Chapter 6 – Quadrilaterals

ATTACH 6.1 – INVESTIGATING PARALLELOGRAMS

6.1 – Parallelograms on the Coordinate Plane

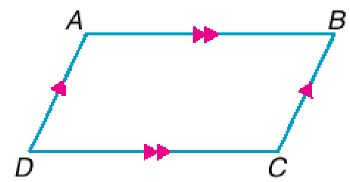
Objectives:

- Show that a quadrilateral is a parallelogram on the coordinate plane
- Identify and verify parallelograms

❖ Parallelograms

➢ A parallelogram is a quadrilateral with both pairs of opposite sides parallel.

▪ $\overline{AB} \parallel \overline{DC}$ & $\overline{AD} \parallel \overline{BC}$



DISTANCE FORMULA:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

MIDPOINT FORMULA:

$$(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

SLOPE FORMULA:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

❖ Proving Parallelograms on the Coordinate Plane

➢ Show that both pairs of opposite sides are parallel.

▪ How using coordinates?

➢ Show that both pairs of opposite sides are congruent.

▪ How using coordinates?

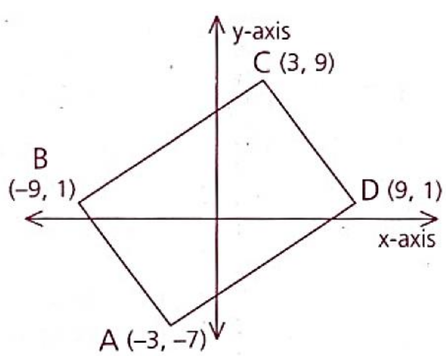
➢ Show that ONE pair of opposite sides is both parallel AND congruent.

➢ Show that the diagonals bisect each other.

▪ How using coordinates?

EXAMPLE:

1. Show that *ABCD* is a parallelogram.



6.2 – Properties of Parallelograms

<u>Objectives:</u>

- Know and prove the properties of parallelograms
- Apply the properties of parallelograms to find side lengths, segment lengths, and angle measures

❖ Parallelograms

➢ A <u>parallelogram</u> is a quadrilateral with both pairs of opposite sides parallel.

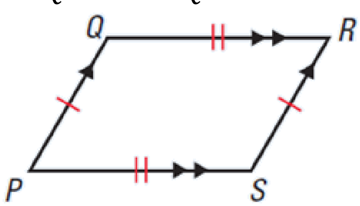
▪ $\overline{AB} \parallel \overline{DC}$ & $\overline{AD} \parallel \overline{BC}$

❖ Properties of Parallelograms

➢ If a quadrilateral is a parallelogram, then…

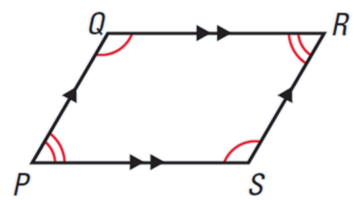
▪ Both pairs of opposite sides are congruent

- $\overline{PQ} \cong \overline{SR}$ & $\overline{QR} \cong \overline{PS}$



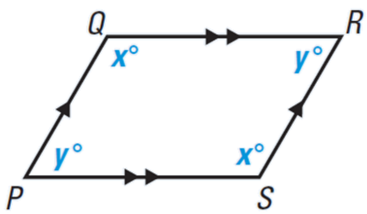
▪ Both pairs of opposite angles are congruent

- $\angle Q \cong \angle S$ & $\angle P \cong \angle R$



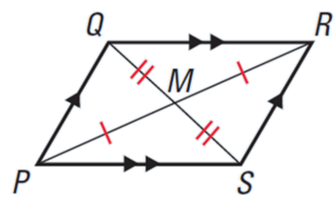
▪ Consecutive angles are supplementary

- $x° + y° = 180°$



▪ The diagonals bisect each other

- $\overline{QM} \cong \overline{MS}$ & $\overline{PM} \cong \overline{MR}$

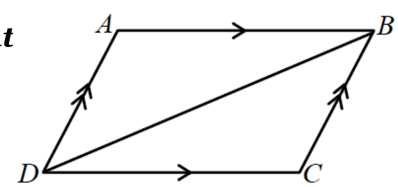


THE SUM OF ALL FOUR ANGLES IN ANY QUADRILATERAL IS 360°.

❖ ***Prove that both pairs of opposite angles are congruent***

Given: *ABCD* is a parallelogram

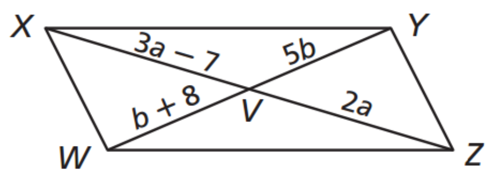
Prove: $\angle A \cong \angle C$ & $\angle ABC \cong \angle ADC$



STATEMENTS	REASONS
1. *ABCD* is a parallelogram	1. Given
2. $\overline{AB} \parallel \overline{DC}$ & $\overline{AD} \parallel \overline{BC}$	2. Definition of parallelogram

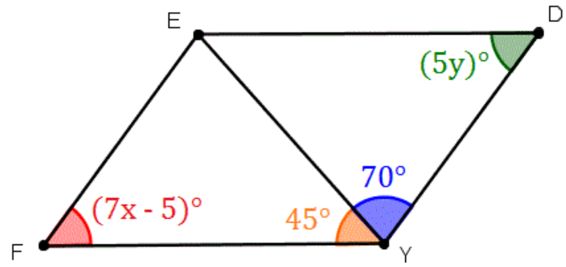
EXAMPLES: USING THE PROPERTIES OF PARALLELOGRAMS

1. *WXYZ* is a parallelogram. Find each measure.
   a. *WV*
   b. *YW*
   c. *ZV*
   d. *ZX*



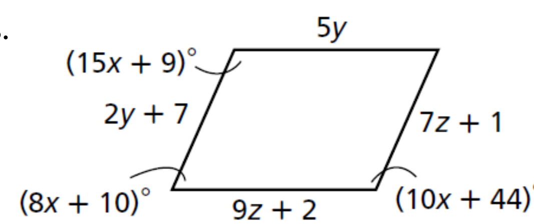
The diagonals bisect each other.

2. *FEDY* is a parallelogram. Find the value of each variable.



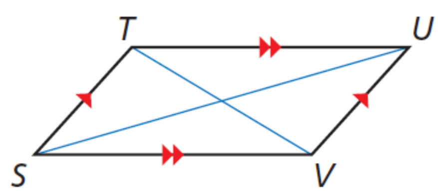
Opposite angles are congruent; consecutive angles are supplementary.

3. For the given parallelogram, find the value of the variables.



Opposite sides and angles are congruent; consecutive angles are supplementary.

4. In *STUV*, $m\angle TSU = 32°$, $m\angle USV = x^2$, $m\angle TUV = 12x$, and $\angle TUV$ is an acute angle. Find the value of x (that makes sense) and $m\angle USV$.



Opposite angles are congruent.
Remember, since both pairs of opposite sides are parallel, those "parallel/angle" pair relationships exist as well.

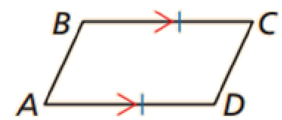
6.3 – PROVING QUADRILATERALS ARE PARALLELOGRAMS

Objectives:

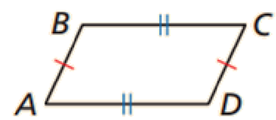
- Prove that a quadrilateral is a parallelogram
- Identify and verify parallelograms

❖ Conditions for Parallelograms

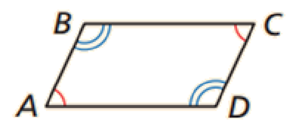
- If both pairs of opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram. (Definition)
- If one pair of opposite sides of a quadrilateral is parallel and congruent, then the quadrilateral is a parallelogram.
  - If $\overline{BC} \parallel \overline{AD}$ and $\overline{BC} \cong \overline{AD}$, then $ABCD$ is a parallelogram.



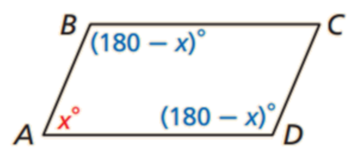
- If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
  - If $\overline{BC} \cong \overline{AD}$ and $\overline{AB} \cong \overline{CD}$, then $ABCD$ is a parallelogram.



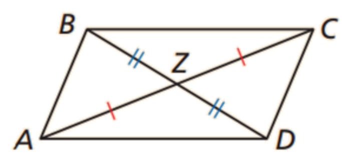
- If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
  - If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $ABCD$ is a parallelogram.



- If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral is a parallelogram
  - If $\angle A$ is supplementary to $\angle B$ and $\angle A$ is supplementary to $\angle D$, then $ABCD$ is a parallelogram.

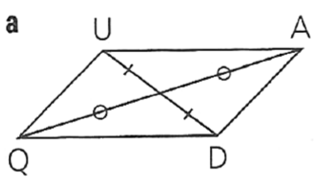


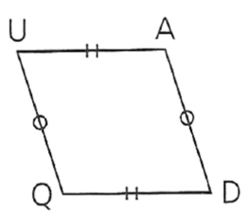
- If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
  - If $\overline{AZ} \cong \overline{ZC}$ and $\overline{BZ} \cong \overline{ZD}$, then $ABCD$ is a parallelogram.

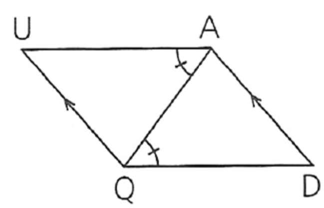


EXAMPLES: IDENTIFYING PARALLELOGRAMS

1. For each quadrilateral $QUAD$, state the property or definition that proves that $QUAD$ is a parallelogram.

a


c


e


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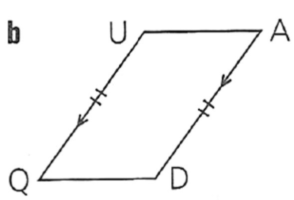

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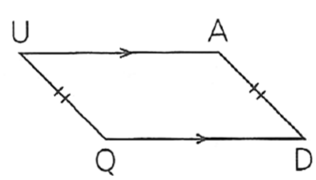
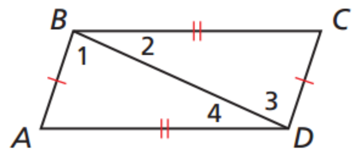


PROVE THIS PROPERTY:

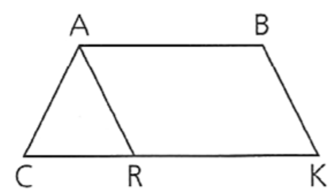
- If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

2. Given: $\overline{AB} \cong \overline{CD}$
   $\overline{BC} \cong \overline{DA}$
   Prove: $ABCD$ is a parallelogram



STATEMENTS	REASONS

3. Given: $\triangle CAR$ is isosceles w/base $\overline{CR}$
   $\overline{AC} \cong \overline{BK}$
   $\angle C \cong \angle K$
   Prove: $BARK$ is a parallelogram

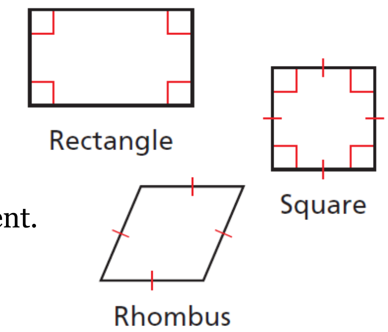


STATEMENTS	REASONS

6.4 – Rectangles, Rhombi, & Squares

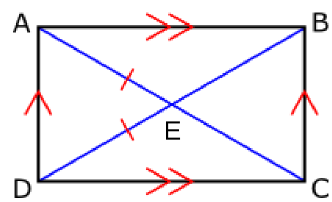
Objectives:

- Apply the properties of rectangles, rhombi, and squares to find side lengths, segment lengths, and angle measures
- Find areas of rectangles, rhombi, and squares



❖ Special Parallelograms
  - ➢ Rectangle
    - A parallelogram in which at least one angle is a right angle.
  - ➢ Rhombus
    - A parallelogram in which at least two consecutive sides are congruent.
  - ➢ Square
    - A parallelogram that is BOTH a rectangle and a rhombus.

❖ Properties of Rectangles
  - ➢ **All properties of a parallelogram apply!**
  - ➢ All angles are right angles
    - $\angle DAB, \angle ABC, \angle BCD, \angle CDA$ are right angles
  - ➢ The diagonals are congruent
    - $\overline{AC} \cong \overline{BC}$

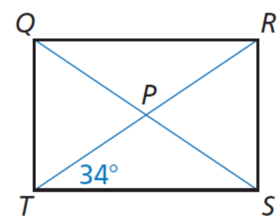


$$A = bh$$

1. Given: $QRST$ is a rectangle, $m\angle PTS = 34°$, $QS = 10$

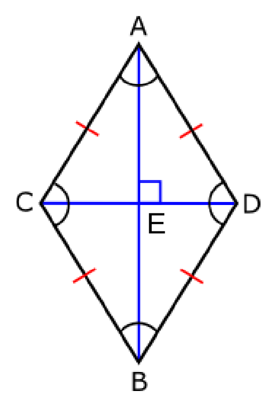
$m\angle QTR =$ _______ $m\angle QRT =$ _______ $m\angle RPS =$ _______

$QP =$ _______ $RT =$ _______ $RP =$ _______



❖ Properties of Rhombuses
  - ➢ **All properties of a parallelogram apply!**
  - ➢ All sides are congruent—that is, a rhombus is equilateral
    - $\overline{AC} \cong \overline{BC} \cong \overline{AD} \cong \overline{BD}$
  - ➢ The diagonals bisect the vertex angles
    - $\overline{AB}$ bisects $\angle CAD$ & $\angle CBD$
    - $\overline{CD}$ bisects $\angle ACB$ & $\angle ADB$
  - ➢ The diagonals are perpendicular bisectors of each other
    - $\overline{AB} \perp \overline{CD}$
    - $\overline{AB}$ & $\overline{CD}$ bisect each other
  - ➢ The diagonals divide the rhombus into four congruent right triangles

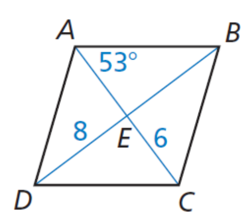
$$A = \frac{1}{2}d_1 d_2$$



2. Given: $ABCD$ is a rhombus, $m\angle BAC = 53°$, $DE = 8$, $EC = 6$

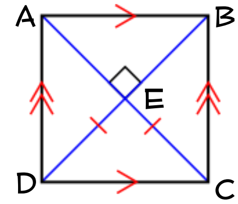
$m\angle DAC =$ _______ $m\angle AED =$ _______ $m\angle ADC =$ _______

$DB =$ _______ $AE =$ _______ $CD =$ _______



❖ Properties of Squares
- **All properties of a parallelogram apply!**
- **All the properties of a rectangle apply!**
- **All the properties of a rhombus apply!**
- The diagonals form four isosceles right triangles.

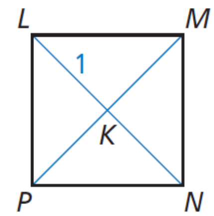
$A = s^2$



3. Given: *LMNP* is a square, *LK* = 1

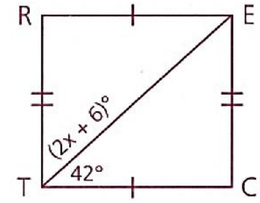
$m\angle MKN$ = ______ $m\angle LMK$ = ______ $m\angle LPK$ = ______

KN = ______ *LN* = ______ *MP* = ______

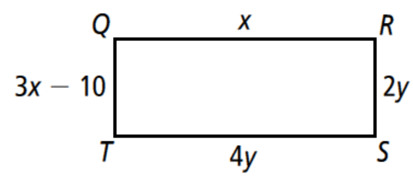


EXAMPLES

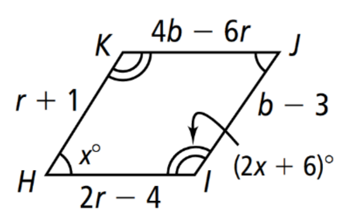
4. In order for *RECT* to be a rectangle, what must the value of *x* be?



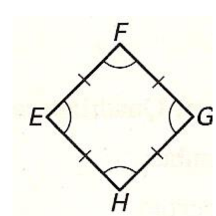
5. Given: Rectangle *QRST*
Set up and solve a system of equations to find the value of the variables.



6. Given: Rhombus *HIJK*
   a. Find the value of the variables: *b, r,* & *x*.
   b. Find $m\angle J$ & $m\angle K$.



7. Given: *EFGH* is a square with a perimeter of 36, $EH = x + 6$, $m\angle F = 2y - 4$
Find the values of x & y.



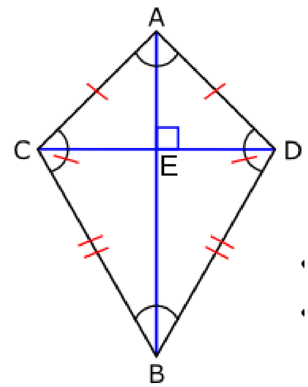
6.5 – Kites & Trapezoids

Objectives:

- Apply the properties of kites and trapezoids to find side lengths, segment lengths, and angle measures
- Find areas of kites and trapezoids

❖ Kites
   ➢ A quadrilateral with two pairs of consecutive congruent sides with opposite sides that are NOT congruent.

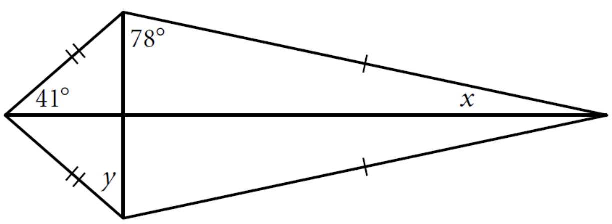
❖ Properties of Kites
   ➢ The diagonals are perpendicular to each other
      ▪ $\overline{AB} \perp \overline{CD}$
   ➢ One diagonal is the perpendicular bisector of the other
      ▪ $\overline{AB}$ bisects $\overline{CD}$
   ➢ One of the diagonals bisects a pair of opposite angles
      ▪ $\overline{AB}$ bisects $\angle CAD$ & $\angle CBD$
   ➢ One pair of opposite angles are congruent
      ▪ $\angle ACB \cong \angle ADB$



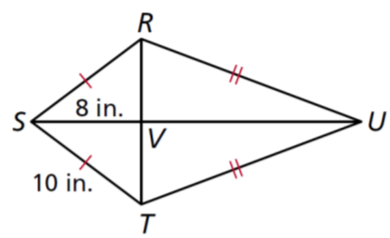
$$A = \frac{1}{2} d_1 d_2$$

EXAMPLES: USING THE PROPERTIES OF KITES

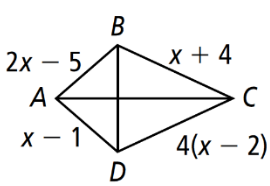
1. Find the values of x and y in the kite shown.



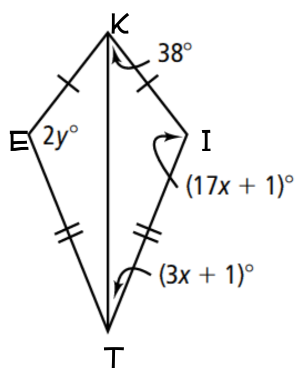
2. Find RV in the kite shown.



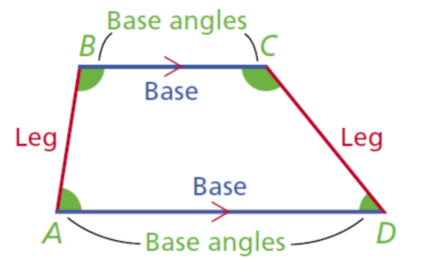
3. Given: Kite $ABCD$
   Find the value of x.



4. Given: Kite $KITE$
   Find the values of x and y in the kite shown.

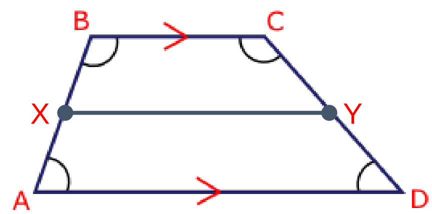


- Trapezoids
  - A quadrilateral with exactly one pair of parallel sides.
    - $\overline{BC} \parallel \overline{AD}$
  - Properties of Trapezoids
    - Consecutive non-base angles are supplementary.
      - $\angle A$ is supplementary to $\angle B$
      - $\angle C$ is supplementary to $\angle D$

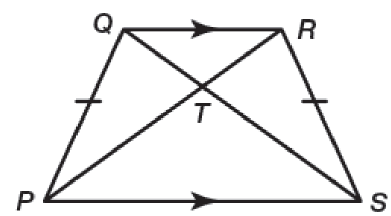


- Midsegment of a Trapezoid
  - Parallel to the bases
  - Length is half the sum of the lengths of the bases:

$$XY = \frac{1}{2}(AD + BC)$$



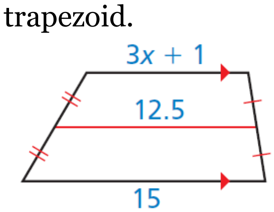
- Isosceles Trapezoids
  - A trapezoid with congruent non-parallel sides (legs)
    - $\overline{QP} \cong \overline{RS}$
  - Properties of Isosceles Trapezoids
    - All properties of a trapezoid apply
    - The base angles are congruent.
      - $\angle QPS \cong \angle RSP$
      - $\angle PQR \cong \angle SRQ$
    - The diagonals are congruent.
      - $\overline{QS} \cong \overline{RP}$



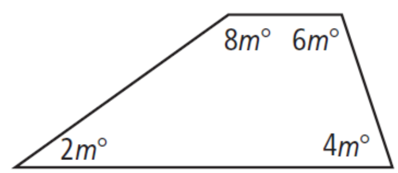
$$\boldsymbol{A = \frac{1}{2}h(b_1 + b_2)}$$

EXAMPLES: USING THE PROPERTIES OF TRAPEZOIDS

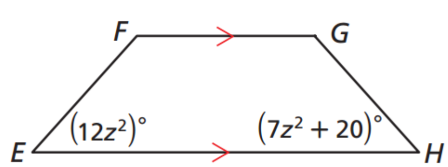
5. Find the value of x in the trapezoid.



6. Is the quadrilateral a trapezoid? Explain your reasoning.



7. Given: Isosceles trapezoid $EFGH$
   a. Find the value of z.
   b. Find $m\angle G$.



6.6.D1 – PROVING SPECIAL QUADRILATERALS IN THE COORDINATE PLANE

Objective:

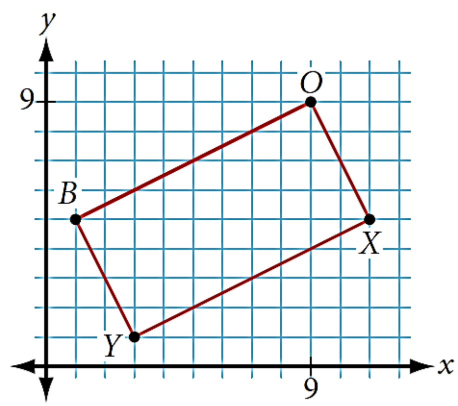
- Use the distance, slope, and midpoint formulas to prove that a figure graphed in the coordinate plane is special quadrilateral: rectangle, rhombus, square, kite, or trapezoid

	Distance Formula	Midpoint Formula	Slope Formula
Formula	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	$m = \frac{y_2 - y_1}{x_2 - x_1}$
When to Use It	To determine whether • sides are congruent • diagonals are congruent	To determine • the coordinates of the midpoint of a side • whether diagonals bisect each other	To determine whether • opposite sides are parallel • diagonals are perpendicular • sides are perpendicular

EXAMPLES: QUADRILATERALS IN THE COORDINATE PLANE

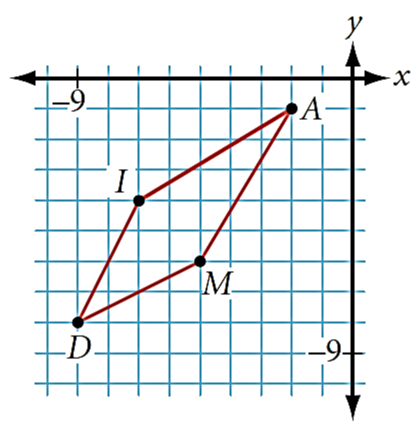
1. Show that *BOXY* is a rectangle.

   (Remember, you must first show that *BOXY* is a parallelogram.)



Use the diagonals to determine whether a parallelogram with the given vertices is a rectangle, rhombus, or square. Give all the names that apply.

2. $A(-2,-1), I(-7,-4), D(-9,-8), M(-5,-6)$

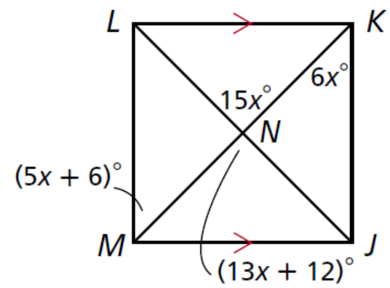


6.6.D2 – PROOFS WITH SPECIAL QUADRILATERALS

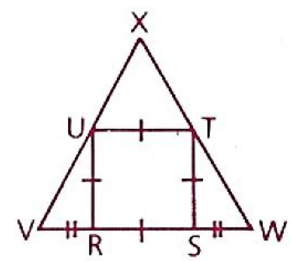
Objective:

- Prove that a quadrilateral is a special quadrilateral: rectangle, rhombus, square, kite, or trapezoid

1. Write and solve an equation to find the value of x.
   a. Is *JKLM* a parallelogram? Explain.
   b. Is *JKLM* a rectangle? Explain.
   c. Is *JKLM* a rhombus? Explain.



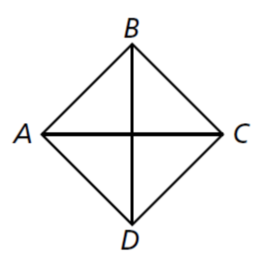
2. Given: *RSTU* is a square; $\overline{VR} \cong \overline{SW}$
   a. Is *VWTU* an isosceles trapezoid?
   b. Is $\triangle VWX$ an isosceles triangle?
   c. Is $\triangle UTX$ an isosceles triangle?



3. Determine if the conclusion is valid and explain your reasoning. If the conclusion is NOT valid, tell what additional information is needed to make it valid.

Given: $\overline{AB} \cong \overline{CD}$
$\overline{BC} \cong \overline{AD}$
$\overline{AD} \perp \overline{DC}$
$\overline{AC} \perp \overline{BD}$

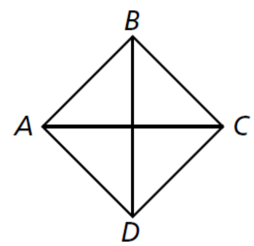
Conclusion: *ABCD* is a square



4. Determine if the conclusion is valid and explain your reasoning. If the conclusion is NOT valid, tell what additional information is needed to make it valid.

Given: $\overline{AB} \cong \overline{BC}$

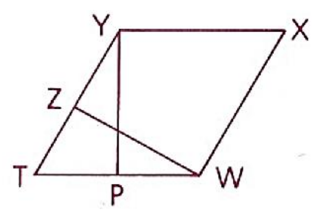
Conclusion: *ABCD* is a rhombus



5. In rhombus MATH, the coordinates of the endpoints of the diagonal $\overline{MT}$ are $M(0,-1)$ and $T(4,7)$. Write an equation of the line that contains diagonal $\overline{AH}$. Using the given information, explain how you know that your line contains diagonal $\overline{AH}$.

6. Given: *YTWX* is a parallelogram
$\overline{YP} \perp \overline{TW}$
$\overline{ZW} \perp \overline{TY}$
$\overline{TP} \cong \overline{TZ}$

Prove: *TWXY* is a rhombus



STATEMENTS	REASONS
1. *YTWX* is a parallelogram	Given
2. $\overline{YP} \perp \overline{TW}$	Given
3.	
4. $\overline{ZW} \perp \overline{TY}$	Given
5.	
6.	
7. $\overline{TP} \cong \overline{TZ}$	Given
8.	
9. $\triangle TYP \cong \triangle TWZ$	
10.	CPCTC
11. *TWXY* is a rhombus	Definition of rhombus - *A parallelogram with at least one pair of consecutive sides congruent*