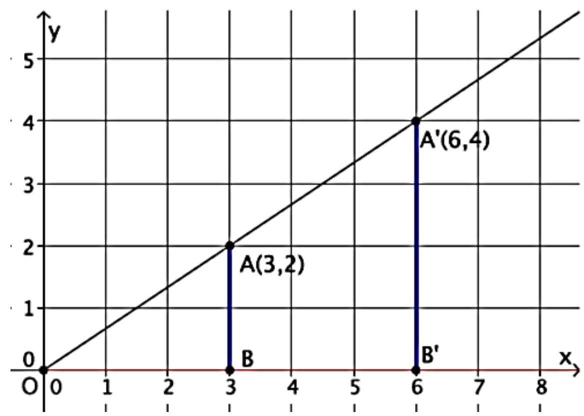
- What is the scale factor of the dilation above?

- Similarity
  - In a dilation, the image and pre-image are **SIMILAR** because they have the same shape – but are different sizes.
    - Corresponding side lengths are **PROPORTIONAL**
      - Similarity ratio – The ratio of any side length in the first figure to the corresponding side length in the second figure.
    - Corresponding angles are **CONGRUENT**

EXAMPLE 1: Find the scale factor of the dilation and then set up and solve a proportion to find the value of the variable.



- Dilations in the Coordinate Plane with center $(0, 0)$
  - The graph below represents a dilation from center $(0, 0)$ by scale factor $k = 2$.



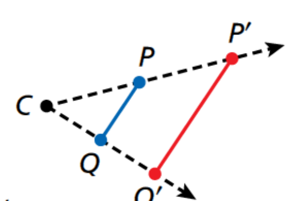
- Compare the pre-image, $\overline{AB}$, to the image $\overline{A'B'}$. What effect does the dilation have on the coordinates of dilated points?

- Complete the conjecture: The image of $(\boldsymbol{a}, \boldsymbol{b})$ after a dilation with center at the origin and scale factor $\boldsymbol{k}$ has the coordinates __________________.

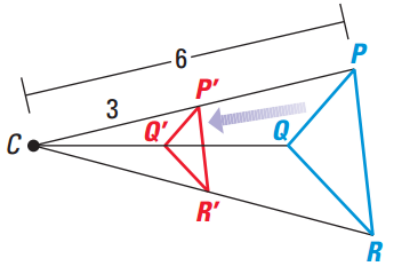
Dilations

A dilation, or *similarity transformation*, is a transformation in which the lines connecting every point *P* with its image *P′* all intersect at a point *C*, called the **center of dilation**. $\frac{CP'}{CP}$ is the same for every point *P*.

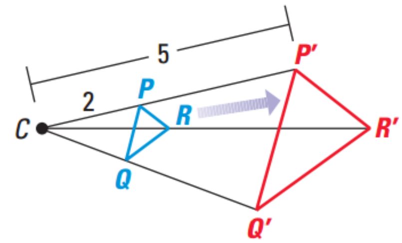
The scale factor *k* of a dilation is the ratio of a linear measurement of the image to a corresponding measurement of the preimage. In the figure, $k = \frac{P'Q'}{PQ}$.



- A dilation with $k > 1$ is an enlargement. A dilation with $0 < k < 1$ is a reduction.



Reduction: $k = \frac{CP'}{CP} = \frac{3}{6} = \frac{1}{2}$



Enlargement: $k = \frac{CP'}{CP} = \frac{5}{2}$

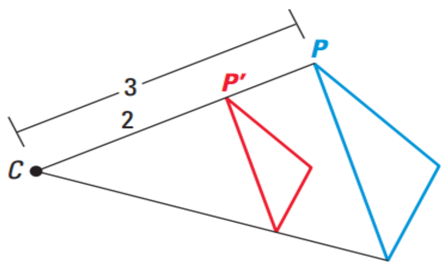
SCALE FACTOR

$$k = \frac{\text{image}'}{\text{preimage}}$$

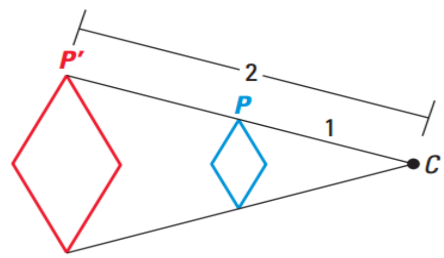
EXAMPLES –

2. Identify the dilation as an enlargement or a reduction and find its scale factor.

a.



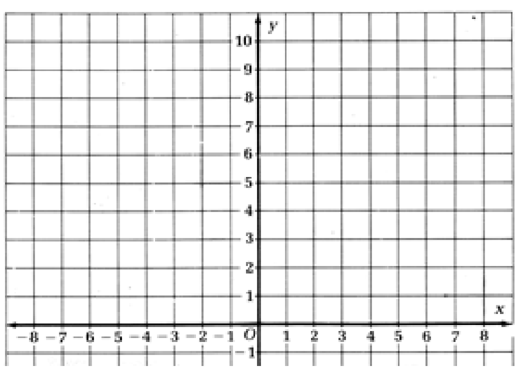
b.



Apply the dilation to the polygon with the given vertices. Name the coordinates of the image points. Describe the dilation: center, scale factor, enlargement or reduction.

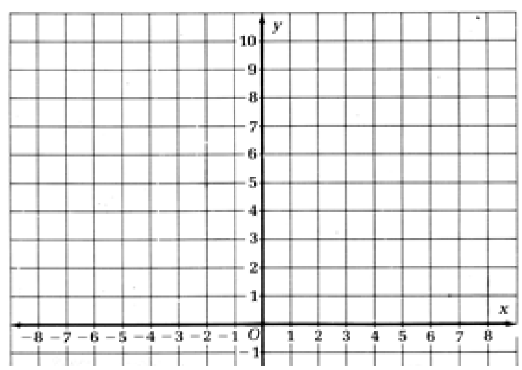
3. $D: (x, y) \rightarrow (2x, 2y)$

$A(2, 1), B(2, 3), C(5, 1)$



4. $D: (x, y) \rightarrow \left(\frac{2}{3}x, \frac{2}{3}y\right)$

$P(-6, 3), Q(-3, 9), R(3, 6)$

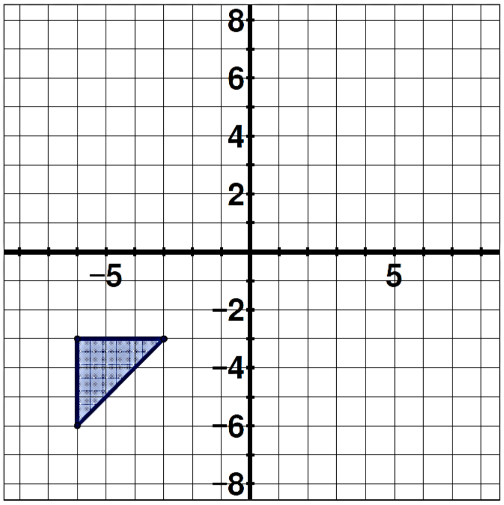


❖ Constructing Dilations – Center NOT at the Origin

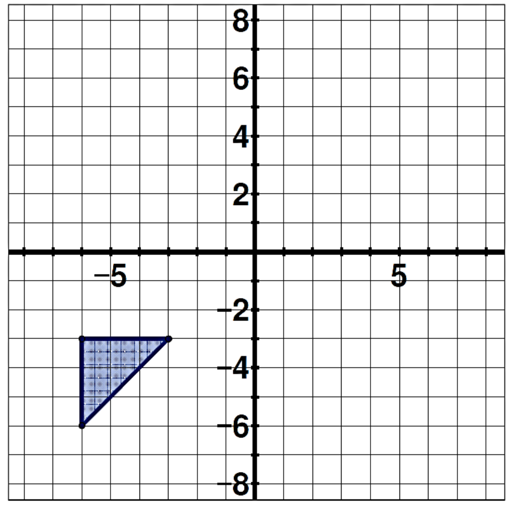
1. Plot the center point on the coordinate plane.
2. Determine the coordinates of one pre-image point.
   a. What is the vertical distance from the center of dilation to the pre-image point?
   b. What is the horizontal distance from the center of dilation to the pre-image point?
3. Multiply the vertical and horizontal distances by the scale factor to determine the new vertical and horizontal distances.
   a. Using these distances, plot your new point starting from the center of dilation.
4. Repeat for all vertices.

EXAMPLES –

5. Dilate by a scale factor of $^1/_3$ with center of dilation at $(3, 0)$.

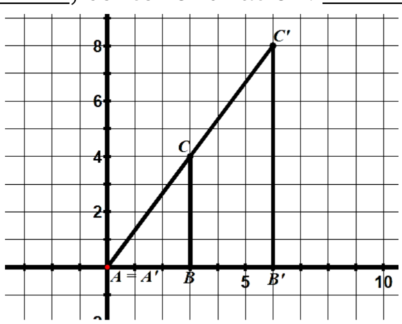


6. Dilate by a scale factor of $^4/_3$ with center of dilation at $(0, -6)$.

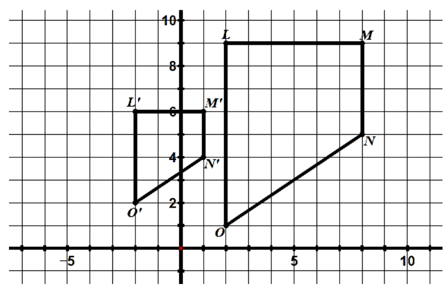


In the following problems, one figure has been dilated to obtain the new figure. Determine the scale factor AND the center of dilation.

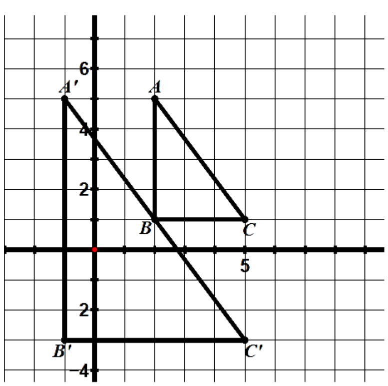
7. $k =$ ______; center of dilation: __________



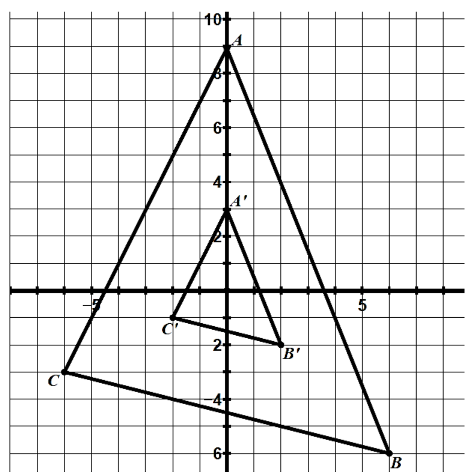
8. $k =$ ______; center of dilation: __________



9. $k =$ ______; center of dilation: __________



10. $k =$ ______; center of dilation: __________



4.3 – SIMILARITY TRANSFORMATIONS

<u>OBJECTIVES:</u>

- EXPLORE THE BASIC CONCEPT OF SIMILARITY AS ILLUSTRATED THROUGH TRANSFORMATIONS
- PROVE THAT FIGURES ARE SIMILAR USING TRANSFORMATIONS

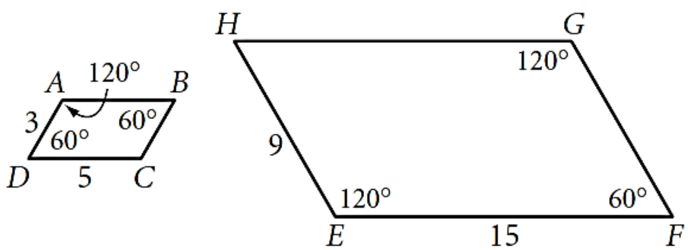
❖ Similarity

- ➢ In a dilation, the image and pre-image are **SIMILAR** because they have the same shape – but are different sizes.
  - ▪ Corresponding side lengths are **PROPORTIONAL**
    - Similarity ratio – The ratio of any side length in the first figure to the corresponding side length in the second figure.
  - ▪ Corresponding angles are **CONGRUENT**
- ➢ The order of the vertices in a similarity statement is important. It identifies the corresponding angles and the corresponding sides.

similarity statement	congruent angles	corresponding sides
$ABCD \sim EFGH$	$\angle A \cong \angle E$ $\angle B \cong \angle F$ $\angle C \cong \angle G$ $\angle D \cong \angle H$	$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$

<u>**EXAMPLE:** – Finding Unknown Lengths in Similar Polygons</u>

2. Is $ABCD \sim EFGH$? Explain your reasoning.

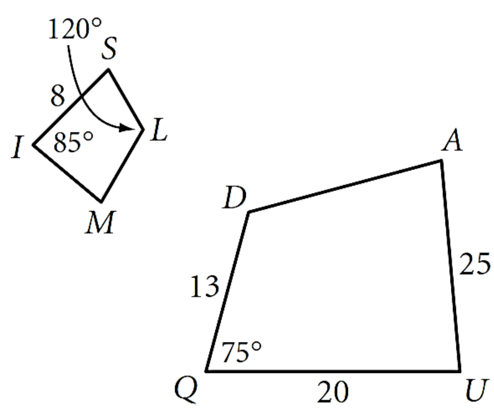


3. The quadrilaterals below are similar: $QUAD \sim SIML$

   a. If $QUAD$ is the pre-image, what is the scale factor?

   b. Find $m\angle D$, $m\angle U$, and $m\angle A$.

   c. Use the scale factor to find the unknown sides lengths SL and MI.

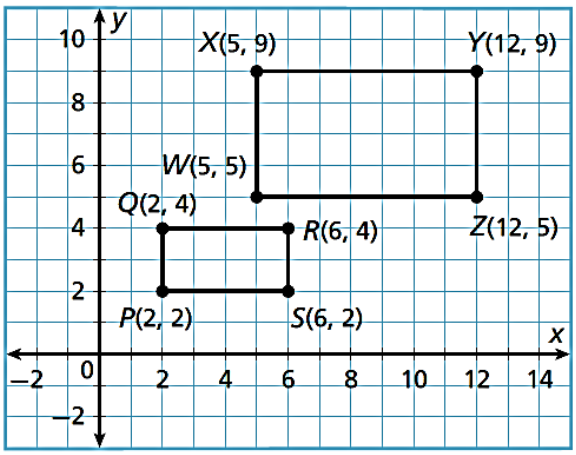


- ❖ Similarity Transformations
  - ➢ A dilation or composition of one or more dilations and one or more congruence transformations.
  - ➢ Two figures are SIMILAR if and only if there is a similarity transformation that maps one figure to the other figure.

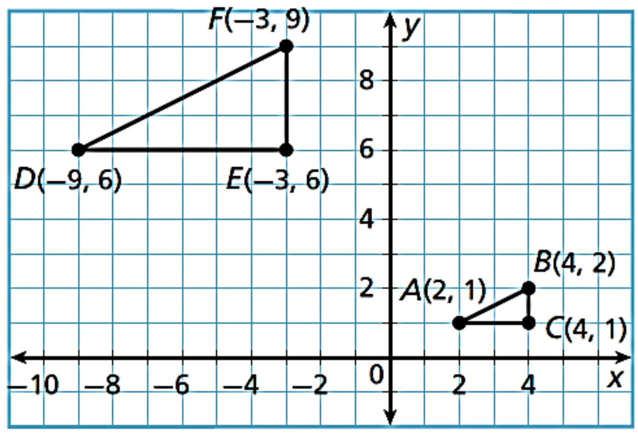
EXAMPLES – <u>Determining Whether Polygons are Similar</u>

Use the definition of similarity in terms of similarity transformations to determine whether the two figures are similar. Explain your answer.

3. Is $PQRS \sim WXYZ$?

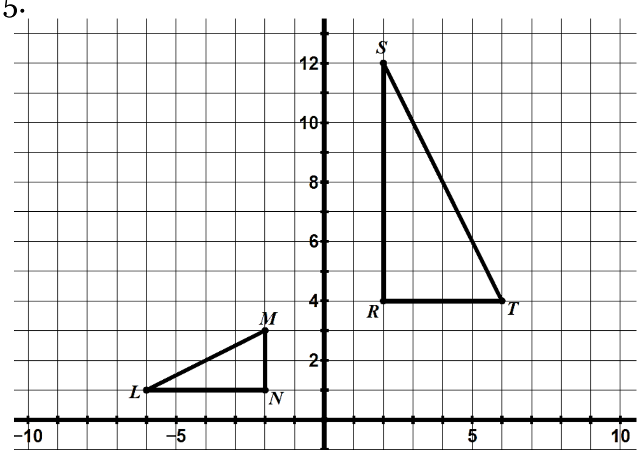


4. Is $\triangle DEF \sim \triangle ACB$?

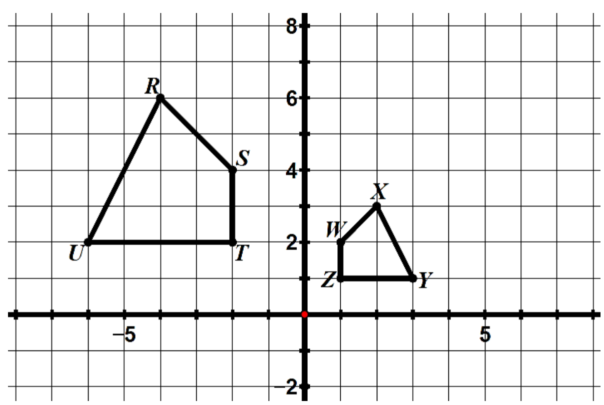


The figures shown are similar. Assume that the figure on the left is the pre-image. List the sequence of transformations that verifies the similarity of the two figures.

5.



6.



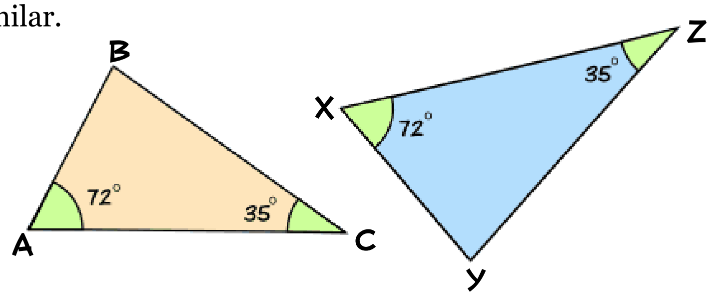
4.4 – Triangle Similarity

<u>OBJECTIVES:</u>

- GIVEN TWO TRIANGLES, EXPLAIN WHY THEY ARE SIMILAR
- GIVEN TWO TRIANGLES AND A SIMILARITY THEOREM, DETERMINE THE ADDITIONAL INFORMATION NEEDED TO PROVE SIMILARITY
- GIVEN TWO TRIANGLES, DETERMINE WHETHER THEY ARE SIMILAR
- USE SCALE FACTOR AND PROPORTIONS TO FIND MISSING SEGMENT LENGTHS IN TRIANGLES

❖ Angle-Angle Similarity Postulate (AA)

➢ If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.



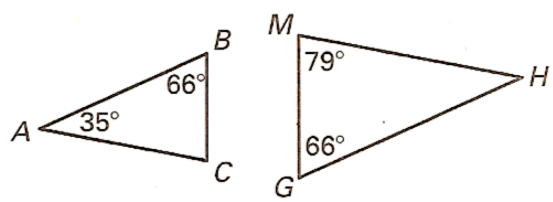
$\angle A \cong \angle X$

$\angle C \cong \angle Z$

$\triangle ABC \sim \triangle XYZ$

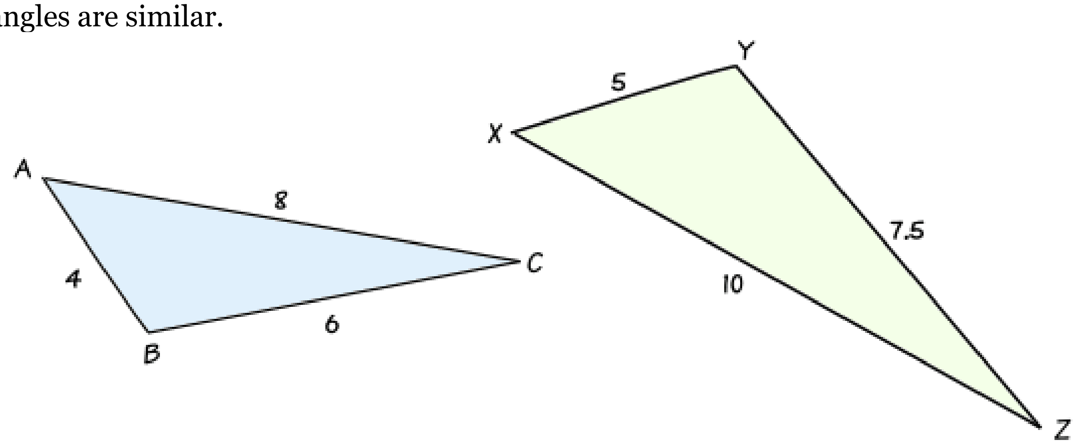
by the AA Similarity Theorem

EXAMPLE: ***Determine whether the triangles are similar. Explain your reasoning. If they are similar write a similarity statement.***



❖ Side-Side-Side Similarity Postulate (SSS)

➢ If the three sides of one triangle are proportional to the corresponding sides of another, then the triangles are similar.



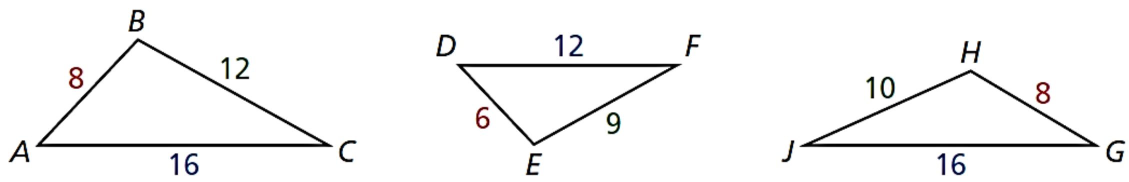
$$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$$

$$\frac{4}{5} = \frac{6}{7.5} = \frac{8}{10} \rightarrow \frac{4}{5}$$

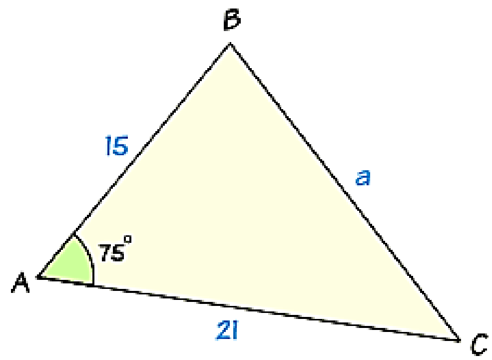
$\triangle ABC \sim \triangle XYZ$

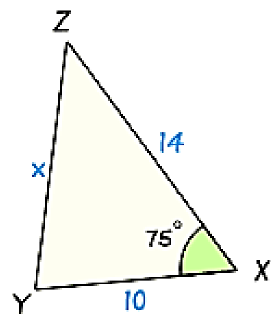
by the SSS Similarity Theorem

EXAMPLE: ***Is either $\triangle DEF$ or $\triangle GHJ$ similar to $\triangle ABC$? Explain your reasoning.***



- ❖ Side-Angle-Side Similarity Postulate (SAS)
  - ➢ If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides that include these angles are proportional, then the triangles are similar.





$$\frac{AB}{XY} = \frac{AC}{XZ}$$

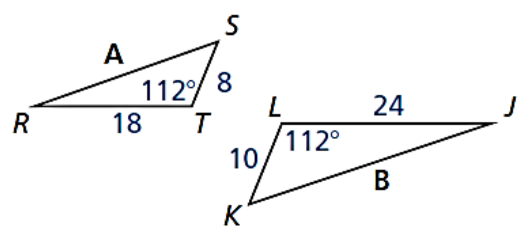
$$\frac{15}{10} = \frac{21}{14} \rightarrow \frac{3}{2}$$

$\angle A \cong \angle X$

$\triangle BAC \sim \triangle YXZ$

by the SAS Similarity Theorem

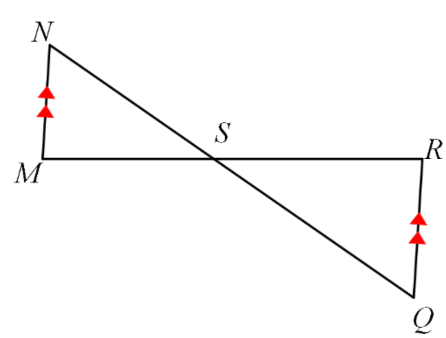
EXAMPLE: ***Determine whether the two triangles are similar. Explain your reasoning. If they are similar, write a similarity statement.***



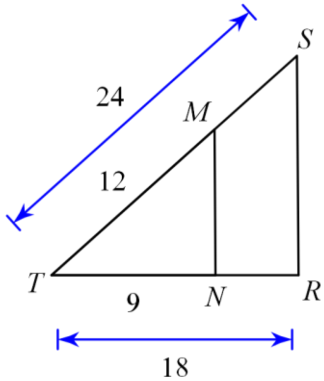
EXAMPLES – <u>Determining Similarity</u>

Are the two triangles similar? Explain your reasoning. If they are similar, identify the similarity theorem and complete the similarity statement.

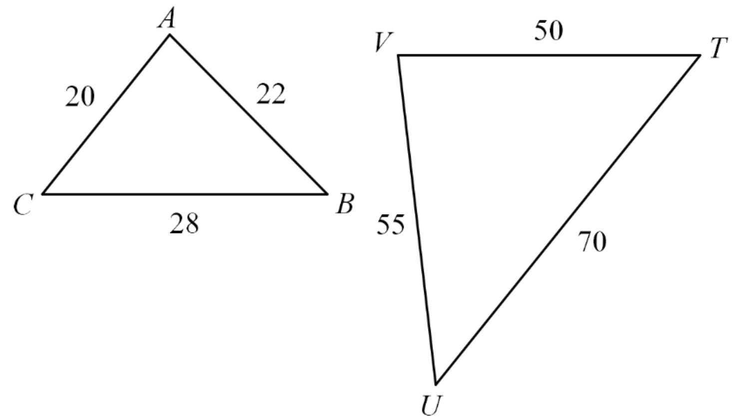
1. $\triangle SRQ \sim$ __________



2. $\triangle TSR \sim$ ___________

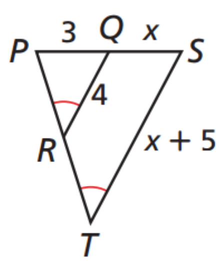


3. $\triangle ABC \sim$ ___________

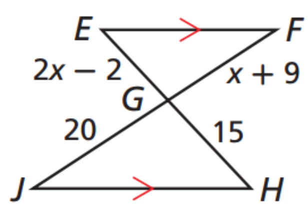


EXAMPLES – The triangles are similar. Set up and solve a proportion to find the value of the variable.

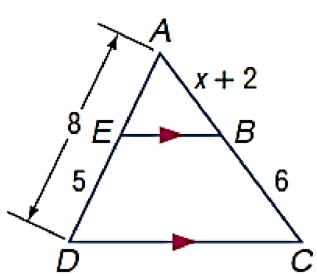
4. $\triangle PQR \sim \triangle PST$



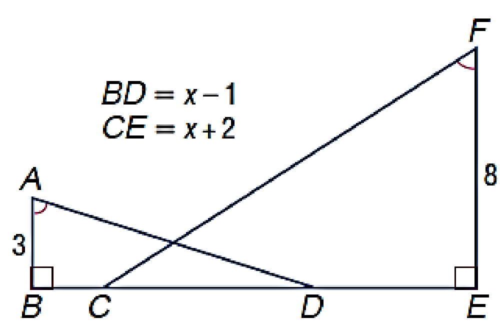
5. $\triangle EFG \sim \triangle HJG$



6. $\triangle AEB \sim \triangle ADC$



7. $\triangle ABC \sim \triangle FEC$



4.5 – Similarity in Right Triangles

<u>OBJECTIVE:</u>

- USE THE RIGHT TRIANGLE/ALTITUDE SIMILARITY THEOREM AND THE GEOMETRIC MEAN TO FIND MISSING SIDE AND SEGMENTS LENGTHS OF SIMILAR RIGHT TRIANGLES AND SOLVE APPLICATION PROBLEMS

❖ Geometric Mean

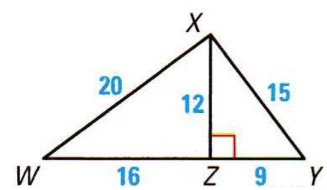
- ➢ For any two positive numbers a and b, the geometric mean is the positive number x such that: $\frac{a}{x} = \frac{x}{b}$

EXAMPLES – Find the geometric mean between the following numbers. If necessary, express as a radical in simplest form.

1. 5 and 8
2. 4 and 18

❖ Introduction

- ➢ Explain <u>how</u> $\triangle XZW \sim \triangle YZX \sim \triangle YXW$



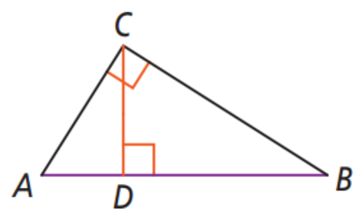
❖ Right Triangle/Altitude Similarity Theorem

- ➢ If an altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

❖ Right Triangle Altitude/Hypotenuse Theorem

- ➢ The measure of the altitude (drawn from the vertex of the right angle of a right triangle to its hypotenuse) is the geometric mean between the measures of the two segments of the hypotenuse.
  - Let $\boldsymbol{h}$ **= altitude** $\overline{CD}$
  - Let $\boldsymbol{x}$ **&** $\boldsymbol{y}$ **= the two segments of the hypotenuse**: $\overline{AD}$ & $\overline{BD}$

  GEOMETRIC MEAN: $\frac{CD}{AD} = \frac{BD}{CD} \Leftrightarrow \frac{\boldsymbol{h}}{\boldsymbol{x}} = \frac{\boldsymbol{y}}{\boldsymbol{h}}$



❖ Right Triangle Altitude/Leg Theorem

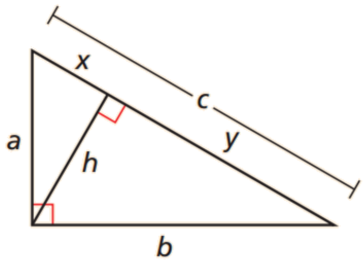
- ➢ If the altitude is drawn to the hypotenuse of a right triangle, each leg of the right triangle is the geometric mean of the hypotenuse and the segment of the hypotenuse adjacent to the leg.
  - Let $\boldsymbol{a}$ **= leg** $\overline{AC}$ and $\boldsymbol{x}$ **= segment** $\overline{AD}$
  - Let **c = hypotenuse** $\overline{AB}$

  GEOMETRIC MEAN: $\frac{AC}{AB} = \frac{AD}{AC} \Leftrightarrow \frac{\boldsymbol{a}}{\boldsymbol{c}} = \frac{\boldsymbol{x}}{\boldsymbol{a}}$

  - Let $\boldsymbol{b}$ **= leg** $\overline{BC}$ and $\boldsymbol{y}$ **= segment** $\overline{BD}$
  - Let **c = hypotenuse** $\overline{AB}$

  GEOMETRIC MEAN: $\frac{BC}{AB} = \frac{BD}{BC} \Leftrightarrow \frac{\boldsymbol{b}}{\boldsymbol{c}} = \frac{\boldsymbol{y}}{\boldsymbol{b}}$

- You can label a diagram as shown below.
  - The legs are a & b, the hypotenuse is c, and the altitude is h.
    - Notice that segment x is adjacent to leg a, and that segment y is adjacent to leg b.



<u>Altitude/Hypotenuse</u>

$$\frac{h}{x} = \frac{y}{h} \Rightarrow h^2 = xy$$

<u>Altitude/Leg</u>

$$\frac{a}{c} = \frac{x}{a} \Rightarrow a^2 = cx$$

$$\frac{b}{c} = \frac{y}{b} \Rightarrow b^2 = cy$$

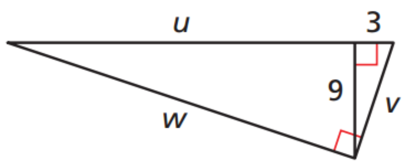
<u>Additional Formulas</u>

$$x + y = c$$

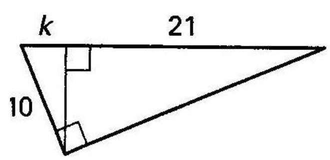
You can also use the Pythagorean Theorem for any of the right triangles.

EXAMPLES – Find the value of the variable(s). If necessary, express as a radical in simplest form.

3.



4.



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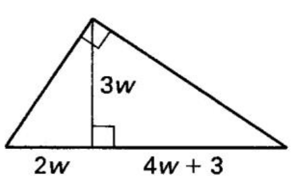


4.6 – Proportionality Theorems

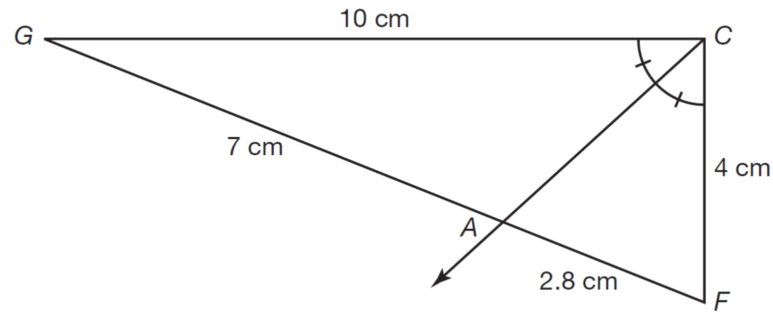
OBJECTIVES:

- USE THE ANGLE BISECTOR/PROPORTIONAL SIDE THEOREM TO CALCULATE SEGMENT LENGTHS IN TRIANGLES
- USE THE TRIANGLE PROPORTIONALITY THEOREM TO CALCULATE SEGMENT LENGTHS IN TRIANGLES
- USE THE PROPORTIONAL SEGMENTS THEOREM AND THE TRIANGLE MIDSEGMENT THEOREM TO DETERMINE MISSING SEGMENT LENGTHS

❖ Introduction:

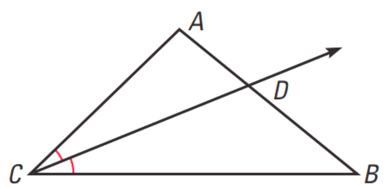
➢ Given: $\overrightarrow{CA}$ bisects $\angle GCF$

- Calculate various segment ratios.
- What do you notice?



❖ Angle Bisector/Proportional Side Theorem

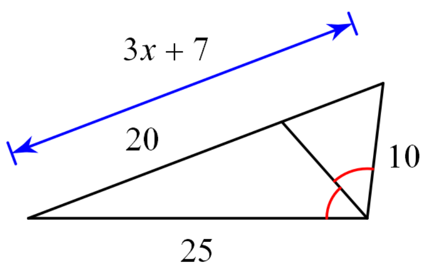
➢ A bisector of an angle in a triangle divides the opposite side into two segments whose lengths are in the same ratio as the lengths of the sides adjacent to the angle.



If $\overrightarrow{CD}$ bisects $\angle ACB$, then $\frac{AD}{DB} = \frac{CA}{CB}$.

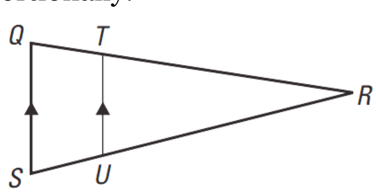
EXAMPLES – Use the Angle Bisector/Proportional Side Theorem to set up and solve a proportion to find the value of x.

1.



❖ Triangle Proportionality Theorem

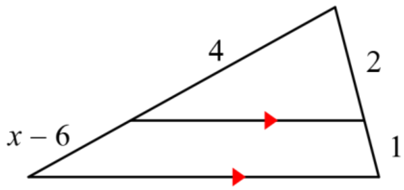
➢ If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.



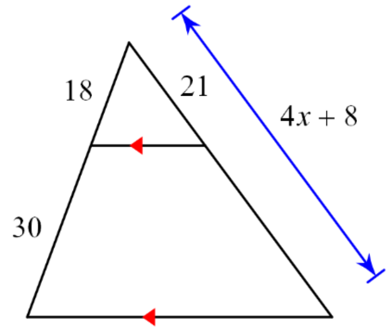
If $\overline{TU} \parallel \overline{QS}$, then $\frac{RT}{TQ} = \frac{RU}{US}$.

EXAMPLES – Use the Triangle Proportionality Theorem to set up and solve a proportion to find the value of x.

2.



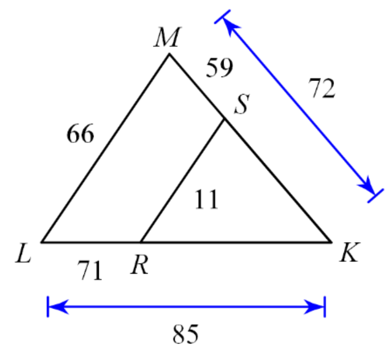
3.



❖ Converse of the Triangle Proportionality Theorem

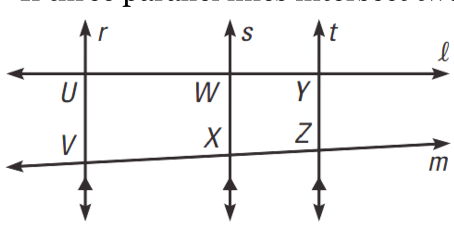
➢ If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

4. Use the Converse of the Triangle Proportionality Theorem and determine whether $\overline{LM} \parallel \overline{RS}$. *Are the segments of the sides proportional?*



❖ Proportional Segments Theorem

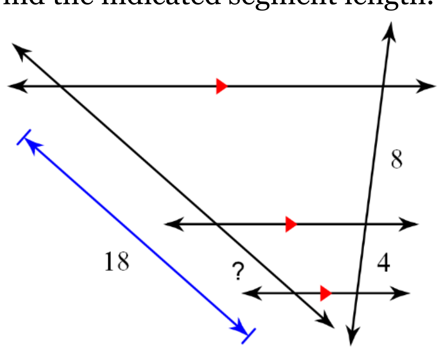
➢ If three parallel lines intersect two transversals, then they divide the transversals proportionally.



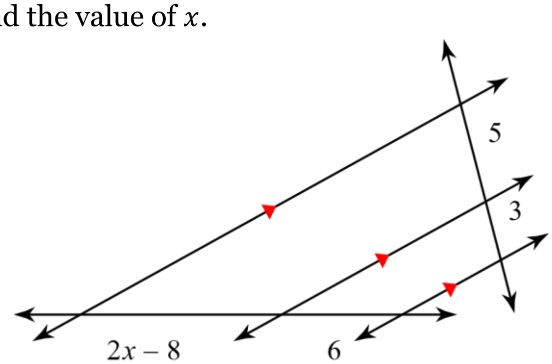
If $r \parallel s$ and $s \parallel t$, and ℓ and m intersect r, s, and t, then $\frac{UW}{WY} = \frac{VX}{XZ}$.

EXAMPLES – Use the Proportional Segments Theorem to set up and solve a proportion.

5. Find the indicated segment length.



6. Find the value of x.

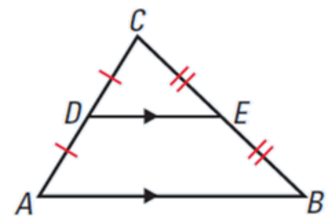


- Midsegments
  - Midsegment (of a triangle)—A segment that connects the midpoints of two sides of a triangle.

- The Triangle Midsegment Theorem
  - The midsegment of a triangle is parallel to the third side of the triangle and is half the measure of the third side of the triangle.

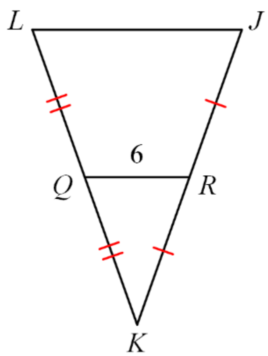
    In $\triangle ABC$, if $CD = DA$ and $CE = EB$,

    then $\overline{DE} \parallel \overline{AB}$ and $DE = \frac{1}{2}AB$.

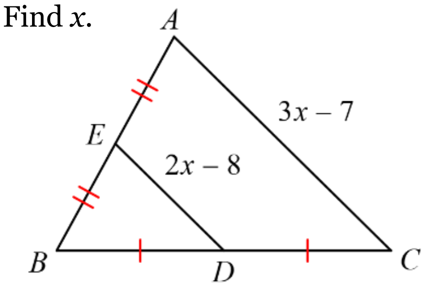


EXAMPLES – Use the Triangle Midsegment Theorem to find the indicated segment length.

7. Find LJ



8. Find x.



4.7 – APPLICATIONS WITH SIMILAR FIGURES

OBJECTIVES:

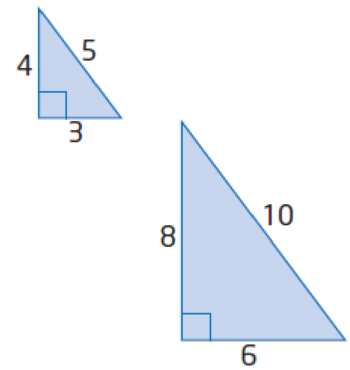
- APPLY PROPERTIES OF SIMILAR FIGURES TO CALCULATE INDIRECT MEASUREMENTS IN REAL WORLD CONTEXTS
- USE PROPORTIONS TO SOLVE PROBLEMS INVOLVING PERIMETER AND AREA OF SIMILAR FIGURES

- Indirect Measurement
  - At times, measuring something directly is impossible, or physically undesirable. When these situations arise, indirect measurement, the technique that uses proportions to calculate measurement, can be implemented.
    - Your knowledge of similar triangles can be very helpful in these situations.

EXAMPLES – Set up and solve a proportion that models the scenario.

1. A 5-foot tall student stands near a flagpole. The flagpole and the student are perpendicular to the ground. The sun's rays strike the flagpole and the student at the same angle. The flagpole casts a 15-foot shadow, and the student casts a 2-foot shadow. Use indirect measurement to find the height of the flagpole.

❖ Investigation: Perimeters & Areas of Similar Figures



a. Find the scale factor, k, for the pair of triangles.

b. Find the perimeter of the first triangle.

c. Find the perimeter of the second triangle.

d. Find the ratio of the perimeters.

e. What do you notice about the scale factor and the ratio of the perimeters?

f. *Complete the conjecture*: If the scale factor of two similar figures is $\frac{a}{b}$, then the ratio of their perimeters is ______.

g. Find the area of the first triangle.

h. Find the area of the second triangle.

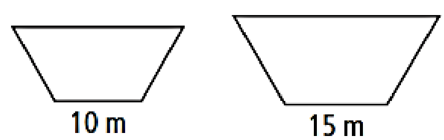
i. Find the ratio of the areas.

j. What do you notice about the scale factor and the ratio of the areas?

k. *Complete the conjecture*: If the scale factor of two similar figures is $\frac{a}{b}$, then the ratio of their areas is ______.

EXAMPLES:

2. The figures in each pair are similar. Compare the first figure to the second. Give the ratio of the perimeters and the ratio of the areas.



3. The figures in each pair are similar. The area of the larger pentagon: 135 cm^2. Find the area of the other figure to the nearest whole number.