Chapter 5 Notes Packet

Name: Backer

TRIANGLES & CONGRUENCE

5.1 CLASSIFYING TRIANGLES

Objectives:

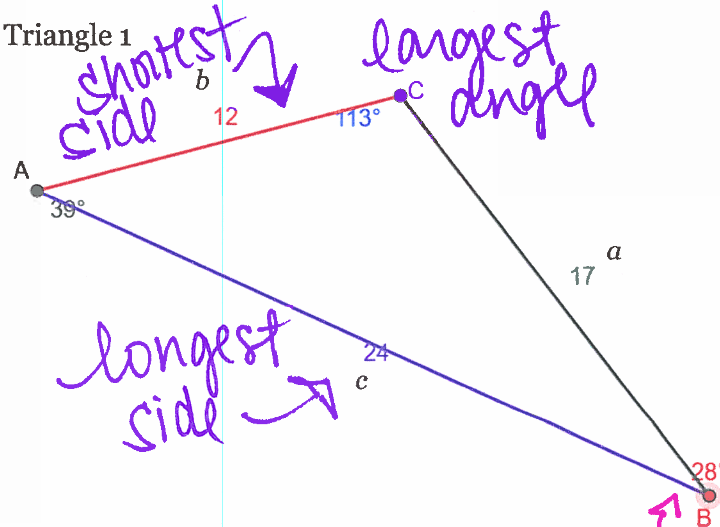
- Classify a triangle given the locations of its vertices graphed on the coordinate plane
- Identify the relationship between the side lengths of a triangle and the measures of its interior angles

❖ Triangles can be classified in two ways: by their angle measures or by their side lengths.

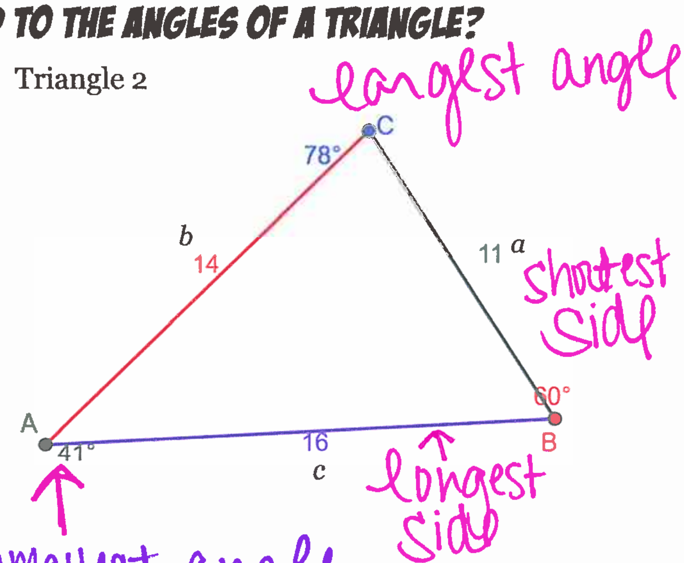
- By sides:
  - Equilateral = 3 congruent sides
  - Isosceles = at least 2 congruent sides
  - Scalene = no congruent sides
- By angles:
  - Acute = 3 acute angles
  - Right = 1 right angle
  - Obtuse = 1 obtuse angle
  - Equiangular = 3 congruent angles

HOW ARE THE SIDES OF A TRIANGLE RELATED TO THE ANGLES OF A TRIANGLE?

Triangle 1



Triangle 2



Identify (by letter):

	LARGEST ANGLE	LONGEST SIDE	SMALLEST ANGLE	SHORTEST SIDE
TRIANGLE 1	$\angle C$	c or $\overline{AB}$	$\angle B$	b or $\overline{AC}$
TRIANGLE 2	$\angle C$	c or $\overline{AB}$	$\angle A$	a or $\overline{BC}$

Calculate and then compare:

	c^2	$a^2 + b^2$	$c^2 <,=,> a^2 + b^2$	ACUTE OR OBTUSE?
TRIANGLE 1	576 >	$12^2+17^2=433$	c^2 > $a^2 + b^2$	obtuse
TRIANGLE 2	256 <	$11^2+14^2=317$	c^2 < $a^2 + b^2$	acute

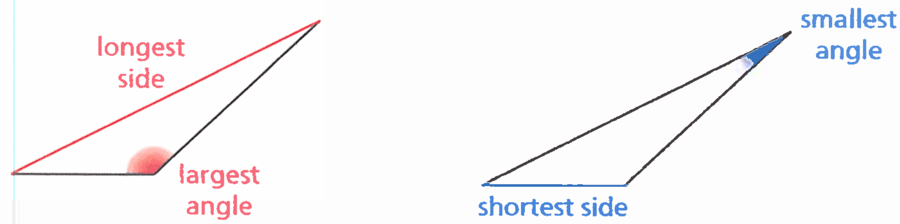
Make a conjecture about the relationship between a triangle's sides and its angles.

the largest angle is opposite the longest side
the smallest angle is opposite the shortest side
if $c^2 < a^2+b^2$, the Δ is acute
if $c^2 > a^2+b^2$, the Δ is obtuse

- Pythagorean Inequalities
  - For $\triangle ABC$, with c as the length of the longest side…

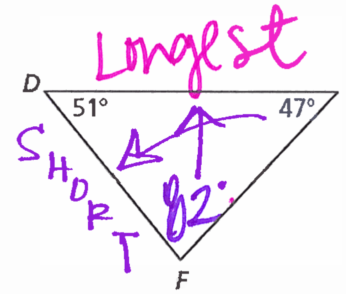
Acute: $c^2 < a^2 + b^2$ Right: $c^2 = a^2 + b^2$ Obtuse: $c^2 > a^2 + b^2$

- Triangle Side/Angle Theorems
  - The positions of the longest and shortest sides of a triangle are related to the positions of the largest and smallest angles.



EXAMPLES:

1. List the sides of $\triangle DEF$ in order from shortest to longest.



$\overline{DF}$, $\overline{EF}$, $\overline{DE}$

2. Consider a triangle with side lengths 5.3, 6.7, and 7.8. Is the triangle acute, right, or obtuse?

c

$7.8^2 \quad 5.3^2 + 6.7^2$

$60.84 < 72.98$

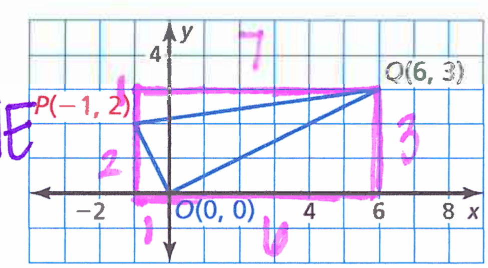
$c^2 < a^2 + b^2 \rightarrow$ acute

- Classifying a Triangle in the Coordinate Plane
  - To determine if the triangle is scalene, isosceles, or equilateral, use the Distance Formula to determine the length of each side.
  - To determine if the triangle is right, use the slope formula, or the Pythagorean Theorem.
    - Perpendicular lines form right angles and have slopes that are opposite reciprocals

DISTANCE FORMULA	SLOPE FORMULA	PYTHAGOREAN THEOREM
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	$m = \dfrac{y_2 - y_1}{x_2 - x_1}$	$a^2 + b^2 = c^2$

EXAMPLE:

3. Classify $\triangle OPQ$ by its sides. Then determine whether it is a right triangle.



$OP = \sqrt{1^2 + 2^2} = \sqrt{5}$

$OQ = \sqrt{6^2 + 3^2} = \sqrt{45}$ SCALENE

$PQ = \sqrt{1^2 + 7^2} = \sqrt{50}$

$(\sqrt{50})^2 \quad (\sqrt{5})^2 + (\sqrt{45})^2$

$50 = 50$ Right

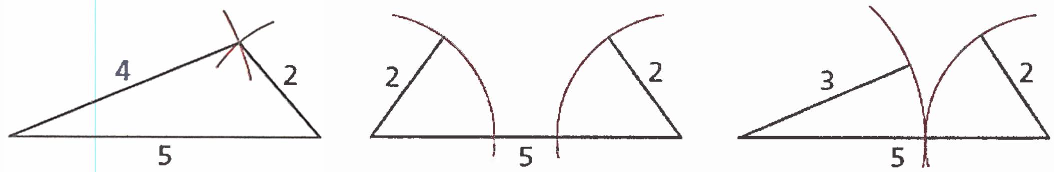
$m_{OP} = \frac{-2}{1} = -2$

$m_{OQ} = \frac{3}{6} = \frac{1}{2}$ opp. Recips

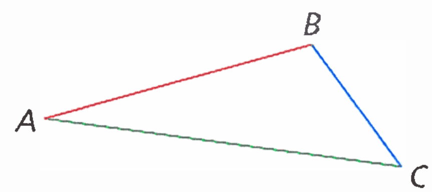
$\overline{OP} \perp \overline{OQ}$

HOW ARE ANY TWO SIDES OF A TRIANGLE RELATED TO THE THIRD SIDE?

- Three Segments One Triangle?
  - Not every group of three segments can be used to form a triangle. The lengths of the segments must fit a certain relationship.
    - Three attempted triangle constructions using segments with given lengths are shown below. Only the first group of segments forms a triangle.



- Triangle Inequality Theorem
  - The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
    - $AB + BC > AC$
    - $AC + BC > AB$
    - $AB + AC > BC$



EXAMPLES:

4. Can a triangle have sides with the given lengths? Explain your reasoning.

   a. 5 feet, 10 feet, 15 feet

   $5 + 10 > 15$

   $\cancel{15 > 15}$

   Not a $\triangle$

   b. $n + 6, n^2 - 1, 3n; n = 4$

   10, 15, 12

   $10 + 15 > 12 \rightarrow 25 > 12$

   $10 + 12 > 15 \rightarrow 22 > 15$

   $15 + 12 > 10 \rightarrow 27 > 10$

   $\triangle$

5. A triangle has one side of length 14 and another side of length 9. Find the range of the possible lengths of the third side.

   $14 + 9 > C$ $14 + C > 9$ $C + 9 > 14$

   $23 > C$ $C > -5$ $C > 5$

   $5 < C < 23$

5.2 PROPERTIES OF TRIANGLES

Objectives:

- Prove and apply the Triangle Sum Theorem and the Exterior Angle Theorem
- Prove and apply the Isosceles Triangle Base Angles Theorem and its Converse

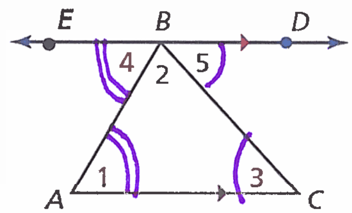
❖ Triangle Sum Theorem

➢ **The sum of the measures of the angles of a triangle is 180 degrees.**

PROOF

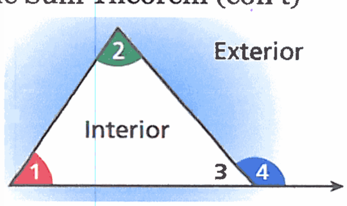
Given: $\overleftrightarrow{BD} \parallel \overleftrightarrow{AC}$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180°$



Statements	Reasons
1. $\overleftrightarrow{BD} \parallel \overleftrightarrow{AC}$	1. Given
2. $\angle 1 \cong \angle 4$	2. Alt. int. ∠s are ≅
3. $m\angle 1 = m\angle 4$	3. Definition of congruent angles
4. $\angle 3 \cong \angle 5$	4. Alt. int. ∠s are ≅
5. $m\angle 3 = m\angle 5$	5. Definition of congruent angles
6. $\angle EBD$ is a straight angle	6. Assumed from diagram
7. $m\angle EBD = 180°$	7. Definition of straight angle
8. $m\angle 2 + m\angle 4 + m\angle 5 = m\angle EBD$	8. Angle Addition Postulate
9. $m\angle 1 + m\angle 2 + m\angle 3 = 180°$	9. Substitution

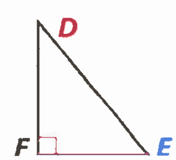
❖ Triangle Sum Theorem (con't)



$m\angle 1 + m\angle 2 + m\angle 3 =$ 180°

$m\angle 3 + m\angle 4 =$ 180°

Linear Pair Postulate



- Corollary: The acute angles of a right triangle are complementary: $m\angle D + m\angle E = 90°$

EXAMPLES:

1. In $\triangle LMN$, $m\angle L = 8x$, $m\angle M = m\angle N = 6x - 1$. Use the Triangle Sum Theorem to set up and solve an equation to find the value of x.

$8x + 6x - 1 + 6x - 1 = 180°$

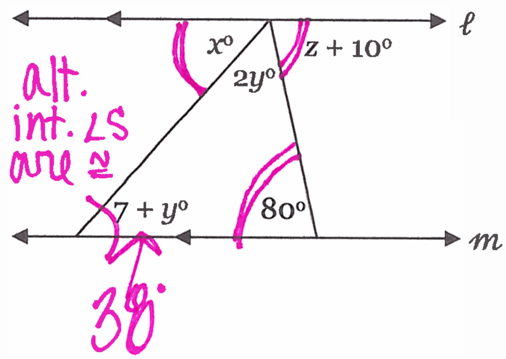
$20x - 2 = 180°$

$20x = 182$

$x = 9.1$

2. Given: $\ell \parallel m$

Find the values of the variables.



△ sum theorem to find y

$2y+7+y+80=180°$

$3y+87=180$

$y=31$ $\quad x=38$

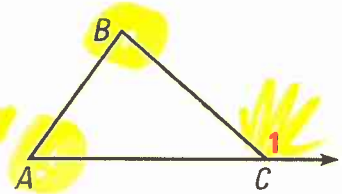
alt. int. ∠s are ≅

$z+10=80 \rightarrow z=70$

❖ **TRIANGLE EXTERIOR-ANGLE THEOREM**

> **The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles.**

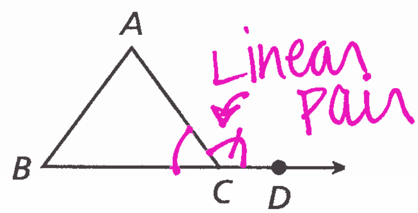
- $m\angle A + m\angle B = m\angle 1$



PROOF

Given: $\triangle ABC$ with exterior angle $\angle ACD$

Prove: $m\angle ACD = m\angle A + m\angle B$

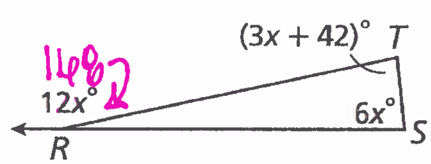


Statements	Reasons
1. $\triangle ABC$ with exterior angle $\angle ACD$	1. Given
2. $m\angle A + m\angle B + m\angle ACB = 180°$	2. Triangle Sum Theorem
3. ∠ACB & ∠ACD form a linear pair	3. Assumed from the diagram
4. ∠ACB is supp. to ∠ACD	4. Linear Pair Postulate
5. $m\angle ACB + m\angle ACD = 180°$	5. Definition of supplementary angles
6. $m\angle A + m\angle B + \cancel{m\angle ACB} = \cancel{m\angle ACB} + m\angle ACD$	6. Substitution
7. $m\angle A + m\angle B = m\angle ACD$	7. Subtraction Property of Equality

combine these

EXAMPLE:

3. Use the Triangle Exterior-Angle Theorem to set up and solve an equation to find the value of x. Then find $m\angle TRS$.



ext. ∠R = m∠T + m∠S

$12x = 3x + 42 + 6x$

$12x = 9x + 42$

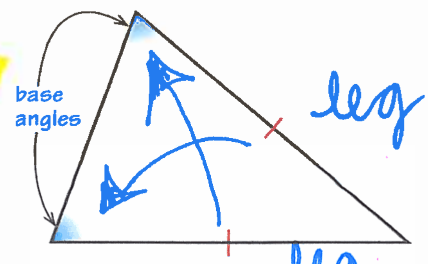
$3x = 42$

$x = 14$ $\quad m\angle TRS = 12°$

- Isosceles Triangle Base Angles Theorem
  - If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

    If △, then △.
- Converse of the Isosceles Triangle Base Angles Theorem
  - If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

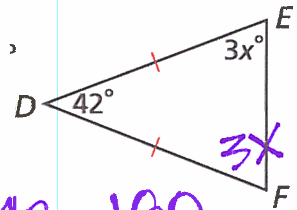
    If △, then △.



The **base angles** of an isosceles triangle are opposite the congruent sides.

EXAMPLES:

4. Find $m\angle F$ and then set up and solve an equation to find the value of x.

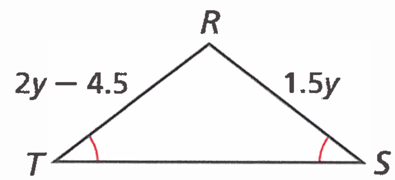


$6x + 42 = 180$

$6x = 138$

$x = 23$

5. Set up and solve an equation to find the value of y and then find RS.



$RS = 13.5$

$2y - 4.5 = 1.5y$

$-4.5 = -0.5y$

$9 = y$

6. In △RST (above), $m\angle S = 2x^2 + 50$ and $m\angle T = 25x$. Find the value of x, that makes sense.

$2x^2 + 50 = 25x$

$2x^2 - 25x + 50 = 0$

$(2x-5)(x-10) = 0$

$x = 2.5$ or ~~$x = 10$~~

100

−20 −5

−25

$2x^2$	$-20x$	$2x$
$-5x$	50	-5
x	-10	

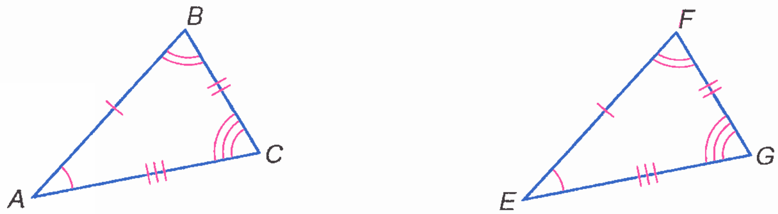
5.3 CONGRUENCE & TRANSFORMATIONS

Objectives:

- Identify corresponding sides and corresponding angles of congruent triangles
- Use the definition of congruence in terms of rigid motions to prove that two figures are congruent

❖ Congruent Triangles

- Triangles that are the same shape and size are congruent triangles.
- Each triangle has three angles and three sides. If all six of the corresponding parts of two triangles are congruent, then the triangles are congruent.



- The corresponding sides and angles can be determined from any congruence statement by following the order of the letters. *ORDER IS IMPORTANT!

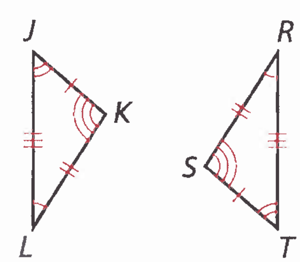
$\triangle ABC \cong \triangle EFG$ ⇨

$\angle A \cong \angle E$ $\quad \angle B \cong \angle F$ $\quad \angle C \cong \angle G$

$\overline{AB} \cong \overline{EF}$ $\quad \overline{BC} \cong \overline{FG}$ $\quad \overline{AC} \cong \overline{EG}$

EXAMPLES:

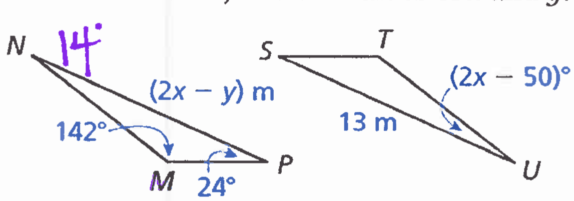
1. Write a congruence statement for the triangles. Identify all six pairs of congruent corresponding parts.



Congruent statement: $\triangle LJK \cong \triangle RTS$

CORRESPONDING CONGRUENT SIDES	CORRESPONDING CONGRUENT ANGLES
$\overline{LJ} \cong \overline{RT}$	$\angle L \cong \angle R$
$\overline{JK} \cong \overline{TS}$	$\angle J \cong \angle T$
$\overline{LK} \cong \overline{RS}$	$\angle K \cong \angle S$

2. $\triangle MNP \cong \triangle TUS$, find the values of x and y.



$\angle U \cong \angle N$

$2x-50=14$

$2x=64$

$x=32$

$\overline{NP} \cong \overline{US}$

$2x-y=13$

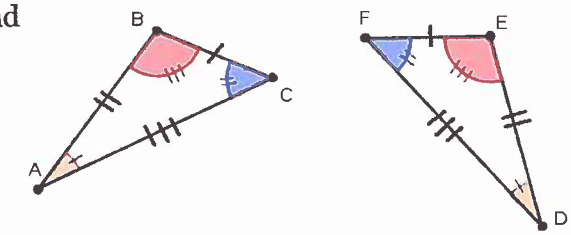
$2(32)-y=13$

$64-y=13$

$+y=+51$

$m\angle A = 14°$ ← use $\triangle$ Sum Theorem

3. $\triangle ABC \cong \triangle DEF$, $m\angle B = 148°$, $m\angle C = 18°$, $m\angle D = 3x + y$, and $m\angle F = 5x - y$. Set up and solve a system of equations to find the values of x and y.



$\angle D \cong \angle A$ $\quad$ $\angle F \cong \angle C$

$3x + y = 14$ $\quad$ $5x - y = 18$

add these

$8x = 32$

$x = 4$ → $3 \cdot 4 + y = 14$

$12 + y = 14$

$y = 2$

❖ Transformation Rules

➢ Translations
- Right: $x + h$
- Left: $x - h$
- Up: $y + k$
- Down: $y - k$

➢ Rotations
- 90° counterclockwise: $(-y, x)$
- 180°: $(-x, -y)$
- 270° counterclockwise: $(y, -x)$

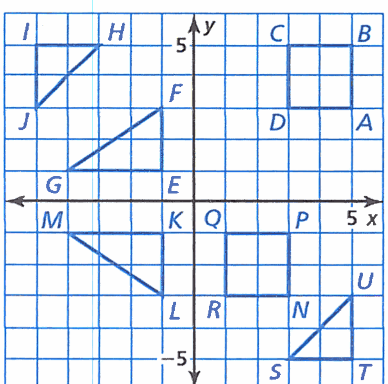
➢ Reflections
- Over the x-axis: $(x, -y)$
- Over the y-axis: $(-x, y)$

❖ CONGRUENCE & TRANSFORMATIONS

➢ Two figures are congruent if one figure can be made to carry onto the second figure using one or more rotations, reflections, and/or transformations.

EXAMPLES:

4. Identify any congruent figures in the coordinate plane. Explain your reasoning.



$ABCD \cong$ NPQR

CONGRUENCE TRANSFORMATION: Left 2 & down 4

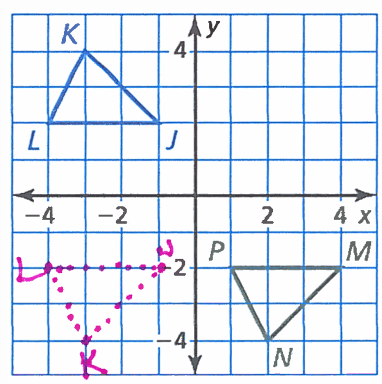
$\triangle HIJ \cong$ $\triangle$UTS

CONGRUENCE TRANSFORMATION: Rotate 180°

$\triangle GEF \cong$ $\triangle$MKL

CONGRUENCE TRANSFORMATION: Reflect over x-axis

5. Describe a congruence transformation that maps $\triangle JKL$ to $\triangle MNP$.



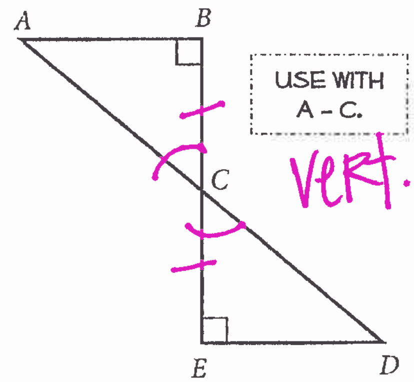
Reflect over x-axis
Right 5

5.4 TRIANGLE CONGRUENCE THEOREMS

Objectives:

- Use the triangle congruence theorems – SSS, SAS, ASA, or AAS – to prove triangle congruency
- Determine whether there is enough information to prove whether two triangles are congruent by SSS, SAS, ASA, or AAS

ACCESSING PRIOR KNOWLEDGE



USE WITH A – C.

Vert. ∠s

a. If C is the midpoint of $\overline{BE}$, then what two segments are congruent?

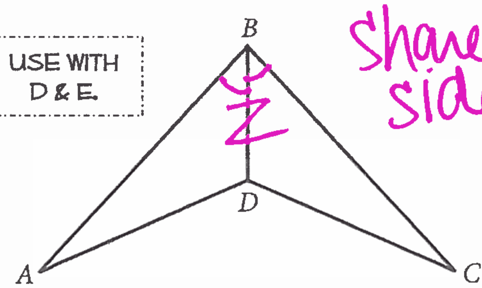
$\overline{BC} \cong \overline{CE}$

b. If $\overline{BE}$ & $\overline{AD}$ intersect at C, what two angles must be congruent and why?

$\angle BCA \cong \angle ECD$
Vertical ∠s are ≅

c. Name two other congruent angles and explain why they are congruent.

$\angle B \cong \angle E$
Right ∠s are ≅



USE WITH D & E.

shared side

d. If $\overrightarrow{BD}$ bisects $\angle ABC$, then what two angles are congruent?

$\angle ABD \cong \angle CBD$

e. Why is $\overline{BD} \cong \overline{BD}$?

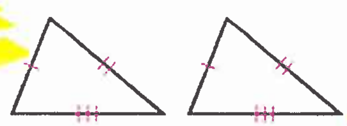
Reflexive

❖ Triangle Congruence Theorems

➢ Proving triangles congruent could be a very tedious task if we had to verify the congruence of every one of the six pairs of corresponding parts.

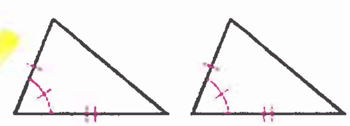
➢ Triangles have some special properties that will enable us to prove two triangles are congruent by comparing only three specially chosen pairs of corresponding parts.

SSS
SIDE-SIDE-SIDE



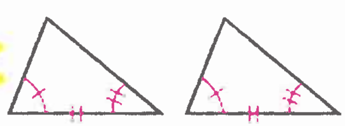
Three pairs of congruent sides

SAS
SIDE-ANGLE-SIDE



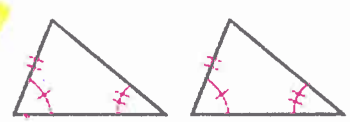
Two pairs of congruent sides and one pair of congruent angles (angles between the pairs of sides)

ASA
ANGLE-SIDE-ANGLE



Two pairs of congruent angles and one pair of congruent sides (sides between the pairs of angles)

AAS
ANGLE-ANGLE-SIDE



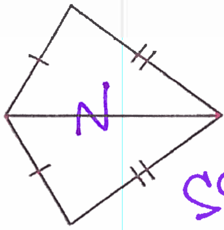
Two pairs of congruent angles and one pair of congruent sides (sides not between the pairs of angles)

BE ON THE LOOKOUT FOR: VERTICAL ANGLES & SHARED SIDES

EXAMPLES:

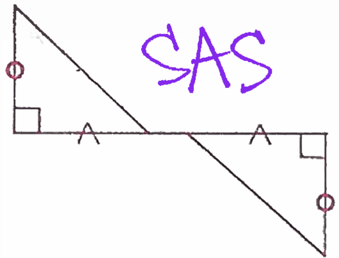
1. Are the triangles shown congruent? If so, state the method – SSS, SAS, ASA or AAS – that would prove congruence.

a



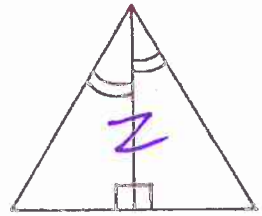
SSS

c



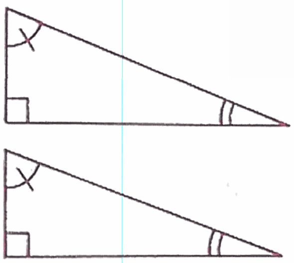
SAS

e



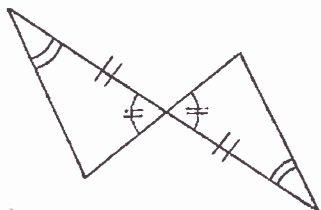
ASA

b



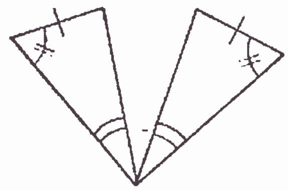
not ≅

d



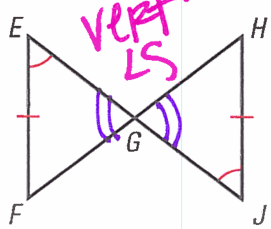
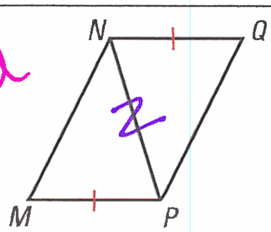
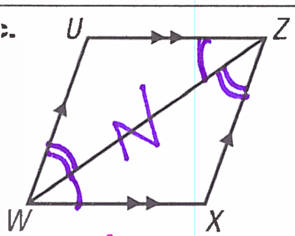
ASA

f



AAS

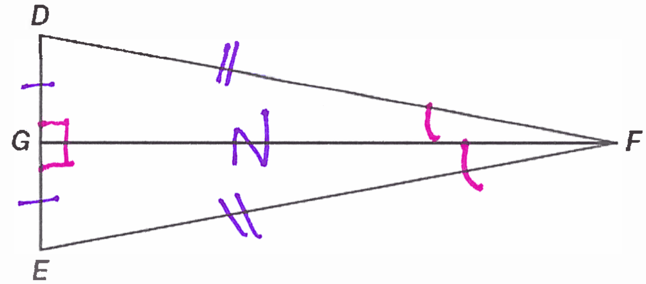
Does the diagram given enough information to show that the triangles are congruent? If so, state the method – SSS, SAS, ASA or AAS – you would use.

	WHAT I KNOW IS CONGRUENT:	CONGRUENT? / METHOD
2. 	$\overline{EF} \cong \overline{JH}$ $\angle E \cong \angle J$ $\angle EGF \cong \angle JGH$ (vert. ∠s are ≅)	CONGRUENT? yes METHOD AAS
3. shared side 	$\overline{MP} \cong \overline{QN}$ $\overline{NP} \cong \overline{NP}$	CONGRUENT? no METHOD
4. 	$\angle UZW \cong \angle XWZ$ $\angle UWZ \cong \angle XZW$ (alt. int. ∠s are ≅) $\overline{WZ} \cong \overline{WZ}$	CONGRUENT? yes METHOD ASA

parallel lines

Alt. int. ∠s are ≅

Determine which given information results in $\triangle DFG \cong \triangle EFG$. State the appropriate congruence theorem if the triangles can be proven congruent, or state that there is not enough information if additional information is needed to determine congruent triangles.



5. G is the midpoint of $\overline{DE}$ & $\overline{DE} \perp \overline{FG}$

SAS

6. $\triangle DEF$ is isosceles with $\overline{FD} \cong \overline{FE}$

not $\cong$

7. $\overline{FG}$ bisects $\angle DFE$, $\overline{FD} \cong \overline{FE}$

SAS

8. $\overline{FG}$ bisects $\angle DFE$, $\angle DGF$ is a right angle

ASA

5.5.D1 CONGRUENT TRIANGLE PROOFS

Objectives:

- Use the triangle congruence theorems – SSS, SAS, ASA, or AAS – to prove triangle congruency
- Draw conclusions from the given information

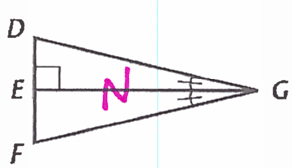
❖ Methods of Proving Triangles Congruent

SSS SAS ASA AAS

EXAMPLES:

Determine whether you could prove that the triangles are congruent. If so, write a congruence statement & identify the congruence theorem you would use.

1.

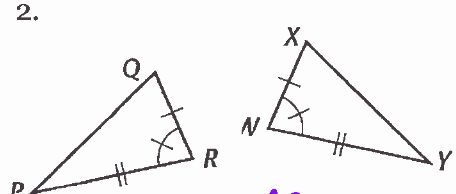


CONGRUENT? yes

$\triangle DEG \cong$ $\triangle FEG$

CONGRUENCE THEOREM: ASA

2.

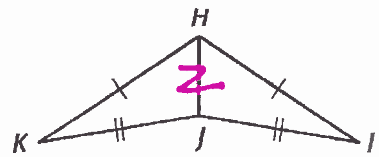


CONGRUENT? yes

$\triangle QRP \cong$ $\triangle XNY$

CONGRUENCE THEOREM: SAS

3.



CONGRUENT? yes

$\triangle KHJ \cong$ $\triangle IHJ$

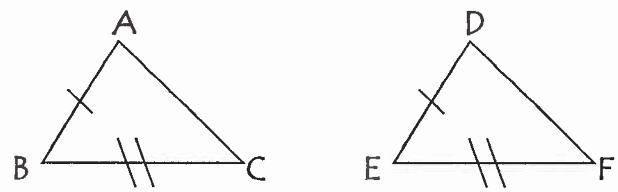
CONGRUENCE THEOREM: SSS

Problems 4 – 6: You are given the congruent angles and sides shown by the tick marks. Name the additional pair of congruent sides or angles needed to prove that the triangles are congruent by the specified method.

4. $\triangle ABC \cong \triangle DEF$

   a. SSS $\overline{AC} \cong \overline{DF}$

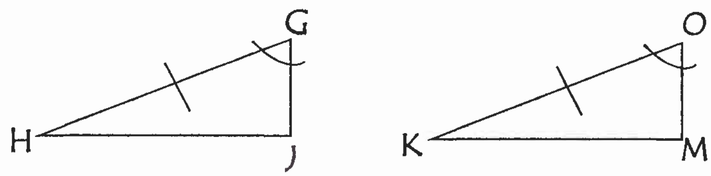
   b. SAS $\angle B \cong \angle E$



5. $\triangle GHJ \cong \triangle OKM$

   a. ASA $\angle H \cong \angle K$

   b. AAS $\angle J \cong \angle M$

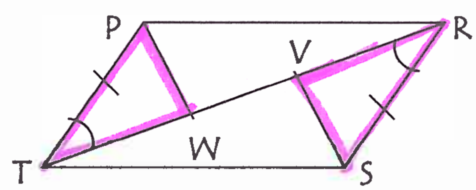


6. $\triangle PWT \cong \triangle SVR$

   a. SAS $\overline{TW} \cong \overline{RV}$

   b. ASA $\angle TPW \cong \angle RSV$

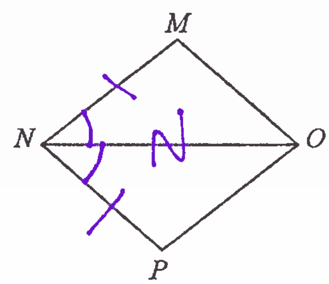
   c. AAS $\angle PWT \cong \angle SVR$



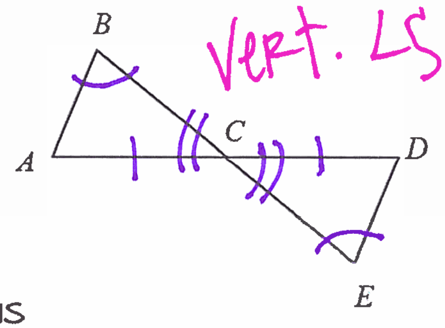
7. Given: $\overline{MN} \cong \overline{PN}$

   $\overrightarrow{NO}$ bisects $\angle MNP$

   Prove: $\triangle MNO \cong \triangle PNO$



	STATEMENTS	REASONS
S	1. $\overline{MN} \cong \overline{PN}$	1. Given
	2. $\overrightarrow{NO}$ bisects $\angle MNP$	2. Given
A	3. $\angle MNO \cong \angle PNO$	3. Def. of bisects
S	4. $\overline{NO} \cong \overline{NO}$	4. Reflexive
	5. $\triangle MNO \cong \triangle PNO$	5. SAS



8. Given: $\angle B \cong \angle E$

C is the midpoint of $\overline{AD}$

Prove: $\triangle ABC \cong \triangle DEC$

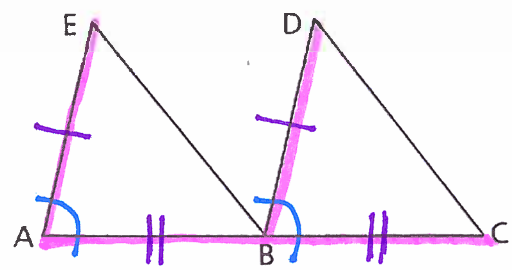
STATEMENTS	REASONS
A 1. $\angle B \cong \angle E$	1. Given
2. C is the midpoint of $\overline{AD}$	2. Given
S 3. $\overline{AC} \cong \overline{DC}$	3. Def. of midpoint
4. $\angle BCA$ & $\angle ECD$ are vertical ∠s	4. Assumed from diagram
A 5. $\angle BCA \cong \angle ECD$	5. Vert. ∠s are ≅
6. $\triangle ABC \cong \triangle DEC$	6. AAS

9. Given: B is the midpoint of $\overline{AC}$

$\overline{EA} \cong \overline{DB}$

$\overline{EA} \parallel \overline{DB}$ → corrs. ∠s are ≅

Prove: $\triangle AEB \cong \triangle BDC$



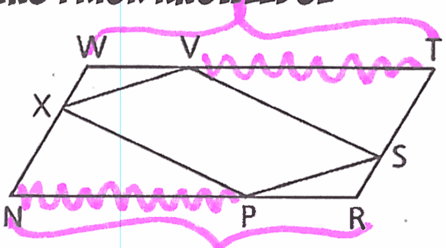
STATEMENTS	REASONS
1. B is the midpoint of $\overline{AC}$	1. Given
S 2. $\overline{AB} \cong \overline{BC}$	2. Def. of midpoint
S 3. $\overline{EA} \cong \overline{DB}$	3. Given
4. $\overline{EA} \parallel \overline{DB}$	4. Given
A 5. $\angle A \cong \angle DBC$	5. Corrs. ∠s are ≅
6. $\triangle AEB \cong \triangle BDC$	6. SAS

5.5.D2 CONGRUENT TRIANGLE PROOFS

Objectives:

- Use the triangle congruence theorems – SSS, SAS, ASA, or AAS – to prove triangle congruency
- Draw conclusions from the given information

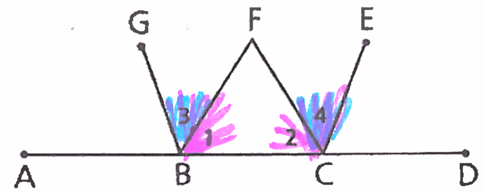
ACCESSING PRIOR KNOWLEDGE



Given: $\overline{WT} \cong \overline{RN}$ & $\overline{VT} \cong \overline{PN}$

Conclusion: $\overline{WV} \cong \overline{RP}$

Reason: Segment Subtraction Property



Given: $\angle 1 \cong \angle 2$ & $\angle 3 \cong \angle 4$

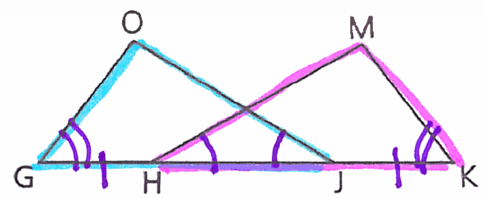
Conclusion: $\angle GBC \cong \angle ECB$

Reason: Angle Addition Property

PROOFS:

1. Given: $\angle GJO \cong \angle KHM$
   $\angle G \cong \angle K$
   $\overline{GH} \cong \overline{KJ}$

   Prove: $\triangle GOJ \cong \triangle KMH$



STATEMENTS	REASONS
A 1. $\angle GJO \cong \angle KHM$	1. Given
A 2. $\angle G \cong \angle K$	2. Given
3. $\overline{GH} \cong \overline{KJ}$	3. Given
4. $\overline{HJ} \cong \overline{HJ}$	4. Reflexive
S 5. $\overline{GJ} \cong \overline{KH}$	5. Segment Add. Prop.
6. $\triangle GOJ \cong \triangle KMH$	6. ASA

HELPFUL HINTS W/OVERLAPPING TRIANGLES

- Draw the triangles separately.
- Outline the two triangles in different colors.
- Be on the lookout for shared sides or shared angles.

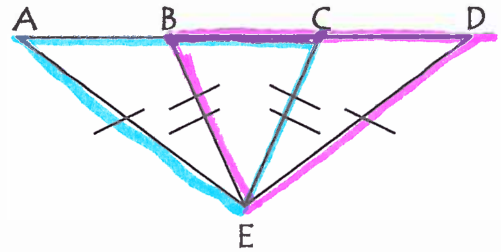
EXAMPLES:

You are given the congruent angles and sides shown by the tick marks. Name the additional pair of congruent sides or angles needed to prove that the triangles are congruent by the specified method.

2. $\triangle AEC \cong \triangle DEB$

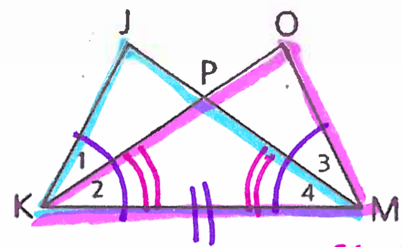
a. SSS $\overline{AC} \cong \overline{DB}$

b. SAS $\angle AEC \cong \angle DEB$



3. Given: $\angle JKM \cong \angle OMK$
$\angle 4 \cong \angle 2$

Prove: $\triangle JKM \cong \triangle OMK$

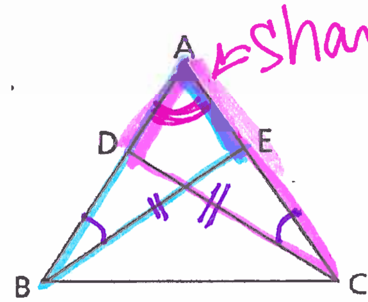


shared side

STATEMENTS	REASONS
A 1. $\angle$ ~~DBC~~ JKM $\cong \angle$ ~~ECB~~ OMK	1. Given
A 2. $\angle 1 \cong \angle 2$	2. Given
S 3. $\overline{KM} \cong \overline{MK}$	3. Reflexive
4. $\triangle JKM \cong \triangle OMK$	4. ASA

4. Given: $\angle ABE \cong \angle ACD$
$\overline{BE} \cong \overline{CD}$

Prove: $\triangle ABE \cong \triangle ACD$

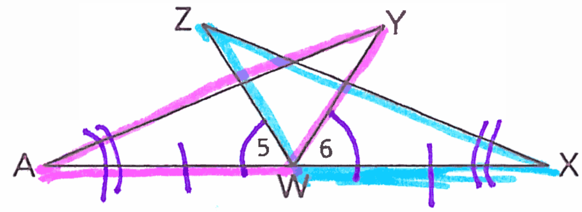


shared angle

STATEMENTS	REASONS
A 1. $\angle ABE \cong \angle ACD$	1. Given
S 2. $\overline{BE} \cong \overline{CD}$	2. Given
A 3. $\angle A \cong \angle A$	3. Reflexive
4. $\triangle ABE \cong \triangle ACD$	4. AAS

4. Given: $\overline{YW}$ bisects $\overline{AX}$
$\angle A \cong \angle X$
$\angle 5 \cong \angle 6$

Prove: $\triangle AWY \cong \triangle XWZ$



STATEMENTS	REASONS
1. $\overline{YW}$ bisects $\overline{AX}$	1. Given
S 2. $\overline{AW} \cong \overline{XW}$	2. Def. of bisects
A 3. $\angle A \cong \angle X$	3. Given
4. $\angle 5 \cong \angle 6$	4. Given
5. $\angle ZWY \cong \angle ZWY$	5. Reflexive
A 6. $\angle AWY \cong \angle XWZ$	6. Angle Add. Prop.
7. $\triangle AWY \cong \triangle XWZ$	7. ASA

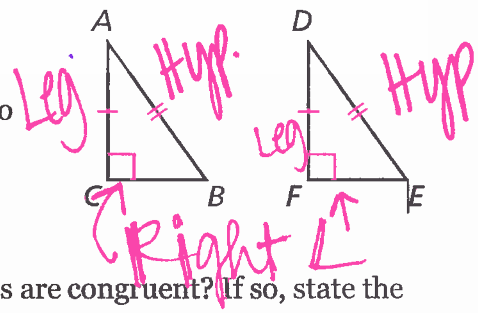
5.6 HL CONGRUENCE THEOREM

Objectives:

- Use the triangle congruence theorems – SSS, SAS, ASA, AAS, or HL – to prove triangle congruency
- Draw conclusions from the given information

❖ **The Hypotenuse-Leg Congruence Theorem**

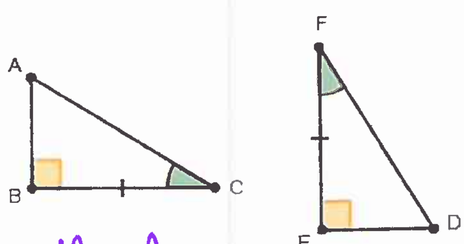
➢ If the hypotenuse and leg of one right triangle are congruent to the hypotenuse and leg of another right triangle, then the triangles are congruent. (HL)



EXAMPLES:

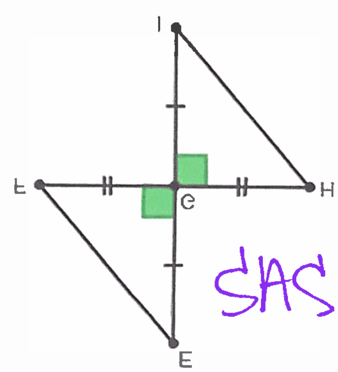
Does the diagram given enough information to show that the triangles are congruent? If so, state the method – SSS, SAS, ASA, AAS, or HL – you would use.

1.



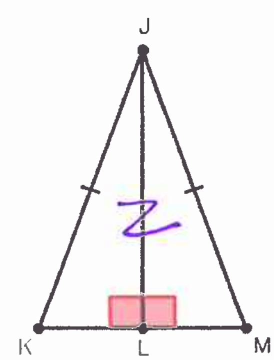
ASA

2.



SAS

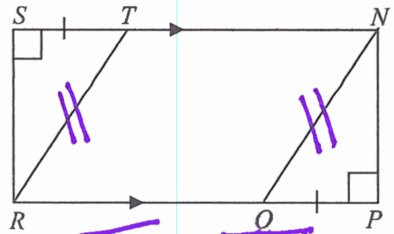
3.



HL

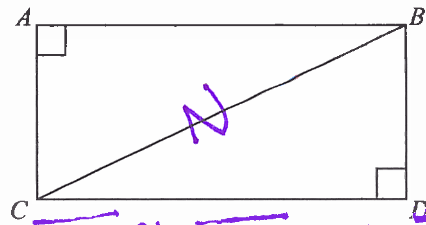
What additional information would you need to prove the triangles congruent by the HL Congruence Theorem?

4. $\triangle STR \cong \triangle PQN$



$\overline{TR} \cong \overline{QN}$

5. $\triangle ABC \cong \triangle DCB$



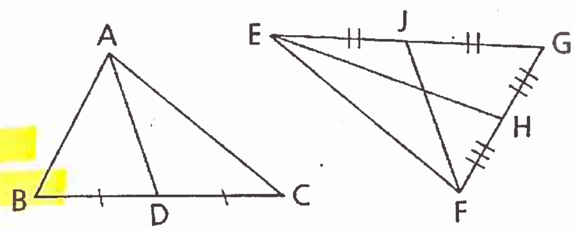
$\overline{AC} \cong \overline{DB}$ OR $\overline{AB} \cong \overline{DC}$

❖ Medians of Triangles

➢ A **MEDIAN** of a triangle is a line segment drawn from any vertex of the triangle to the midpoint of the opposite side.

➢ If a segment is a **MEDIAN**, then it divides into two congruent segments the side to which it is drawn.

- Given: $\overline{AD}$ is a median
- Conclusion: $\overline{BD} \cong \overline{CD}$

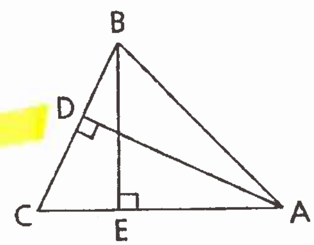


❖ Altitudes of Triangles

➢ An **ALTITUDE** of a triangle is a line segment drawn from any vertex of the triangle to the opposite side, extended if necessary, and perpendicular to that side.

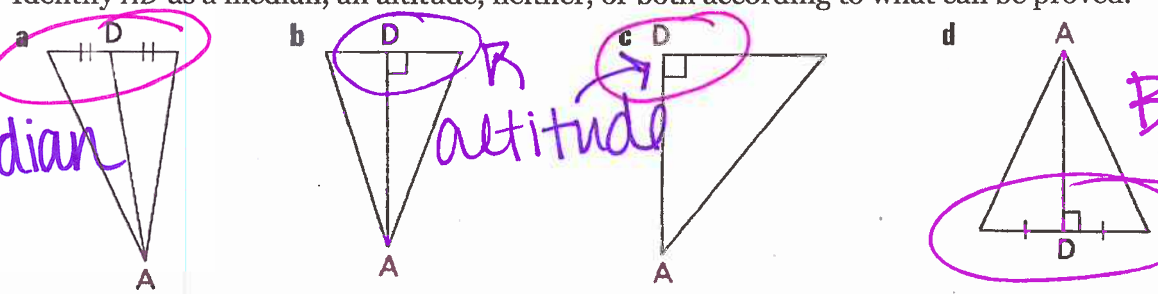
➢ If a segment is an **ALTITUDE**, then it forms right angles with the side to which it is drawn.

- Given: $\overline{AD}$ is an altitude
- Conclusion: $\angle ADC$ & $\angle ADB$ are right angles



EXAMPLES:

6. Identify $\overline{AD}$ as a median, an altitude, neither, or both according to what can be proved.



median

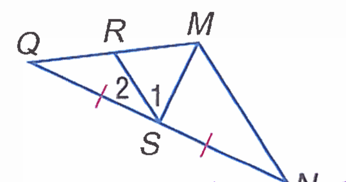
altitude

BOTH

7. Given: $\overline{MS}$ is an altitude AND a median of $\triangle MNQ$

$m\angle 1 = 3x + 11$, $m\angle 2 = 7x + 9$, $QS = 3y - 14$, $SN = 2y + 1$

Find the values of x & y.



alt → ∠MSQ is a Rt ∠

$m\angle 1 + m\angle 2 = m\angle MSQ$

$3x + 11 + 7x + 9 = 90$

$10x + 20 = 90$

$10x = 70$

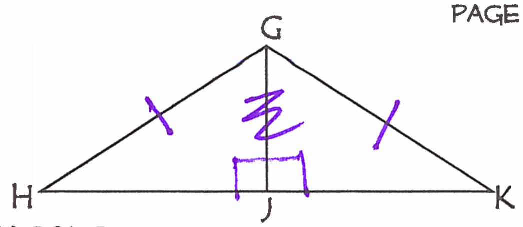
$x = 7$

median → $\overline{QS} \cong \overline{SN}$

$3y - 14 = 2y + 1$

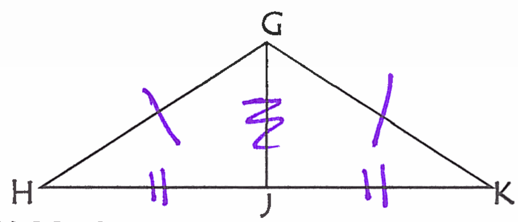
$y = 15$

8. Given: $\overline{GH} \cong \overline{GK}$
$\overline{GJ}$ is an altitude
Prove: $\triangle GHJ \cong \triangle GKJ$



	STATEMENTS	REASONS
HS	1. $\overline{GH} \cong \overline{GK}$	1. Given
	2. $\overline{GJ}$ is an altitude	2. Given
	3. $\angle GJH$ & $\angle GJK$ are Right $\angle$s	3. Def. of altitude
A	4. $\angle GJH \cong \angle GJK$	4. Right $\angle$s are $\cong$
LS	5. $\overline{GJ} \cong \overline{GJ}$	5. Reflexive
	6. $\triangle GHJ \cong \triangle GKJ$	6. HL

9. Given: $\overline{GH} \cong \overline{GK}$
$\overline{GJ}$ is a median
Prove: $\triangle GHJ \cong \triangle GKJ$



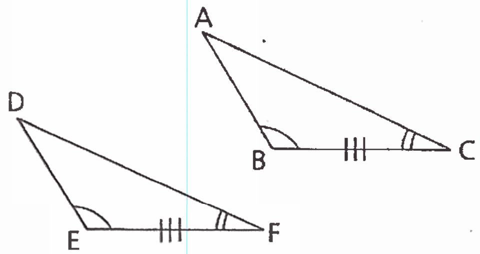
	STATEMENTS	REASONS
S	1. $\overline{GH} \cong \overline{GK}$	1. Given
	2. $\overline{GJ}$ is a median	2. Given
S	3. $\overline{HJ} \cong \overline{KJ}$	3. Def. of median
S	4. $\overline{GJ} \cong \overline{GJ}$	4. Reflexive
	5. $\triangle GHJ \cong \triangle GKJ$	5. SSS

5.7 CPCTC & CIRCLES

Objectives:

- Use the triangle congruence theorems – SSS, SAS, ASA, AAS, or HL – to prove triangle congruency
- Draw conclusions from the given information

We know that $\triangle DEF \cong \triangle ABC$ by ASA. Is $\angle A \cong \angle D$? Is $\overline{DF} \cong \overline{AC}$? Explain your reasoning.



yes, because the $\triangle$s are $\cong$ so all corresponding sides & angles are $\cong$

❖ CPCTC

➢ **Corresponding parts of congruent triangles are congruent.**

- If two triangles are congruent, then each part of one triangle is congruent to the corresponding part of the other triangle.

Must prove $\triangle$s are $\cong$ first!

❖ To use CPCTC in a proof, follow these steps:

➢ Identify two **triangles** in which the segments or angles are corresponding parts.

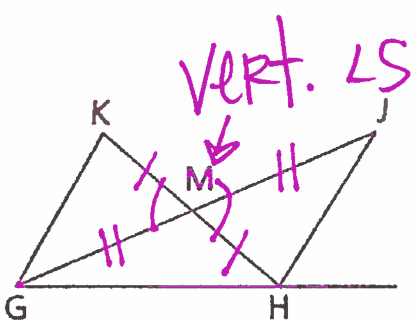
➢ Prove the triangles are congruent.

➢ State the two parts are congruent using CPCTC as the reason.

A PROOF:

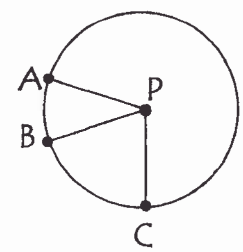
1. Given: $\overline{KH}$ & $\overline{JG}$ bisect each other

   Prove: $\overline{KG} \cong \overline{HJ}$



STATEMENTS	REASONS
1. $\overline{KH}$ & $\overline{JG}$ bisect each other	1. Given
S 2. $\overline{KM} \cong \overline{HM}$	2. Def. of bisects
S 3. $\overline{GM} \cong \overline{JM}$	3. Def. of bisects
4. $\angle KMG$ & $\angle HMJ$ are vert. ∠s	4. Assumed from diagram
A 5. $\angle KMG \cong \angle HMJ$	5. Vert. ∠s are $\cong$
6. $\triangle KMG \cong \triangle HMJ$	6. SAS
7. $\overline{KG} \cong \overline{HJ}$	7. CPCTC

- ❖ Circles
  - A circle is named by its center; this circle is called circle P (or $\odot P$)



  - ➢ Radii
    - Points A, B, and C lie on circle P ($\odot P$)
      - $\overline{PA}$ is called a radius
      - $\overline{PA}, \overline{PB}, \& \overline{PC}$ are called radii
    - Theorem: **All radii of a circle are congruent.**

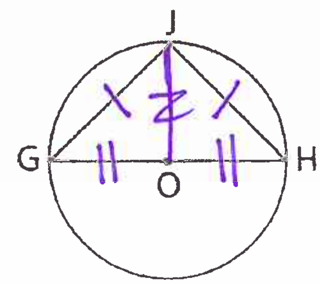
- ❖ Auxiliary Lines
  - ➢ Need there to be line connecting two points? No problem!
    - Auxiliary lines connect two points already in the diagram.
    - Whenever we use an auxiliary line in a proof, we must be able to show that such a line can be drawn & then justify it with the following postulate: Two points determine a line.

A PROOF:

2. Given: $\overline{GJ} \cong \overline{HJ}$

$\odot O$

Prove: $\angle G \cong \angle H$



STATEMENTS	REASONS
S 1. $\overline{GJ} \cong \overline{HJ}$	1. Given
2. $\odot O$	2. Given
S 3. $\overline{GO} \cong \overline{HO}$	3. All Radii are $\cong$
4. DRAW $\overline{JO}$	4. Two pts. det. a line
S 5. $\overline{JO} \cong \overline{JO}$	5. Reflexive
6. $\triangle GOJ \cong \triangle HOJ$	6. SSS
7. $\angle G \cong \angle H$	7. CPCTC

5.8 ISOSCELES TRIANGLES IN PROOFS

Objectives:

- Use the triangle congruence theorems – SSS, SAS, ASA, AAS, or HL – to prove triangle congruency
- Draw conclusions from the given information

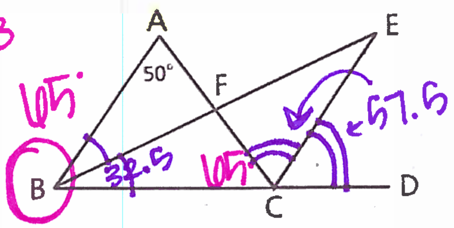
❖ Isosceles Triangles
  - ➢ If at least two sides of a triangle are congruent, then the triangle is an isosceles triangle.

❖ Isosceles Triangle Theorems
  - ➢ Isosceles Triangle Base Angle Theorem
    - ▪ If two sides of a triangle are congruent, then the angles opposite these sides are congruent.
      If △, then △.
  - ➢ Isosceles Triangle Base Angle Converse Theorem
    - ▪ If two angles of a triangle are congruent, then the sides opposite these angles are congruent.
      If △, then △.

EXAMPLES:

1. Given: $\triangle ABC$ is isosceles w/base $\overline{BC}$, $\overrightarrow{BE}$ bisects $\angle ABC$; $\overrightarrow{CE}$ bisects $\angle FCD$

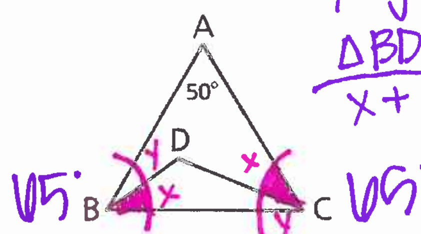
$\angle B \cong \angle ACB$



Find $m\angle ABF$, $m\angle BCE$, & $m\angle E$ = 25°

32.5° 122.5°

2. Given: $\overline{AB} \cong \overline{AC}$
$\angle DBC \cong \angle DCA$



$x+y=65°$

$\triangle BDC$

$x+y+m\angle D=180°$

$65+m\angle D=180$

$m\angle D=115$

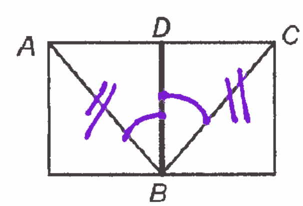
Find $m\angle BDC$

115°

A PROOF:

3. Given: $\angle ABD \cong \angle CBD$
$\triangle ABC$ is isosceles with base $\overline{AC}$

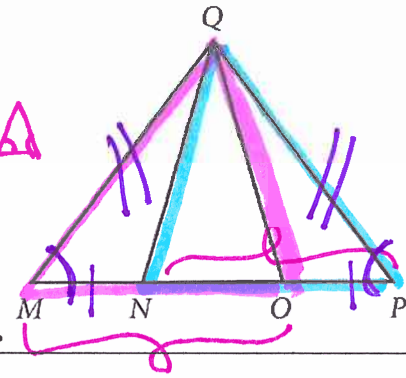
Prove: $\triangle ABD \cong \triangle CBD$



STATEMENTS	REASONS
A 1. $\angle ABD \cong \angle CBD$	1. Given
2. $\triangle ABC$ is isosceles with base $\overline{AC}$	2. Given
S 3. $\overline{AB} \cong \overline{CB}$	3. Def. of isosceles △
S 4. $\overline{DB} \cong \overline{DB}$	4. Reflexive
5. $\triangle ABD \cong \triangle CBD$	5. SAS

4. Given: $\overline{QM} \cong \overline{QP}$ → $\angle M \cong \angle P$
$\overline{MN} \cong \overline{PO}$
If $\triangle$, then $\triangle$

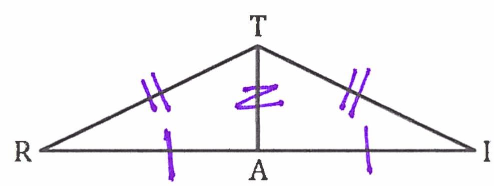
Prove: $\angle QNP \cong \angle QOM$



STATEMENTS	REASONS
S 1. $\overline{QM} \cong \overline{QP}$	1. Given
2. $\overline{MN} \cong \overline{PO}$	2. Given
3. $\overline{NO} \cong \overline{NO}$	3. Reflexive
S 4. $\overline{MO} \cong \overline{PN}$	4. Segment Add. Prop.
A 5. $\angle M \cong \angle P$	5. If $\triangle$, then $\triangle$.
6. $\triangle QNP \cong \triangle QOM$	6. SAS
7. $\angle QNP \cong \angle QOM$	7. CPCTC

5. Given: $\overline{TA}$ is a median of $\triangle RIT$
$\triangle RIT$ is isosceles with base $\overline{RI}$

Prove: $\triangle TRA \cong \triangle TIA$



STATEMENTS	REASONS
1. $\overline{TA}$ is a median of $\triangle RIT$	1. Given
S 2. $\overline{RA} \cong \overline{IA}$	2. Def. of median
3. $\triangle RIT$ is isosceles with base $\overline{RI}$	3. Given
S 4. $\overline{RT} \cong \overline{IT}$	4. Def. of isosceles $\triangle$
S 5. $\overline{TA} \cong \overline{TA}$	5. Reflexive
6. $\triangle TRA \cong \triangle TIA$	6. SSS