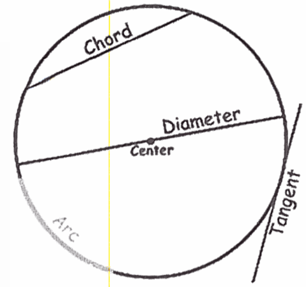
Name: ______________________________

CHAPTER 9: CIRCLES

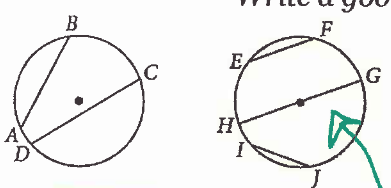
VOCABULARY: PARTS OF A CIRCLE

- Basic Properties & Definitions of Circles
  - A circle is the set of all points in a plane that are a given distance from a given point in the plane
    - The given point is the center of the circle, and the given distance is the radius.
    - A segment that joins the center to a point on the circle is also called a radius.
      - All radii of a circle are congruent.
      - Two circles are congruent if they have congruent radii.

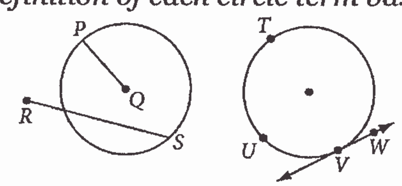


- Chords & Diameters

Write a good definition of each circle term based on your observations.



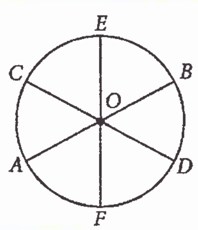
Chords:
$\overline{AB}$, $\overline{CD}$, $\overline{EF}$, $\overline{GH}$, and $\overline{IJ}$



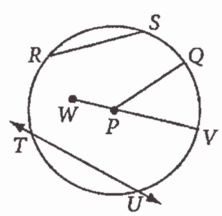
Not chords:
$\overline{PQ}$, $\overline{RS}$, $\overline{TU}$, and $\overleftrightarrow{VW}$

- **CHORDS**
  - Definition:

A segment whose endpoints are ON the circle



Diameters:
$\overline{AB}$, $\overline{CD}$, and $\overline{EF}$



Not diameters:
$\overline{PQ}$, $\overline{RS}$, $\overleftrightarrow{TU}$, and $\overline{VW}$

- **DIAMETER**
  - Definition:

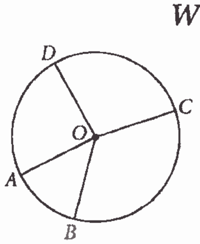
A segment that passes thru the center of the circle

- **Follow-Up Question:**
  - Can a chord of a circle also be a diameter or the circle? Explain why or why not.

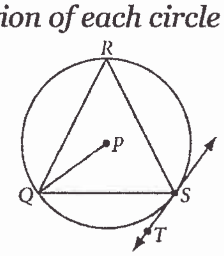
yes HG is a chord & a diameter

- Angles in a Circle
  - Central Angle

Write a good definition of each circle term based on your observations.



$\angle DOC$ intercepts arc $\widehat{DC}$. $\angle AOB$, $\angle BOC$, $\angle COD$, $\angle DOA$, and $\angle DOB$ are central angles of circle O.



$\angle PQR$, $\angle PQS$, $\angle RST$, $\angle QST$, and $\angle QSR$ are not central angles of circle P.

- **CENTRAL ANGLE**
  - Definition:

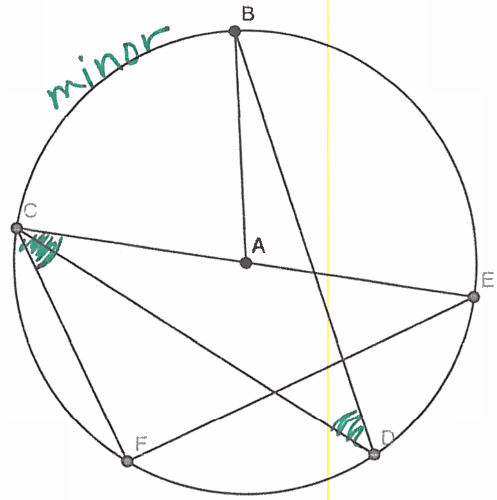
An angle whose vertex is @ the center of the circle

❖ Angles in a Circle (continued)

Write a good definition of the circle term based on your observations.

➢ **INSCRIBED ANGLES**

- Examples: $\angle FCE$, $\angle CDB$, $\angle CEF$, $\angle F$
- Non-examples: $\angle BAC$, $\angle ABD$
- Definition: An angle whose vertex is ON the circle



➢ **MINOR ARCS**

- Examples: $\widehat{BC}$, $\widehat{ED}$ & $\widehat{FC}$
- Non-Examples: $\widehat{CFE}$ & $\widehat{BEF}$
- Definition: A small arc

➢ **MAJOR ARCS**

- Examples: $\widehat{CFE}$ & $\widehat{BEF}$
- Non-Examples: $\widehat{BC}$, $\widehat{ED}$ & $\widehat{FC}$
- Definition: An arc that is larger than a minor arc

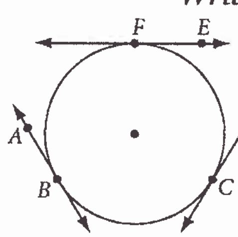
➢ $\angle CDB$ & $\angle CAB$ both **INTERCEPT** $\widehat{BC}$.

Explain what you think it means for an angle to **intercept** an arc.

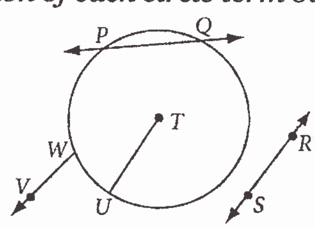
the arc formed when the angle intersects the circle

❖ Lines in a Circle

Write a good definition of each circle term based on your observations.



Tangents:
$\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, and $\overleftrightarrow{EF}$



Not tangents:
$\overleftrightarrow{PQ}$, $\overleftrightarrow{RS}$, $\overline{TU}$, and $\overline{VW}$

➢ **TANGENT**

- Definition: A line that touches the circle

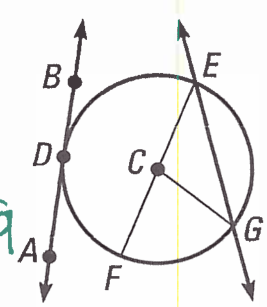
➢ **SECANT**: $\overleftrightarrow{PQ}$

- Definition: A line that passes thru the circle

Given: $\odot Q$

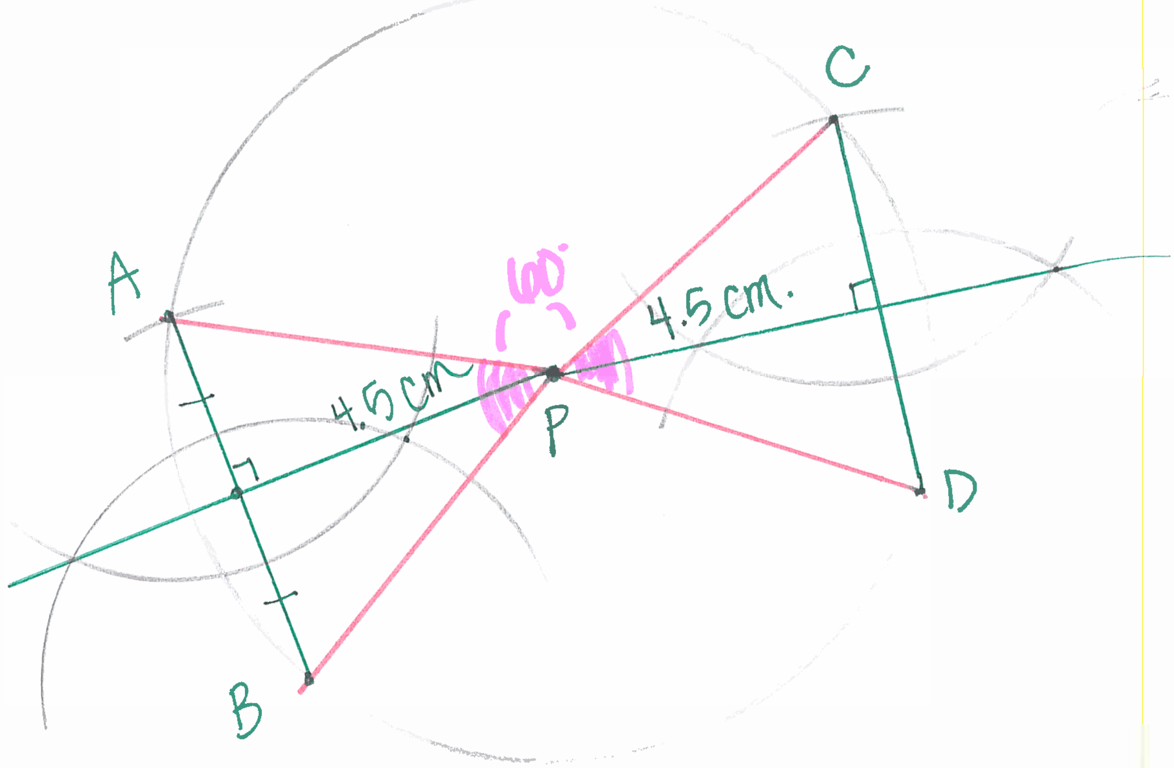
Identify one example of each of the following parts of a circle. Use proper geometric notation.

1. Radius: $\overline{CG}$
2. Diameter: $\overline{EF}$
3. Chord: $\overline{EG}$
4. Central angle: $\angle ECG$
5. Inscribed angle: $\angle FEG$
6. Minor arc: $\widehat{EG}$
7. Major arc: $\widehat{DEG}$
8. A pair of angles that intercept the same arc: $\angle FEG$ & $\angle FCG$
9. Tangent: $\overleftrightarrow{AB}$
10. Secant: $\overleftrightarrow{EG}$



Investigating Radii, Chords, & Central Angles

LESSONS 9.1 & 9.2



- Investigation: Radius-Chord Relationships (Lesson 9.1)
  - Use a compass to construct a circle of any radius, and identify the center as point P.
    - Draw a pair of **congruent chords**, and label one $\overline{AB}$ and the other $\overline{CD}$.
    - Construct the **perpendicular bisector** of $\overline{AB}$.
      - What do you notice about the perpendicular bisector of $\overline{AB}$?
    - Construct the **perpendicular bisector** of $\overline{CD}$.
      - What do you notice about the perpendicular bisector of $\overline{CD}$?

It passes thru the center of the circle.

    - Measure and record the distance from the center of the circle to the **midpoint** of $\overline{AB}$.
    - Measure and record the distance from the center of the circle to the **midpoint** of $\overline{CD}$.
      - What do you notice about these measurements?

they are the same

- Investigation: Congruent Chords & Central Angles (Lesson 9.2)
  - Construct and label four radii: $\overline{PA}, \overline{PB}, \overline{PC}, \& \overline{PD}$.
    - What is true for all **radii** of a circle?

all radii are $\cong$

  - With a protractor, measure $\angle APB$ & $\angle CPD$.
    - These are **central angles** because their vertex is at the center of the circle. These angles also intercept congruent chords. What do you notice about their measurements?

they are $\cong$

Closure:

Using your observations from the investigation, complete the following conjectures and problems 1 – 5.

★ If a radius is perpendicular to a chord, then it __bisects__ the chord.

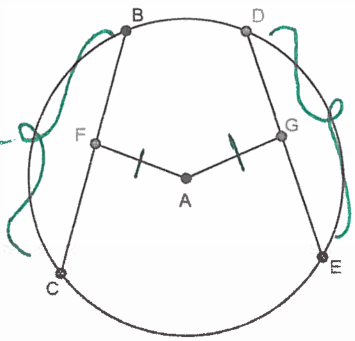
★ The perpendicular bisector of a chord passes through __the center of the circle__

★ If two chords of the same circle are congruent, then they are __the same distance__ from the center of the circle.

★ If two chords of a circle are congruent, then the corresponding central angles are __≅__.

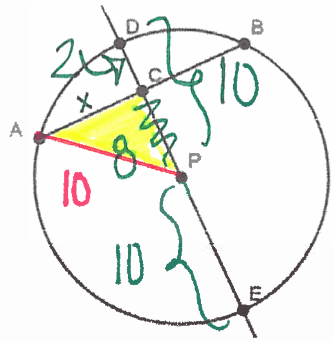
1. Given: $\odot A$, $\overline{AF} \cong \overline{AG}$, $BC = 22$

   Find: DE = 22



2. Given: $\odot P$, $\overline{AB} \perp \overline{DE}$, radius = 10

   Find: AB if $DC = 2$



$x^2 + 8^2 = 10^2$

$x^2 + 64 = 100$

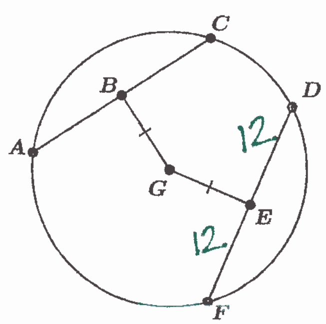
$x^2 = 36$

$x = 6 = AC$

$AB = 2 \cdot 6 = 12$

3. Given: $\odot G$, $\overline{AC} \perp \overline{BG}$, $\overline{DF} \perp \overline{EG}$, $EF = 12$

   Find: AC

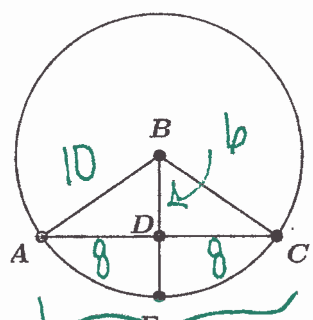


$\overline{AC} \cong \overline{DF}$

$AC = 24$

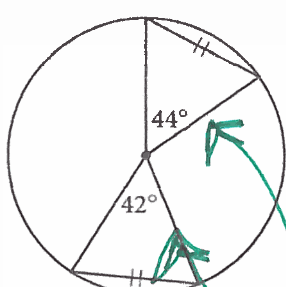
4. Given: $\odot B$, $AB = 10$, $AC = 16$

   Find: DE



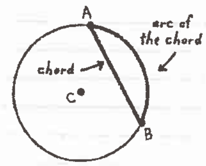
$DE = 10 - 6 = 4$

5. What is wrong with the picture?



If the chords are ≅, then their corrs. central angles are also ≅. These aren't.

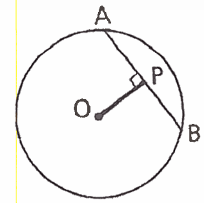
9.1 ~ Radii and Chords



OBJECTIVES:

- Identify the characteristics of the circle
- Identify and describe relationships among radii and chords

❖ RADIUS-CHORD RELATIONSHIPS



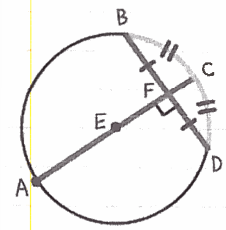
- The distance from the center of a circle to a chord is the measure of the perpendicular segment from the center to the chord.

❖ RADIUS-CHORD THEOREMS

- If a radius (or diameter) is perpendicular to a chord, then it bisects the chord and its arc.
- If a radius of a circle bisects a chord that is not a diameter, then it is perpendicular to that chord.
- The perpendicular bisector of a chord passes through the center of the circle.

If $\overline{AC} \perp \overline{BD}$, then $\overline{BF} \cong \overline{FD}$ and $\overline{BC} \cong \overline{CD}$.

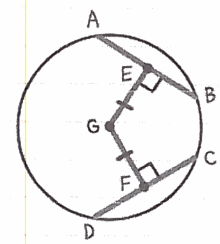
If $\overline{AC}$ is a perpendicular bisector of $\overline{BD}$, then $\overline{AC}$ is a diameter of ⊙E.



❖ THE EQUIDISTANT CHORD THEOREM

- If two chords of the same circle or congruent circles are congruent, then they are equidistant from the center of the circle.

If $\overline{AB} \cong \overline{CD}$, then EG = FG.



EXAMPLES:

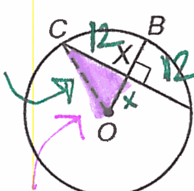
1. Circle O has a radius of 13 inches. Radius $\overline{OB}$ is perpendicular to chord $\overline{CD}$, which is 24 inches long. Find OX.

$x^2+12^2=13^2$
$x^2=25$
$x=5$

OX = 5

Radius bisects chord

13

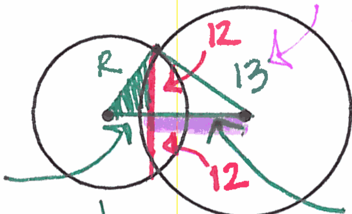


Right Δ → use Pythag Thm.

2. Two circles intersect and have a common chord 24 centimeters long. The centers of the circles are 21 centimeters apart. The radius of one circle is 13 centimeters. Find the radius of the other circle.

$R^2=12^2+16^2$
$R^2=400$
$R=20$

21−5
=16



3. In ⊙O, find CD if $\overline{PA} \cong \overline{QA}$, $DP = x^2+20$, and $XY = 21x$.

$CD = 2 \cdot DP$
$CD = 2(x^2+20) = 2x^2+40$

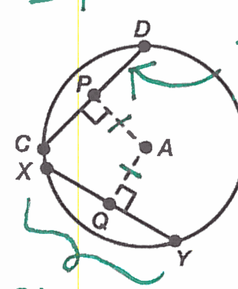
$\overline{CD} \cong \overline{XY}$

$2x^2+40=21x$
$2x^2-21x+40=0$
$(x-8)(2x-5)=0$

80
−5 −16
−21

$2x^2$	$-5x$	x
$-16x$	40	-8
$2x$	-5	

$x = 8$ or 2.5
CD = 168 OR 52.5

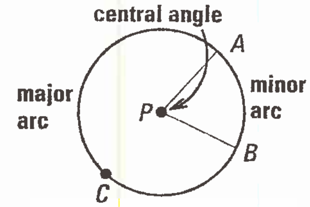


9.2 ~ Chords and Arcs

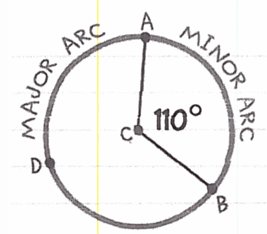
<u>OBJECTIVES:</u>

- Identify the different types of arcs and determine their measures
- Solve practical problems involving circles using the properties of angles, arcs, and chords
- Apply the relationships between congruent arcs, chords, and central angles

❖ Arcs of a Circle



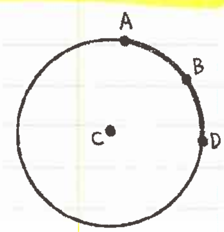
- **CENTRAL ANGLE** – An angle whose vertex is the center of the circle
  - $\angle APB$ is a central angle
- **SEMICIRCLE** – An arc whose endpoints are the endpoints of the diameter
- **MINOR ARC** – An arc that is smaller than a semicircle
  - Named with the two endpoints of the arc
  - Arc measure = measure of the central angle
  - $\widehat{AB}$ is a minor arc; $m\widehat{AB} = 110°$



- **MAJOR ARC** – An arc that is larger than a semicircle
  - Named with three letters, the first and last being the endpoints; the middle is any other point on the arc
  - Arc measure = 360 minus the measure of the minor arc
  - $\widehat{ADB}$ is a major arc; $m\widehat{ADB} = 360 - 110 = 250°$

❖ Congruent Arcs
- In the same or in congruent circles, two arcs are congruent if and only if their corresponding central angles are congruent.

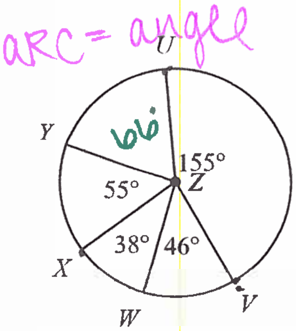
❖ Adjacent Arcs – Arcs of a circle that have exactly one point in common are *adjacent arcs*.
- **ARC ADDITION POSTULATE**
  - The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.



$m\widehat{AD} = m\widehat{AB} + m\widehat{BD}$

EXAMPLES:

arc = angle



1. Find $m\angle YZU$.

   66°

2. Find $m\widehat{VXU}$.

   $360 - 155 = 205°$

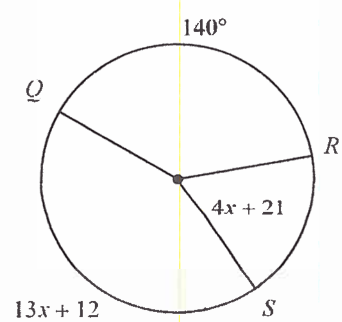
3. Find $m\widehat{XU}$.

   $55 + 66 = 121°$

4. Find $m\widehat{XVU}$.

   $360° - 121° = 239°$

5. Set up and solve an equation to find the value of x. Then find $m\widehat{SQ}$.



$13x + 12 + 4x + 21 + 140 = 360$

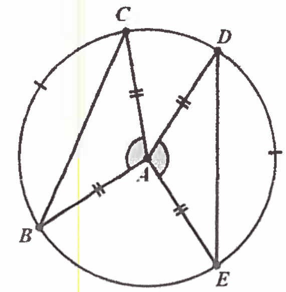
$17x + 173 = 360$

$17x = 187$

$x = 11$ $\quad m\widehat{SQ} = 155°$

❖ CONGRUENT ARCS – CHORDS – CENTRAL ANGLES CONJECTURE

- In the same circle (or in congruent circles)
  - Congruent arcs have congruent chords
  - Congruent chords have congruent central angles
  - Congruent central angles have congruent arcs

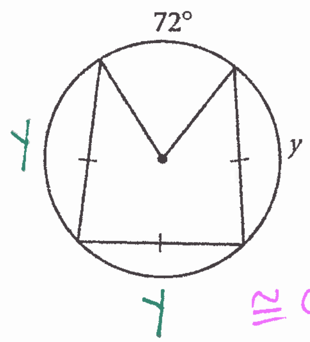


CONGRUENT ARCS	↔	CONGRUENT CHORDS	↔	CONGRUENT CENTRAL ANGLES
$\widehat{BC} \cong \widehat{DE}$		$\overline{BC} \cong \overline{DE}$		$\angle BAC \cong \angle DAE$

EXAMPLES:

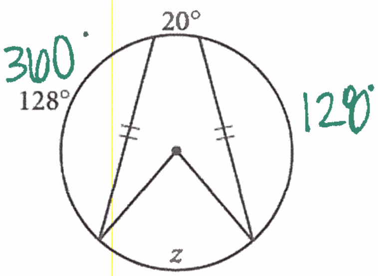
6. Find y.

$3y + 72 = 360$
$3y = 288$
$y = 96°$



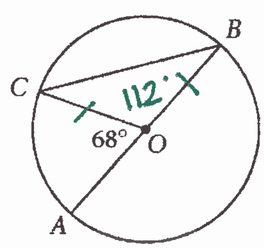
7. Find z.

$z + 20 + 2(128) = 360°$
$z = 84°$



≅ chords ↔ ≅ arcs

8. $\overline{AB}$ is a diameter. Find $m\widehat{AC}$ and $m\angle B$.



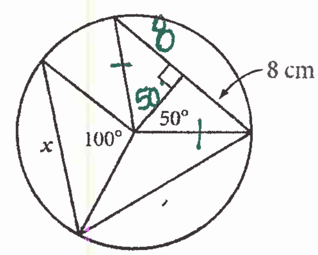
* All Radii are ≅
$\overline{BO} \cong \overline{CO}$
thus $\angle B \cong \angle C$

$180 - 112 = \frac{68}{2} = 34° = m\angle B$

$m\widehat{AC} = 68°$

9. Find x.

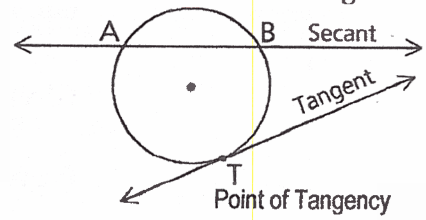
$x = 16$

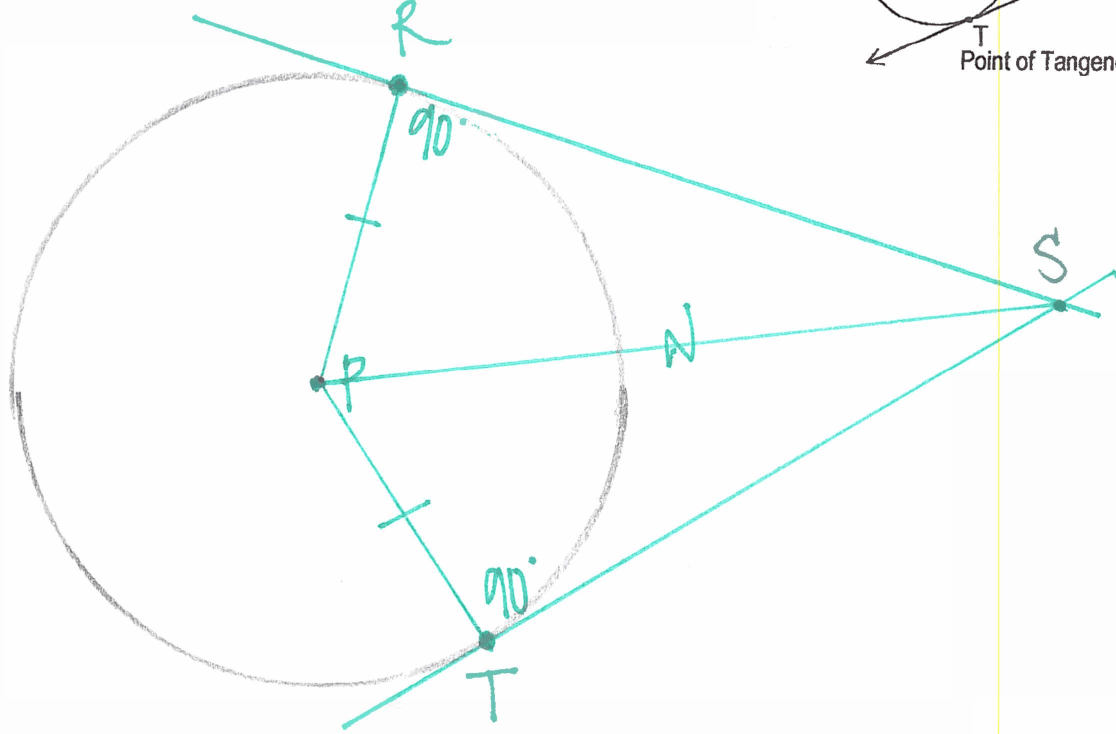


≅ central ∠s
↓
≅ chords

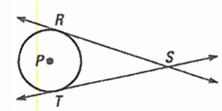
Investigating Tangents

LESSON 9.3





❖ Investigation: Tangents (Lesson 9.3)



- ➢ Use a compass to construct a circle of any radius, and identify the center as point *P*.
- ➢ Use a straight edge and draw two tangent lines, $\overleftrightarrow{RS}$ & $\overleftrightarrow{TS}$, such that the two lines intersect at a point outside of the circle. *Refer to the example shown →*
- ➢ Use a straight edge to draw the radii connecting the center of the circle to the point of tangency.
- ➢ Using a protractor, measure the angles formed by the radius and the tangent line and record in the diagram.
  - ▪ What can we conclude about the segment joining a radius of a circle to the point of tangency?

    Radius $\perp$ to the tangent
    @ the point of tangency

- ➢ Use a straight edge and draw in $\overline{PS}$. Now we've created two triangles: $\triangle PRS$ & $\triangle PTS$
  - ▪ Explain why $\triangle PRS \cong \triangle PTS$.

    $\angle R \cong \angle T$ b/c Right $\angle$s are $\cong$
    $\overline{PR} \cong \overline{PT}$ b/c all Radii are $\cong$
    $\overline{PS} \cong \overline{PS}$ via the Reflexive Prop.
    $\triangle PRS \cong \triangle PTS$ via HL

  - ▪ What can we conclude about the tangent segments $\overline{RS}$ & $\overline{TS}$?

    $\overline{RS} \cong \overline{TS}$ (CPCTC)

Closure:

Using your observations from the investigation, complete the following conjectures and problems 1 – 5.

★ If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

★ If tangent segments are drawn to a circle from an external point, then those segments are congruent.

1. Given: $\odot A$, $\overleftrightarrow{FB}$, $\overleftrightarrow{EC}$ & $\overleftrightarrow{DC}$ are tangents

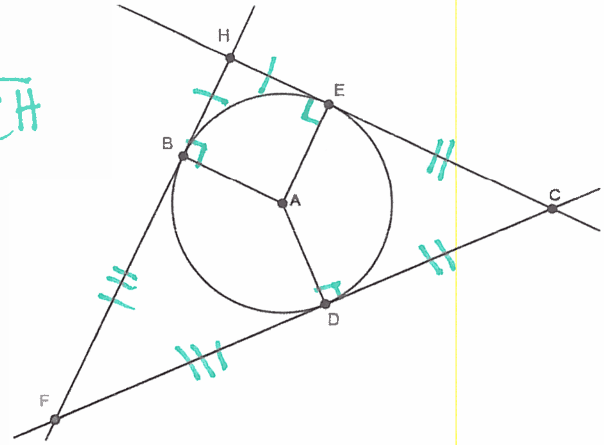
   a. Name all pairs of segments that are perpendicular.

   $\overline{AB} \perp \overline{FH}$, $\overline{AD} \perp \overline{CF}$, $\overline{AE} \perp \overline{CH}$

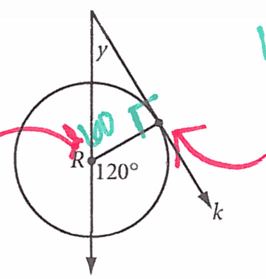
   b. Name all pairs of congruent tangent segments

   $\overline{BH} \cong \overline{EH}$ $\overline{BF} \cong \overline{DF}$

   $\overline{CE} \cong \overline{CD}$



2. Ray k is tangent to $\odot R$; find y.

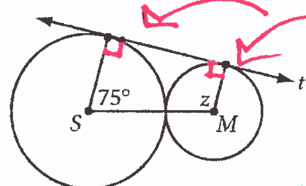


supp ∠S

$y = 30^\circ$

Radius ⊥ tangent

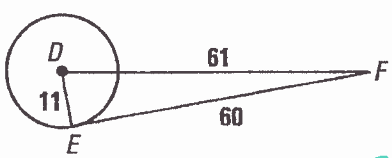
3. Line t is tangent to both circles; find z.



Radius ⊥ tangent

$360 - 2(90) - 75 = 105 = z$

4. Is $\overleftrightarrow{EF}$ is tangent to $\odot D$? Explain your reasoning.



Is ∠E a Rt ∠?

Does the Pythag. Thm. work?

$11^2 + 60^2 \stackrel{?}{=} 61^2$

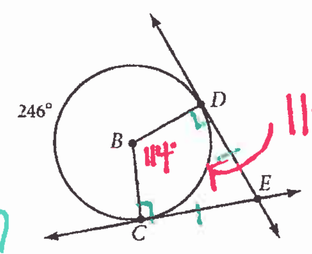
$3721 = 3721$ ✭

yes $\overline{EF}$ is tangent to $\odot D$

b/c $\overline{DE} \perp \overline{EF}$.

5. Given: $\odot B$, $\overleftrightarrow{EC}$ & $\overleftrightarrow{ED}$ are tangents

   Find: $m\widehat{CD}$, $m\angle DBC$, and $m\angle E$



114°

$m\widehat{CD} = 114^\circ$

$m\angle DBC = 114^\circ$

$m\angle E = 360 - 2(90) - 114 = 66^\circ$

What is the connection between $m\widehat{CD}$ & $m\angle E$?

$114^\circ + 66^\circ = 180^\circ$

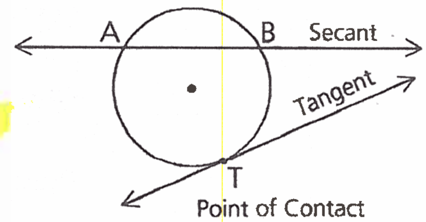
$\widehat{CD}$ & ∠E are supp.

9.3 ~ TANGENTS

OBJECTIVES:

- Identify tangent lines and segments and common internal and common external tangents
- Solve practical problems involving circles using the properties of angles, radii and tangents.

❖ Secants and Tangent Lines

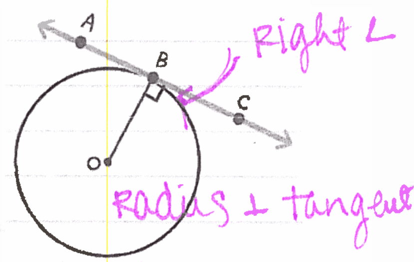


➢ A **SECANT** is a line that intersects a circle at exactly 2 points.

➢ A **TANGENT** is a line in the plane of a circle that intersects the circle at exactly 1 point.

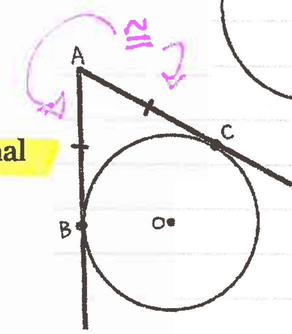
- This point is called the **POINT OF TANGENCY** (or point of contact).

❖ Tangent Theorems



➢ ***TANGENT TO A CIRCLE THEOREM***

- If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.
  - If $\overleftrightarrow{AC}$ is tangent to $\odot O$ at B, then $\overleftrightarrow{AC} \perp \overline{OB}$.

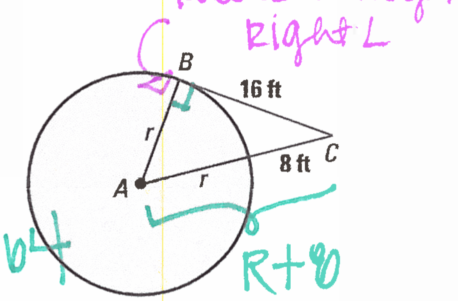


➢ ***TWO-TANGENT THEOREM***

- If two tangent segments are drawn to a circle from an external point, then those segments are congruent.
  - If $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are tangent to $\odot O$, then $\overline{AB} \cong \overline{AC}$.

EXAMPLES:

1. You are standing at C, 8 feet from a grain silo. The distance from you to a point of tangency on the tank is 16 feet. What is the radius of the silo?

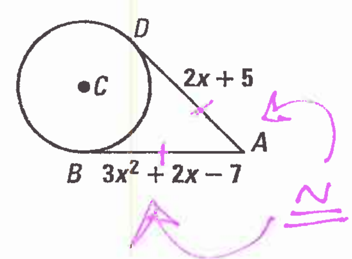


makes $\overline{AB} \perp \overline{BC}$
& ∠B is a right ∠

$R^2 + 16^2 = (R+8)^2$
$R^2 + 256 = R^2 + 16R + 64$
$256 = 16R + 64$
$192 = 16R$
$12 = R$

radius = 12 feet

2. $\overleftrightarrow{AB}$ and $\overleftrightarrow{AD}$ are tangent to $\odot C$. Use the Two-Tangent Theorem to set up and solve an equation to find the value of x (that makes sense).

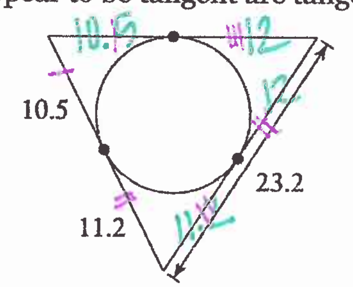


$\overline{AB} \cong \overline{AD}$

$3x^2 + 2x - 7 = 2x + 5$
$3x^2 - 7 = 5$
$3x^2 = 12$
$x^2 = 4$
$x = \pm 2$

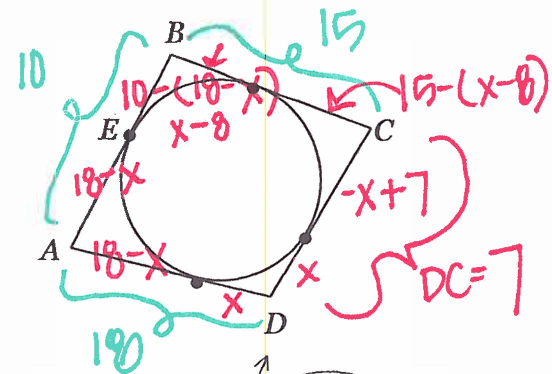
$x = 2$

Use the Two-Tangent Theorem to find the perimeter of the circumscribed polygon. Assume that lines which appear to be tangent are tangent.

3. 

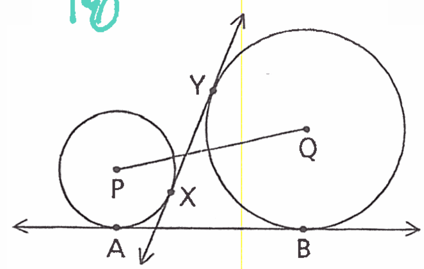
perimeter = 107.4

4. *A walk-around problem:* Quadrilateral $ABCD$ is circumscribed about the circle. $AB = 10$, $BC = 15$, and $AD = 18$. Find CD.



❖ COMMON TANGENTS

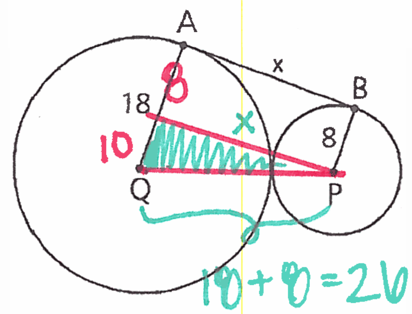
➢ A common tangent is a line tangent to two circles (not necessarily at the same point).
- Common internal tangent if it lies between the circles (intersects the segment joining the centers); line $\overleftrightarrow{XY}$
- Common external tangent if it is not between the circles (does not intersect the segment joining the centers); line $\overleftrightarrow{AB}$



EXAMPLES:

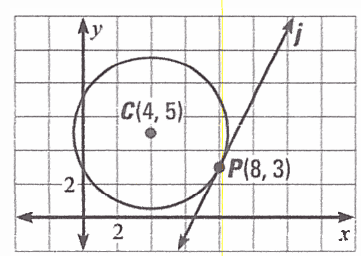
5. $\odot P$ is externally tangent to $\odot Q$
$\odot P$ has a radius of 8 centimeters
$\odot Q$ has a radius of 18 centimeters
Use the "Common-Tangent Procedure" to find the length of the common external tangent, $\overline{AB}$

$x^2 + 10^2 = 26^2$
$x^2 = 576$
$x = 24 = AB$



❖ Common-Tangent Procedure
- ➢ Draw the segment joining the centers.
- ➢ Through the center of the smaller circle, draw a line parallel to the common tangent.
  - This line will intersect the radius of the larger circle to form a rectangle and a right triangle.
- ➢ Use the Pythagorean Theorem and properties of a rectangle to find the length of the common external tangent.

6. In the diagram, line j is tangent to $\odot C$ at P.



a. What is the slope of radius $\overline{CP}$?

$m = \frac{5-3}{4-8} = \frac{2}{-4} = -\frac{1}{2}$

b. What is the slope of line j. Explain.

slope = 2 line $j \perp \overline{CP}$
$\perp$ lines have opp. recip. slopes

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

c. Write an equation for line j.

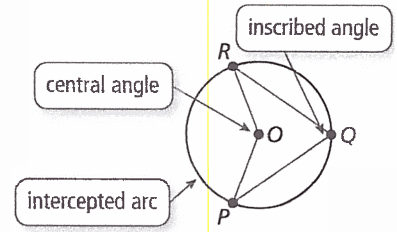
P(8,3)

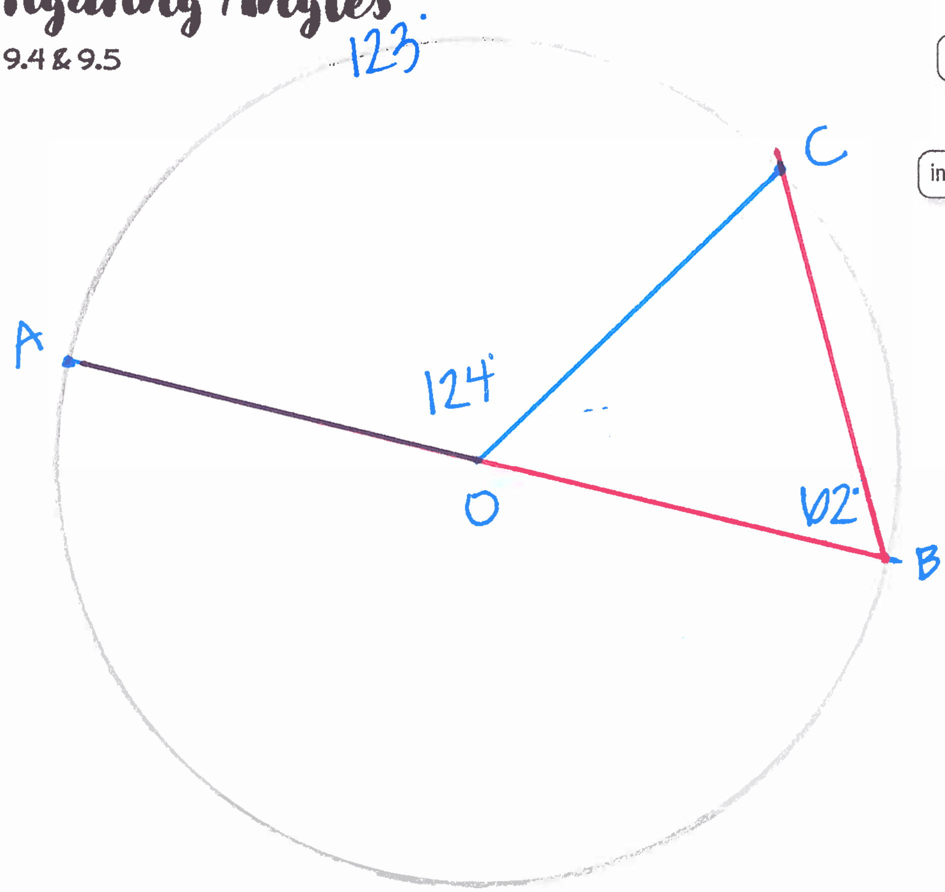
$y - 3 = 2(x - 8)$
$y - 3 = 2x - 16$
$y = 2x - 13$

$$y - y_1 = m(x - x_1)$$

Investigating Angles

LESSONS 9.4 & 9.5



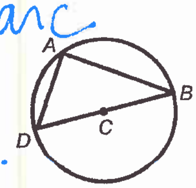


❖ Investigation: Inscribed Angles (Lesson 9.4)

- ➢ Use a compass to construct a circle of any radius, and identify the center as point O.
- ➢ Identify two points on the circle and label them A and C. Then draw $\angle AOC$.
- ➢ Draw a point B on the circle so that $\overline{AB}$ is a diameter. Then draw $\angle ABC$.
  - What angle in your diagram is a central angle? $\angle AOC$
  - What is the intercepted arc of this angle? $\overset{\frown}{AC}$
  - What angle in your diagram is an inscribed angle? $\angle ABC$
  - What is the intercepted arc of this angle? $\overset{\frown}{AC}$
  - What do you notice about the intercepted arcs? same arc
- ➢ Use a protractor and measure the central angle.
  - What is the measure of the intercepted arc? $124°$
- ➢ Use a protractor and measure the inscribed angle. $62°$
  - How does the measure of the inscribed angle compare to the measure of its intercepted arc?

    $62° = \frac{1}{2}(124°)$ angle = 1/2 arc

- ➢ In the diagram shown (at right), $\overline{BD}$ is a diameter and $\angle DAB$ is an inscribed angle. What is $m\angle DAB$? Explain your reasoning.



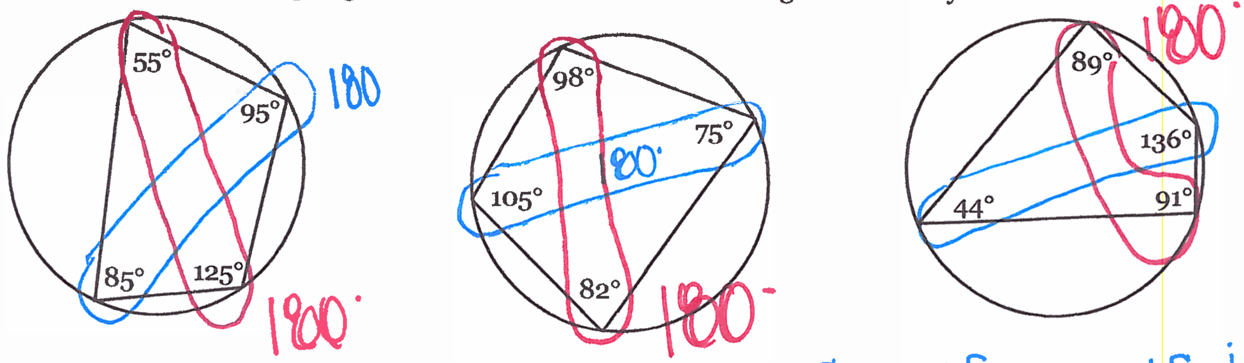
$m\angle DAB = 90°$

$\angle DAB$ intercepts $\overset{\frown}{DB}$ whose measure is $180°$

$\frac{1}{2}(180°) = 90°$

❖ Investigation: Inscribed Quadrilaterals (Lesson 9.4)

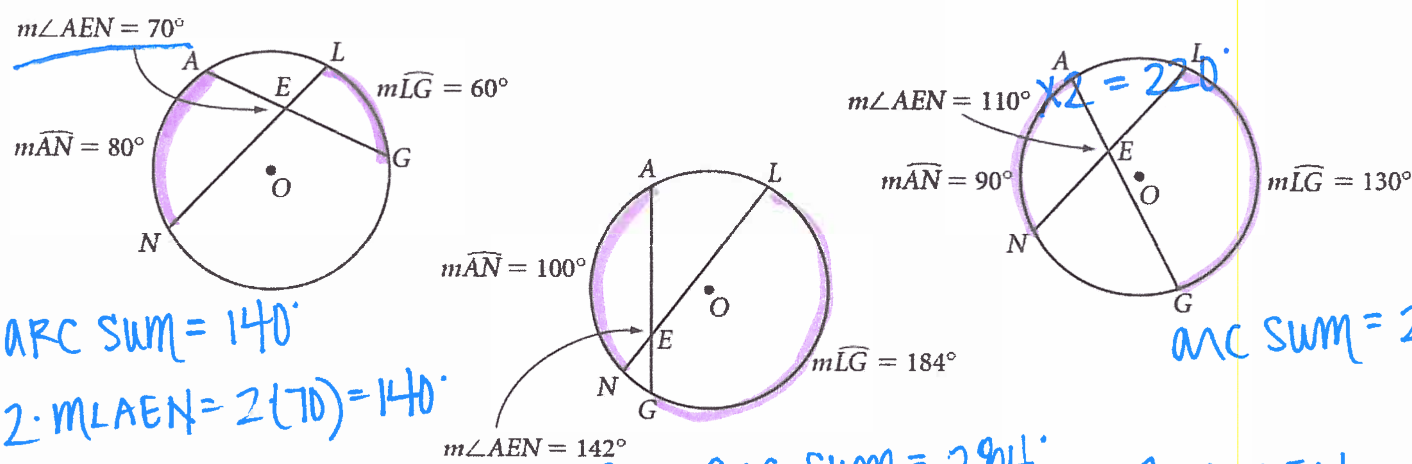
➢ An inscribed quadrilateral is called a cyclic quadrilateral. Four cyclic quadrilaterals are shown with the measures of all four inscribed angles given. Look at the sums of various angles. What do you observe?



Sum of opp. ∠s is 180°

❖ Investigation: Angles Formed by Intersecting Chords (Lesson 9.5)

➢ Use these diagrams to find a relationship between $\angle AEN$ and the intercepted arc measures: $\widehat{AN}$ & $\widehat{LG}$.



arc sum = 140°

2·m∠AEN = 2(70) = 140°

x 2

184°

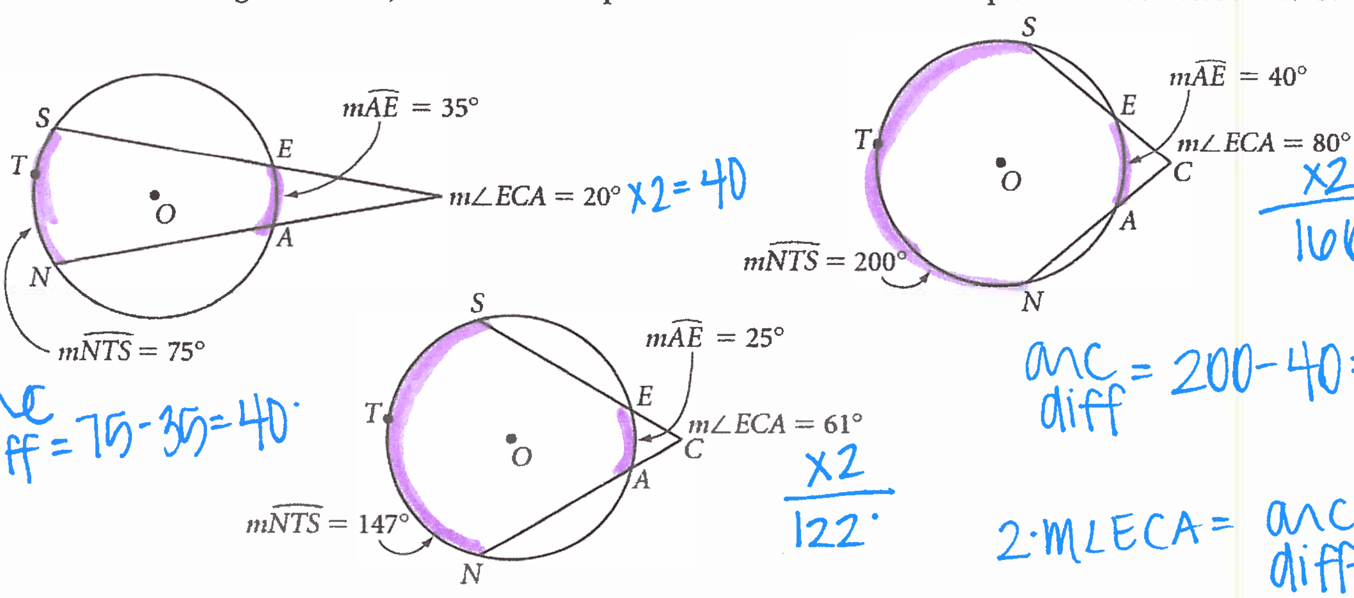
arc sum = 284°

arc sum = 220°

2·m∠AEN = arc sum

❖ Investigation: Angles Formed by Two Secants (Lesson 9.5)

➢ In these diagrams of ⊙O, find a relationship between $\angle ECA$ and the intercepted arc measures: $\widehat{AE}$ & $\widehat{NTS}$.



arc diff = 75 - 35 = 40°

x2

160°

arc diff = 200 - 40 = 160°

x2

122°

2·m∠ECA = arc difference

arc diff = 147 - 25 = 122

Closure:

Using your observations from the investigation, complete the following conjectures and problems 1 – 6.

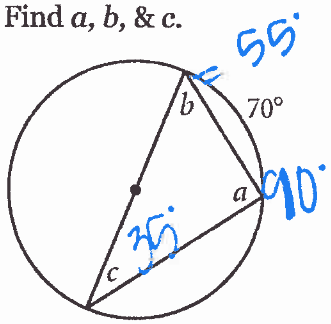
★ The measure of an inscribed angle is Half the measure of its intercepted arc.

★ If a quadrilateral is inscribed in a circle, its opposite angles are Supplementary.

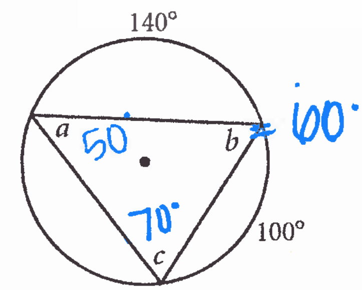
★ The measure of an angle formed by two intersecting chords is half the sum of the two intercepted arcs.

★ The measure of an angle formed by two secants is half the difference of the two intercepted arcs.

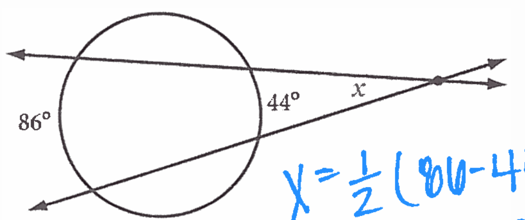
1. Find *a*, *b*, & *c*.



2. Find *a*, *b*, & *c*.



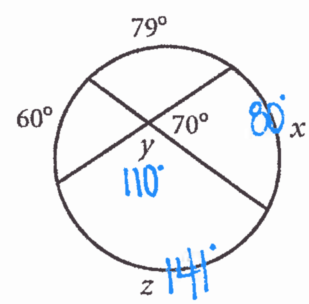
3. Find *x*.



$x = \frac{1}{2}(86 - 44)$

$x = 1/2(42) = 21°$

4.



$2(70) = x + 60°$

$140 = x + 60°$

$80° = x$

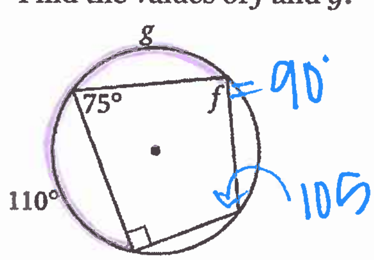
$2(110) = z + 79$

$220 = z + 79$

$141 = z$

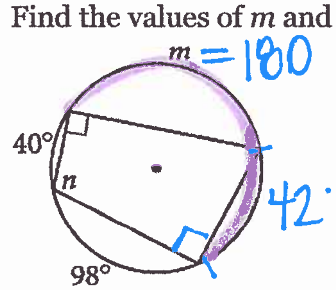
5. Find the values of *f* and *g*.

ARC= 210°



$g = 210 - 110° = 100°$

6. Find the values of *m* and *n*.



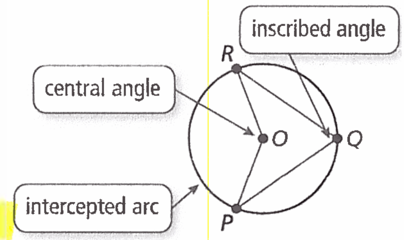
$n = 1/2(180 + 42)$

$n = 1/2(222) = 111°$

9.4 ~ Inscribed Angles and Polygons

OBJECTIVES:
- Determine the measure of inscribed angles
- Apply the Inscribed Right Triangle Theorem & the Inscribed Quadrilateral Theorem
- Solve practical problems involving circles using the properties of angles, chords, radii, and diameters

❖ Inscribed Angles

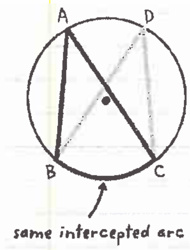


➢ An **INSCRIBED ANGLE** is an angle whose vertex is on a circle and whose sides are determined by two chords.

➢ **INSCRIBED ANGLE THEOREM**
- The measure of an inscribed angle is one-half the measure of its intercepted arc.

angle = 1/2 arc
2·angle = arc

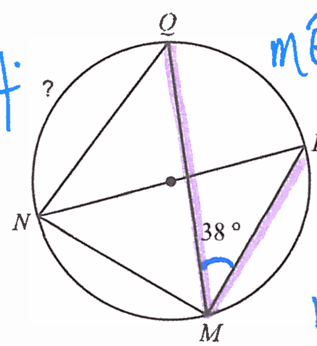
❖ Inscribed Angles Intercepting the Same Arc



➢ If two inscribed angles intercept the same arc, then they are congruent.
- $\angle A$ and $\angle D$ both intercept $\widehat{BC}$, therefore $\angle A \cong \angle D$.

EXAMPLES:

1. Use the Inscribed Angle Theorem to determine the measure of the indicated arc.



104°

$m\widehat{QL} = 2(38) = 76°$

$m\widehat{NQL} = 180$

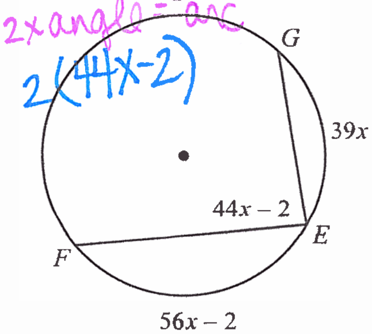
$m\widehat{QN} = 180 - 76$

$m\widehat{QN} = 104°$

2. Use the Inscribed Angle Theorem to set up and solve an equation to find the value of x.

2x angle = arc

$2(44x-2)$



$x = 2$

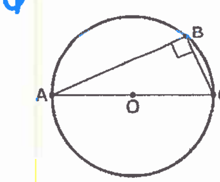
$2(44x-2) + 56x - 2 + 39x = 360°$

$183x - 6 = 360°$

$183x = 366$

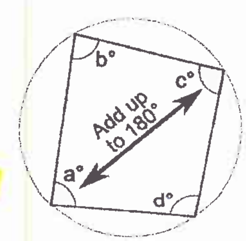
$x = 2$

❖ **INSCRIBED RIGHT TRIANGLE THEOREM**



➢ An angle inscribed in a semicircle is a right triangle.
- If one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.

❖ Cyclic Quadrilateral



➢ A quadrilateral inscribed in a circle is called a **CYCLIC QUADRILATERAL**. Each of its angles is inscribed in the circle, and each of its sides is a chord of the circle.

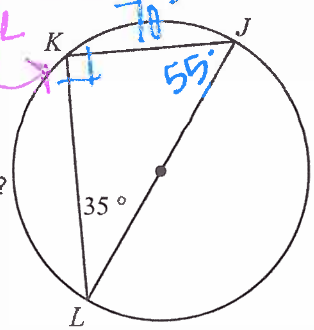
➢ **INSCRIBED QUADRILATERAL THEOREM**
- If a quadrilateral is inscribed in a circle, its opposite angles are supplementary.

➢ If a parallelogram is inscribed in a circle, it must be a rectangle.

EXAMPLES:

3. Use the Inscribed Right Triangle Theorem to find the measure of the indicated arc.

Right ∠



70°

55°

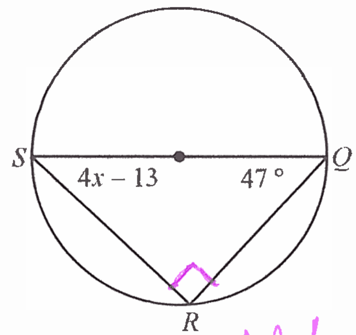
110°

$m\overset{\frown}{JK} = 2(35) = 70°$

$m\overset{\frown}{KL} = 180 - 70$

$= 110°$

4. Use the Inscribed Right Triangle Theorem to set up and solve an equation to find the value of x.



Right ∠

$4x - 13 + 47 = 90$

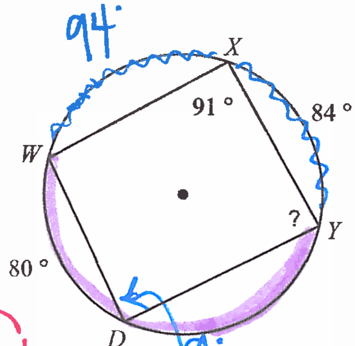
$4x + 34 = 90$

$4x = 56$

$x = 14$

5. Use the Inscribed Quadrilateral Theorem to find the measure of the indicated angle.

94°



$m\overset{\frown}{WXY} = 178$

$m\overset{\frown}{WX} = 178 - 84$

$94°$

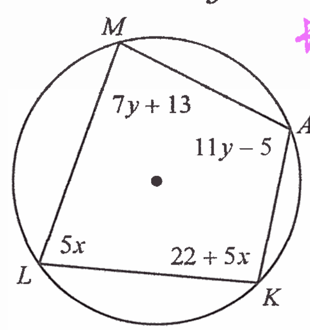
$m\overset{\frown}{WDY} = 182$

89°

$m\angle Y = \frac{1}{2} m\overset{\frown}{DWX}$

$m\angle Y = \frac{1}{2}(80 + 94) = 87°$

6. Use the Inscribed Quadrilateral Theorem to set up and solve a system of equations to find the values of x and y.



* opposite angles are supplementary

$5x + 11y - 5 = 180°$

$22 + 5x + 7y + 13 = 180°$

$5x + 11y = 185°$

Subtract these

$5x + 7y = 145°$

$4y = 40$

$y = 10$

$5x + 11(10) = 185$

$5x + 110 = 185$

$5x = 75$

$x = 15$

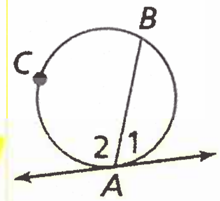
9.5 ~ Angles Related to a Circle

OBJECTIVES:

- Identify and describe the relationships among angles formed by chords, secants, and tangents
- Determine the measure of angles (or arcs) formed by: a chord and a tangent, two chords, two secants, two tangents, a tangent and a secant
- Solve practical problems involving circles using the properties of angles and arcs formed by chords, tangents, and/or secants

❖ **Angles with Vertices *ON* a Circle**

➢ A **TANGENT CHORD ANGLE** is an angle whose vertex is on a circle and whose sides are determined by a tangent and a chord that intersect at the tangent's point of contact



- The measure of a tangent-chord angle (vertex on a circle) is one-half the measure of its intercepted arc.

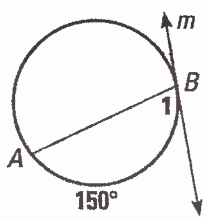
$$m\angle 1 = \frac{1}{2} m\widehat{AB} \leftrightarrow 2m\angle 1 = m\widehat{AB} \qquad m\angle 2 = \frac{1}{2} m\widehat{ACB} \leftrightarrow 2m\angle 2 = m\widehat{ACB}$$

EXAMPLES: angle = 1/2 arc

1. Line m is tangent to the circle. Find $m\angle 1$.

$m\angle 1 = 1/2(150)$

$75°$



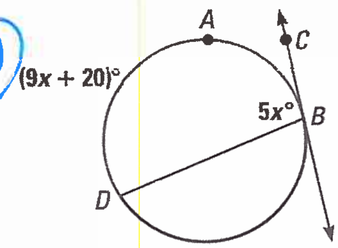
2. $\overleftrightarrow{BC}$ is tangent to the circle. Find $m\angle CBD$.

$2\cdot 5x = 2\cdot\frac{1}{2}(9x+20)$

$10x = 9x+20$

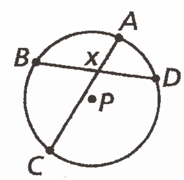
$x = 20$

$m\angle CBD = 100°$



❖ **Angles with Vertices *INSIDE* (but NOT at the Center) of a Circle**

➢ A **CHORD CHORD ANGLE** is an angle formed by two chords that intersect inside a circle, but not at the center.



- The measure of a chord-chord angle is one-half the sum of the measures of the arcs intercepted by the chord-chord angle and its vertical angle.

$$m\angle x = \frac{1}{2}(m\widehat{AB} + m\widehat{CD}) \leftrightarrow 2m\angle x = m\widehat{AB} + m\widehat{CD}$$

EXAMPLES:

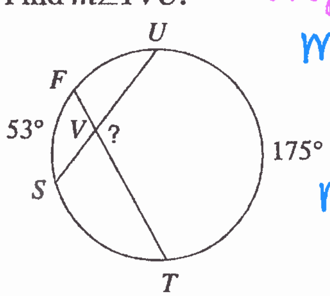
3. Find $m\angle TVU$.

angle = 1/2 (sum)

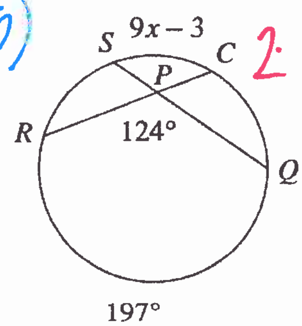
$m\angle TVU = 1/2(53+175)$

$= 1/2(228)$

$m\angle TVU = 114°$



4. Find the value of x.



$2\cdot 124 = 2\cdot\frac{1}{2}(9x-3+197)$

$248 = 9x+194$

$54 = 9x$

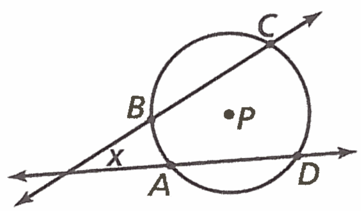
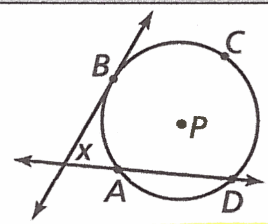
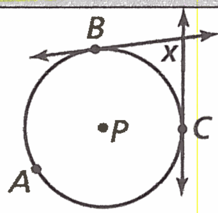
$6 = x$

❖ Angles with Vertices ***OUTSIDE*** a Circle

angle = ½ (difference)

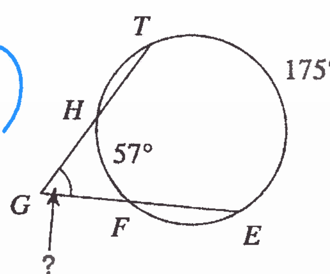
- A **SECANT SECANT ANGLE** is an angle whose vertex is outside a circle and whose sides are determined by two secants.
- A **SECANT TANGENT ANGLE** is an angle whose vertex is outside a circle and whose sides are determined by a secant and a tangent.
  - The measure of a secant-secant angle or a secant-tangent angle (vertex outside a circle) is one-half the difference of the measure of the intercepted arcs.
- A **TANGENT TANGENT ANGLE** is an angle whose vertex is outside a circle and whose sides are determined by two tangents.
  - The measure of a tangent-tangent angle (vertex outside a circle) is one-half the difference of the measure of the intercepted arcs.
  - The sum of the measures of a tangent-tangent angle and its minor arc is 180°.

tan-tan angle + minor arc = 180°

SECANT-SECANT	SECANT-TANGENT	TANGENT-TANGENT
		
$m\angle x = \frac{1}{2}(m\widehat{CD} - m\widehat{AB})$	$m\angle x = \frac{1}{2}(m\widehat{BCD} - m\widehat{AB})$	$m\angle x = \frac{1}{2}(m\widehat{BAC} - m\widehat{BC})$
$2m\angle x = m\widehat{CD} - m\widehat{AB}$	$2m\angle x = m\widehat{BCD} - m\widehat{AB}$	$2m\angle x = m\widehat{BAC} - m\widehat{BC}$
2·angle = difference		$m\angle x + m\widehat{BC} = 180°$

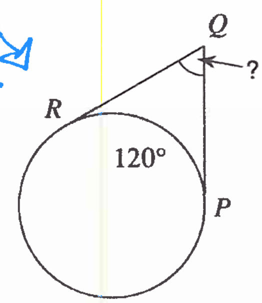
EXAMPLES:

5. Find $m\angle G$.



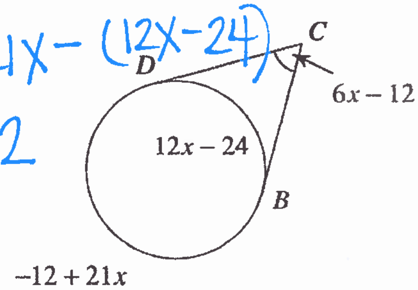
$m\angle G = \frac{1}{2}(175-57)$
$= \frac{1}{2}(118)$
$m\angle G = 59°$

6. Find $m\angle Q$.



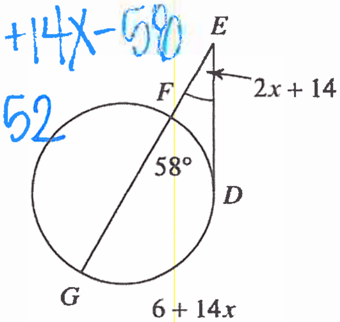
$m\angle Q + 120 = 180°$
$m\angle Q = 60°$

7. Find the value of x.



$2(6x-12) = -12+21x-(12x-24)$
$12x-24 = 9x+12$
$3x = 36$
$x = 12$

8. Find the value of x.



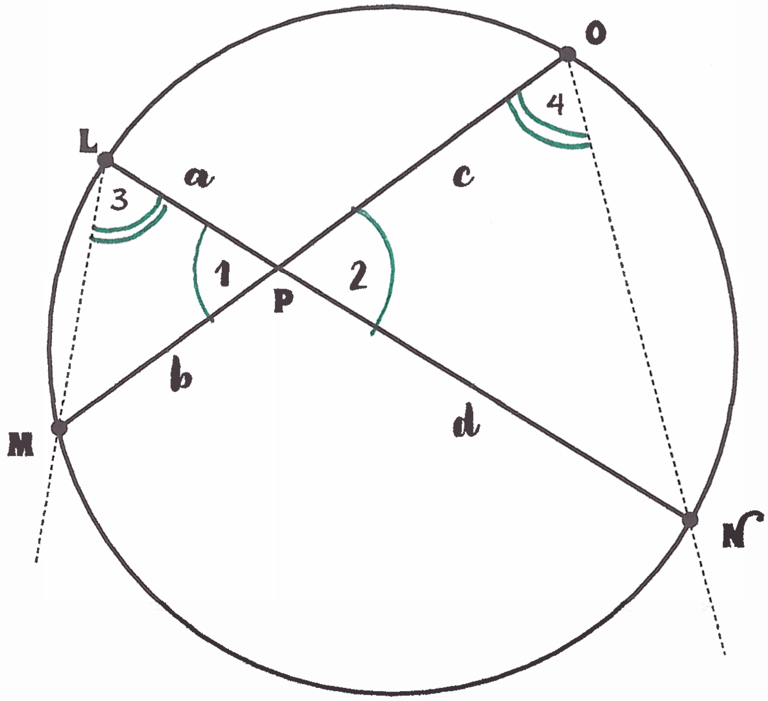
$2(2x+14) = 6+14x-58$
$4x+28 = 14x-52$
$80 = 10x$
$8 = x$

Investigating Segments LESSON 9.6

❖ Investigation: Chord-Chord Segments

➢ $\overline{LN}$ & $\overline{MO}$ are chords that intersect at P

- What is the relationship between the two segments of each chord?



➢ What is the relationship between $\angle 1$ & $\angle 2$?

- How do their measures compare? Vert. $\angle$s are $\cong$ $\angle 1 \cong \angle 2$

➢ Compare inscribed angles $\angle 3$ & $\angle 4$: what do you notice?

- How do their measures compare? Both intercept $\overset{\frown}{MN}$ $\angle 3 \cong \angle 4$

➢ Based on these observations, what is the relationship between $\triangle LPM$ and $\triangle OPN$? Explain your reasoning. $\triangle LPM \sim \triangle OPN$ by AA

➢ Because of this relationship between $\triangle LPM$ and $\triangle OPN$, what is the relationship between their corresponding sides? CORRS. sides are proportional

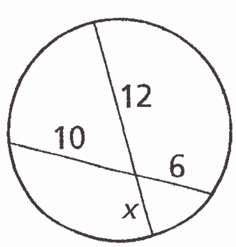
$$\frac{a}{b} = \frac{c}{d}$$

$$ad = bc$$

segs of $\overline{LN}$ segs. of $\overline{MO}$

Apply it:

Find x.



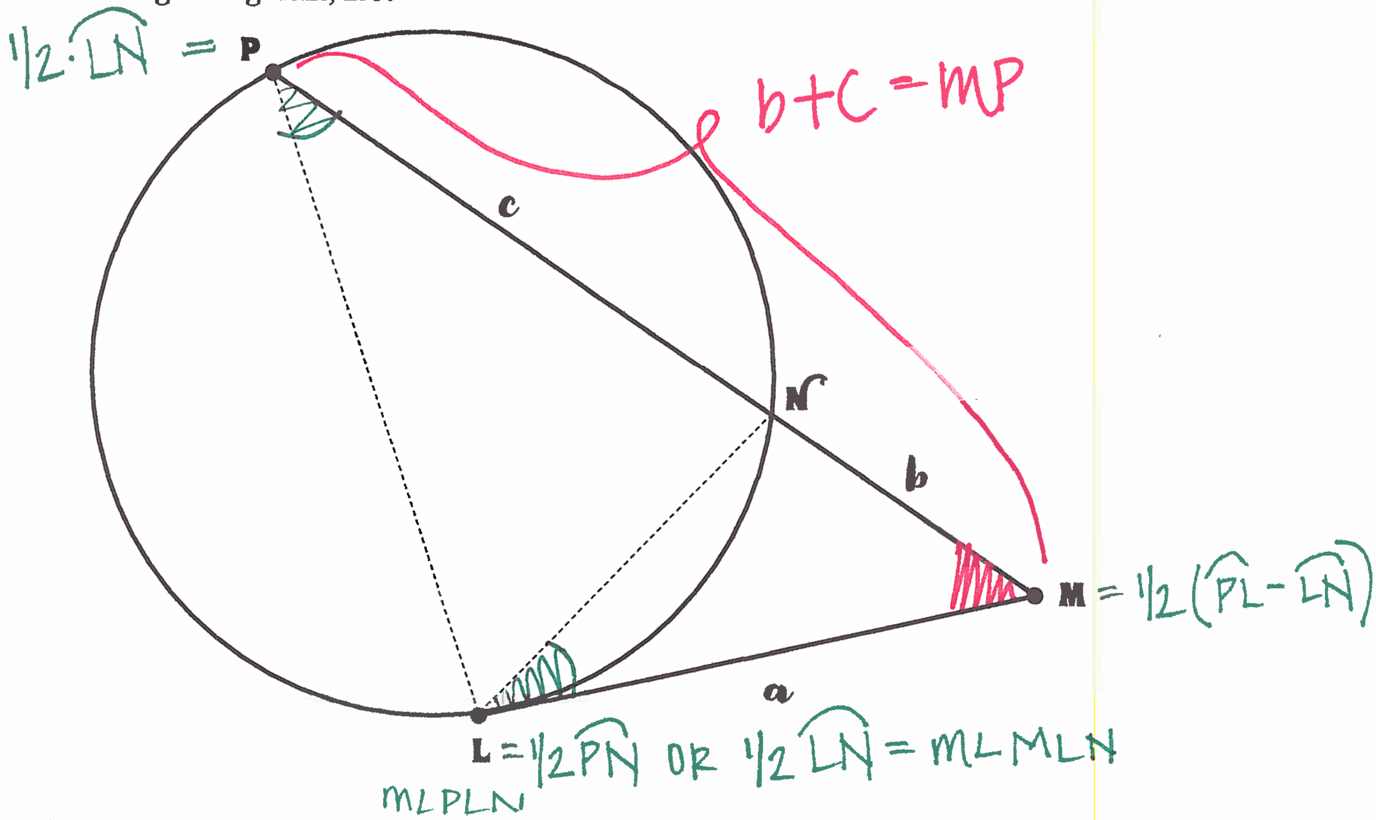
$12x = 60$

$x = 5$

❖ Investigation: Tangent-Secant Segments

➢ $\overline{PM}$ is a secant segment and $\overline{LM}$ is a tangent segment

- What is the relationship between the secant segment, $\overline{PM}$, the external part of the secant segment, b, and the tangent segment, $\overline{LM}$?



Two auxiliary lines have been drawn in. There are now three triangles shown in this diagram. Two of these triangles are similar by the AA Similarity Theorem. Use your knowledge of angle and arc measures to help you determine which two triangles are similar. Then write a proportion that shows the relationship amongst the corresponding sides.

$\angle P \cong \angle MLN$ (Both $= \frac{1}{2}\cdot \overset{\frown}{LN}$)

$\angle M \cong \angle M$ (Reflexive)

$\triangle MLN \sim \triangle MPL$ by AA

$\frac{a}{b} = \frac{b+c}{a} \Rightarrow a^2 = b(b+c)$

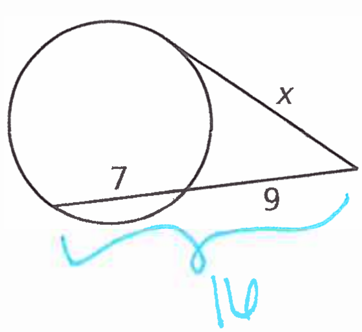
a ↑ tangent seg

b ↑ ext. part

$(b+c)$ ← entire secant seg.

Apply it:

Find x.



tangent^2 = (ext. part)(entire secant)

$x^2 = 9(16)$

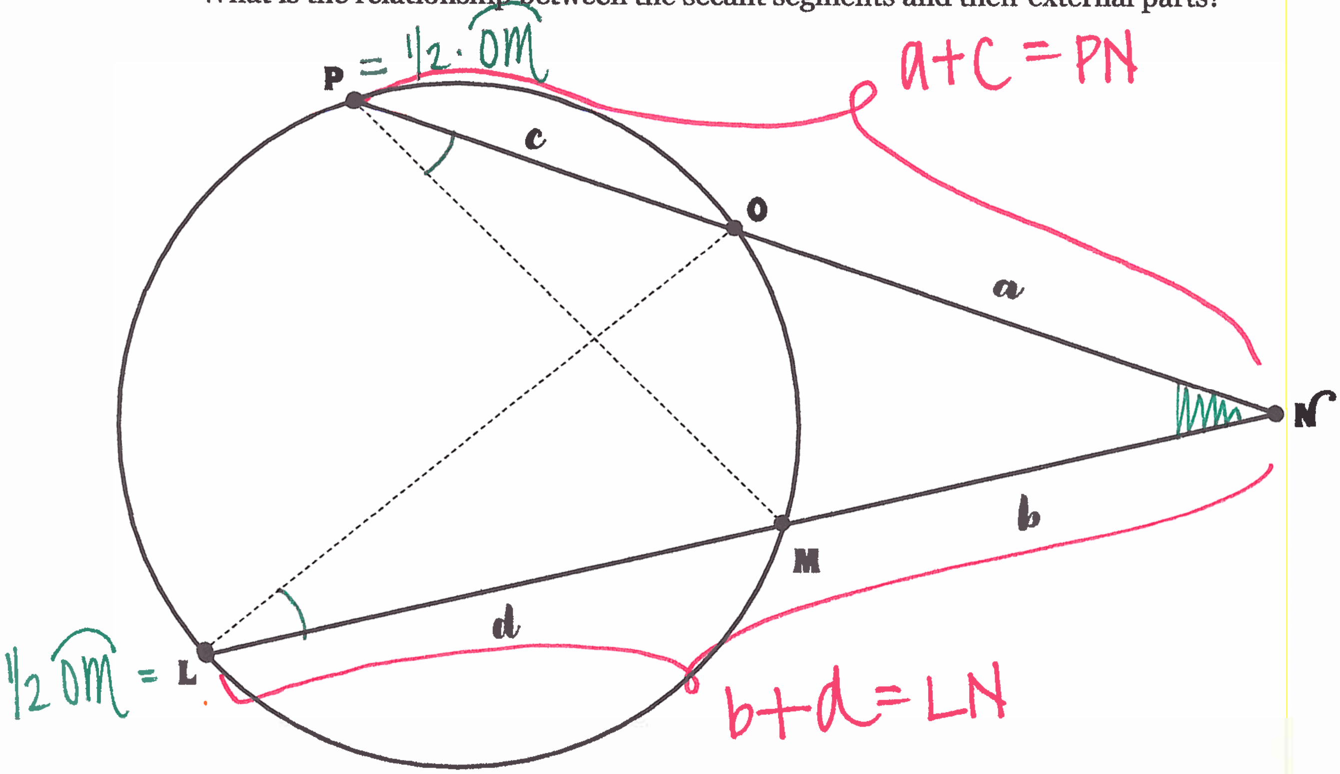
$x^2 = 144$

$x = 12$

❖ Investigation: Secant-Secant Segments

➢ $\overline{PN}$ & $\overline{LN}$ are secant segments

▪ What is the relationship between the secant segments and their external parts?



Two auxiliary lines have been drawn in creating multiple triangles. Two of these triangles are similar by the AA Similarity Theorem. Use your knowledge of angle and arc measures to help you determine which two triangles are similar. Then write a proportion that shows the relationship amongst the corresponding sides.

$\angle P \cong \angle L$ (Both $= \frac{1}{2}\overset{\frown}{OM}$)

$\angle N \cong \angle N$ (Reflexive)

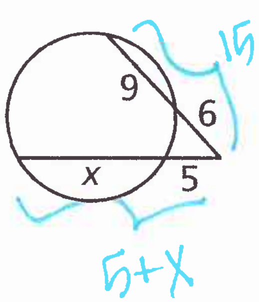
$\triangle LNO \sim \triangle PNM$ by AA

$\frac{b+d}{a} = \frac{a+c}{b} \Rightarrow b(b+d) = a(a+c)$

b: ext. sec. $(b+d)$: whole secant a: ext. sec. $(a+c)$: whole secant

Apply it:

Find x.



$6(15) = 5(5+x)$

$90 = 25 + 5x$

$65 = 5x$

$13 = x$

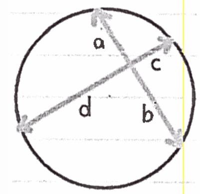
9.6 ~ Segments Related to a Circle

OBJECTIVES:

- Determine the lengths of segments formed by: two chords, two secants, a secant and a tangent
- Solve practical problems involving circles using the Power Theorems

❖ CHORD-CHORD POWER THEOREM

➢ If two chords of a circle intersect inside the circle, then the product of the measures of the segments of one chord is equal to the product of the measures of the segments of the other chord.



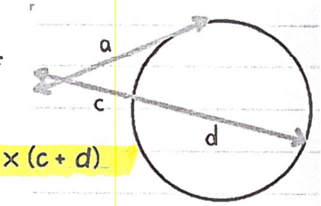
$a \times b = c \times d$

❖ TANGENT-SECANT POWER THEOREM

➢ If a tangent segment and a secant segment are drawn from an external point to a circle, then the square of the measure of the tangent segment is equal to the product of the measures of the entire secant segment and its external part.

- tangent^2 = (whole secant)(external part)

a^2 $(c+d)\cdot c$



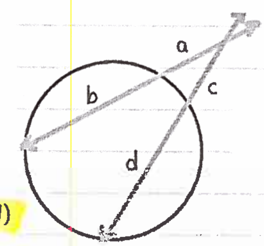
$a^2 = c \times (c + d)$

❖ SECANT-SECANT POWER THEOREM

➢ If two secant segments are drawn from an external point to a circle, then the product of the measures of one secant segment and its external part is equal to the product of the measures of the other secant segment and its external part.

- (whole secant$_1$)(external part$_1$) = (whole secant$_2$)(external part$_2$)

$(a+b)\cdot a = (c+d)\cdot c$

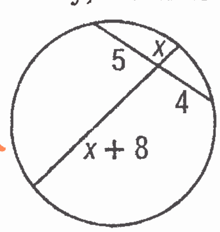


$a \times (a + b) = c \times (c + d)$

EXAMPLES:

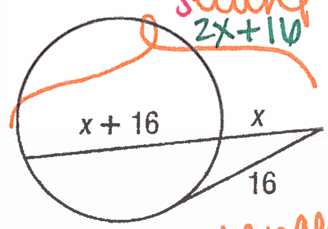
Use the appropriate Power Theorem to set up and solve an equation to find the value of x (that makes sense). If necessary, round to the nearest tenth.

1. chord-chord



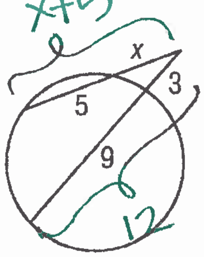
$x(x+8) = 5\cdot 4$
$x^2 + 8x = 20$
$x^2 + 8x - 20 = 0$
$(x+10)(x-2) = 0$
$x = \cancel{-10}$ or 2

2.



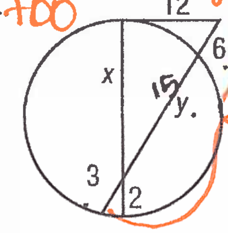
$\tan^2 =$ ext (whole)
$16^2 = x(2x+16)$
$256 = 2x^2 + 16x$
$0 = 2x^2 + 16x - 256$
$0 = 2(x^2 + 8x - 128)$
$0 = 2(x+16)(x-8)$
$x = \cancel{-16}$ or 8

3. sec-sec.



ext · whole = ext · whole
$x(x+5) = 3\cdot 12$
$x^2 + 5x = 36$
$x^2 + 5x - 36 = 0$
$(x+9)(x-4) = 0$
$x = \cancel{-9}$ OR 4

4. chords tangent



$12^2 = 6(y+9)$
$144 = 6y + 54$
$90 = 6y$
$15 = y$

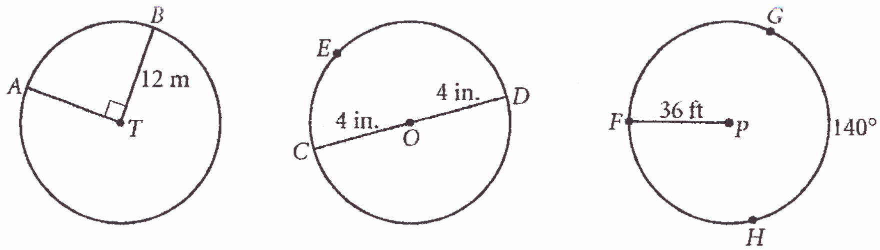
$2x = 3y$
$2x = 3\cdot 15$
$2x = 45$
$x = 22.5$

9.7 ~ Arc Length

OBJECTIVES:

- Distinguish between arc measure and arc length
- Find the length of an arc in given in degrees

INVESTIGATION



	ARC	ARC MEASURE	FRACTION OF CIRCLE	CIRCUMFERENCE Leave in terms of π	COMBINE TO FIND ARC LENGTH
$\odot T$	$\overset{\frown}{AB}$	90°	$\frac{90}{360}=\frac{1}{4}$	$2\pi\cdot12=24\pi$	$\frac{1}{4}\cdot24\pi=6\pi$
$\odot O$	$\overset{\frown}{CED}$	180°	$\frac{1}{2}$	$2\pi\cdot4=8\pi$	$\frac{1}{2}\cdot8\pi=4\pi$
$\odot P$	$\overset{\frown}{GH}$	140°	$\frac{140}{360}=\frac{7}{18}$	$2\pi\cdot36=72\pi$	$\frac{7}{18}\cdot72\pi=28\pi$
	$\overset{\frown}{ARC}$	θ	$\frac{\theta}{360}$	$2\pi R$	$\frac{\theta}{360}\cdot2\pi R$

❖ **ARC LENGTH, *S***

- Arc length is a portion of the circumference of a circle.
- The *length* of an arc is different from the *degree measure* of the arc. Arcs are measured in degrees whereas arc lengths are linear measurements.

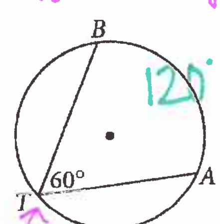
ARC LENGTH FORMULA:

$$S=\frac{\theta}{360}\cdot2\pi R$$

EXAMPLES:

1. If the radius of the circle is 24 cm and $m\angle BTA = 60°$, what is the length of $\overset{\frown}{AB}$?

arc = 2·angle

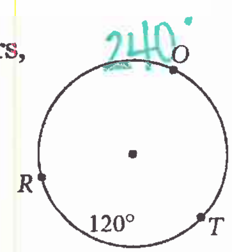


vertex ON circle

$$\frac{120}{360}\cdot2\pi(24)$$

$S=16\pi$ cm.

2. If the length of $\overset{\frown}{ROT}$ is 116π meters, what is the radius of the circle?

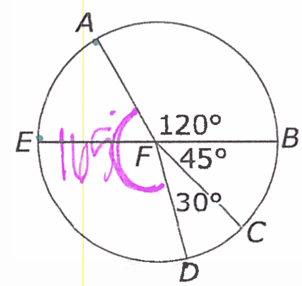


$$116\pi=\frac{240}{360}\cdot2\pi R$$

$$\frac{3}{4\pi}\cdot116\pi=\frac{4\pi}{3}\cdot R\cdot\frac{3}{4\pi}$$

87 m. $=R$

3. $\odot F$ has been divided into sectors. The length of $\overline{FB}$ is 3 units. Find the length of $\overset{\frown}{AED}$ in terms of π.



$m\overset{\frown}{AED}=165°$

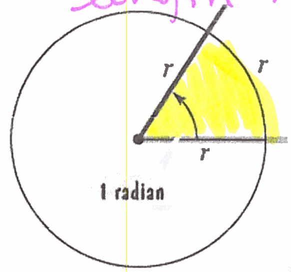
$$S=\frac{165°}{360°}\cdot2\pi\cdot3=\frac{990\pi}{360}$$

$S=2.75\pi$ units

9.8 ~ RADIAN MEASURE

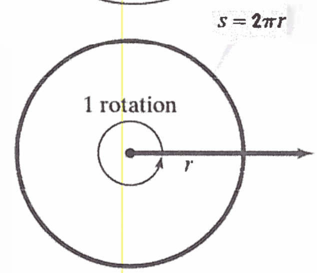
OBJECTIVES:
- Distinguish between degree measure and radian measure
- Find the length of an arc in given in both degree and radian measures

the arc & the radii have length R.



❖ The Problem of Angular Measure
- ➢ Degree units have no mathematical relationship whatsoever to linear units.
  - There are 360 degrees in a circle of radius 1.
  - What relationship does the 360 have to the 1?

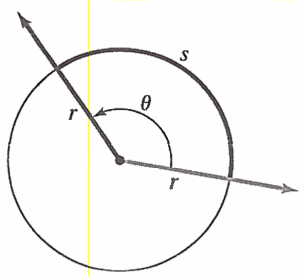
❖ **RADIAN MEASURE**
- ➢ One-radian angle is the measure of the central angle of a circle that intercepts an arc equal in length to the radius of the circle.



❖ **The Relationship between Degrees & Radians**
- ➢ The length of the intercepted arc is equal to the circumference of the circle. The radian measure of this central angle is the circumference of the circle divided by the circle's radius:
$$\theta = \frac{\text{circumference}}{\text{radius}} = \frac{2\pi r}{r} = 2\pi$$
  - Therefore, one complete rotation measures 360 ° and 2π radians. → 180° = π Rads

❖ ***CONVERTING BETWEEN DEGREES AND RADIANS***
- ➢ To convert degrees to radians: $\theta \times \frac{\pi}{180°}$
- ➢ To convert radians to degrees: $\theta \times \frac{180°}{\pi}$

RADIAN MEASURE AND ARC LENGTH



- ➢ If θ is a central angle in a circle of radius r, and if θ is measured IN RADIANS, then the length of the intercepted arc is given by $s = r\theta$.
  - The unit used to describe the length of a circular arc is the same unit that is given in the circle's radius.

EXAMPLES:

1. If $\theta = \pi/2$ & $r = 4$, find the length of the intercepted arc.

$$S = R\theta = 4 \cdot \frac{\pi}{2} = \frac{4\pi}{2} = 2\pi$$

2. A central angle, θ, in a circle of radius 6 inches intercepts an arc of length 15 inches. What is the radian measure of θ?

$S = R\theta$
$15 = 6 \cdot \theta$
$2.5 = \theta$

Convert each angle in degrees to radians

3. $\theta = 150°$ $\times \frac{\pi}{180°}$ → $\frac{5\pi}{6}$

4. $\theta = 18°$ $\times \frac{\pi}{180°}$ → $\frac{\pi}{10}$

Convert each angle in radians to degrees.

5. $\theta = \frac{\pi}{9} \cdot \frac{180}{\pi}$ → 20°

6. $\theta = \frac{7\pi}{6} \cdot \frac{180}{\pi}$ → 210°

9.9 ~ Sectors and Segments of a Circle

OBJECTIVE:

- Find the area of a sector, segment, or annulus of a circle

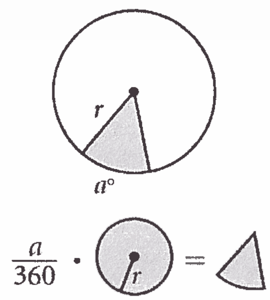
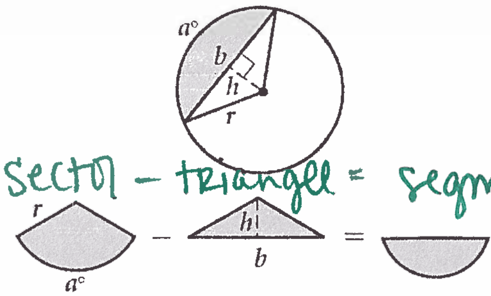
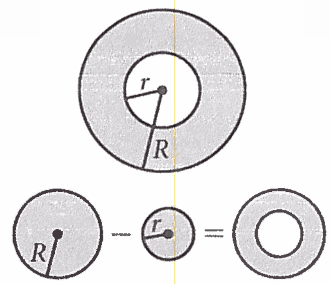
❖ "Slicing" a Circle

➢ A sector of a circle is the region between two radii of a circle and the included arc.

➢ A segment of a circle is the region between a chord of a circle are the included arc.

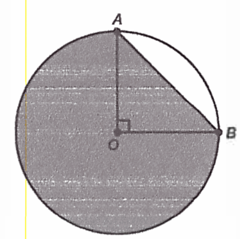
➢ An annulus is the region between two concentric circles.

Look at the "picture equations" and write a formula for finding the area.

AREA OF A SECTOR	AREA OF A SEGMENT	AREA OF AN ANNULUS
	 sector − triangle = segment	
FORMULA: $\frac{\theta}{360}\cdot\pi R^2 =$ area of a sector	FORMULA: $\frac{\theta}{360}\cdot\pi R^2 - \frac{1}{2}bh =$ area of segment	FORMULA: $\pi R^2 - \pi r^2 =$ area of annulus

EXAMPLES:

1. The length of the radius is 10 inches. What is the area of the shaded region. Express your answer in terms of π.

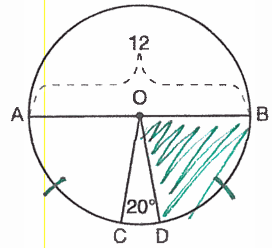


area of sector + area of Δ = area of shade

$\frac{3}{4}\cdot\pi 10^2 + \frac{1}{2}\cdot 10\cdot 10$

$75\pi + 50 =$ shaded area

2. In $\odot O$, diameter $\overline{AB}$ and radii $\overline{OC}$ & $\overline{OD}$ are drawn. The length of $\overline{AB}$ is 12 and the measure of $\angle COD$ is 20°. If $\widehat{AC} \cong \widehat{BD}$, find the area of sector BOD in terms of π.



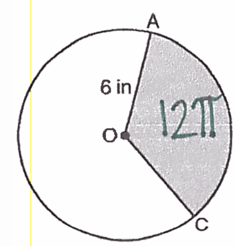
$\angle AOC \cong \angle BOD$

$m\angle BOD = \frac{180-20}{2} = 80°$

$\frac{80°}{360}\cdot\pi\cdot 6^2$

area $= 8\pi$

3. In the diagram of $\odot O$, the area of the shaded sector AOC is 12π square inches and the length of $\overline{OA}$ is 6 inches. Find $m\angle AOC$.



$12\pi = \frac{m\angle AOC}{360°}\cdot\pi\cdot 6^2$

$\frac{12\pi \times 360}{36\pi} = \frac{m\angle AOC\cdot\pi\cdot 36}{36\pi}$

$120° = m\angle AOC$

10.2 ~ Standard Form Equation of a Circle

__OBJECTIVE:__

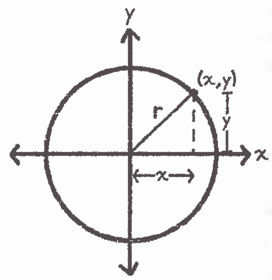
- Use the Pythagorean Theorem to derive the equation of a circle given the center & radius
- Write the standard form equation of a circle given: points on the circle, the center and the circumference or the area, the center and tangents

The Equation of a Circle

For any point (x, y) on the circle,

$x^2 + y^2 = r^2$ (Pythagorean Theorem)

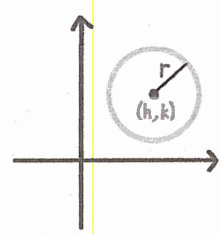
The circle is the shape formed by all (x, y) points where $x^2 + y^2 = r^2$ is true.



If a circle's center is not at the origin, use the standard form equation:

$\boxed{(x - h)^2 + (y - k)^2 = r^2}$

Pythagorean Theorem



The center is (h, k) and the radius is r.

EXAMPLES: Write the standard form equation of the circle described.

1. A circle centered at the origin with radius $\sqrt{22}$

$x^2 + y^2 = (\sqrt{22})^2$

$x^2 + y^2 = 22$

2. A circle centered at $(3, -5)$ with radius 6

$(x-3)^2 + (y+5)^2 = 6^2$

$(x-3)^2 + (y+5)^2 = 36$

MIDPOINT FORMULA:

$$(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

DISTANCE FORMULA:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

❖ Finding the Standard Form Equation of a Circle

➢ ***GIVEN: THE CENTER*** (h, k) ***& A POINT ON THE CIRCLE*** (x, y)

▪ Consider a circle whose center is $(2, -5)$ and that passes through the point $(-7, -1)$.
The radius of a circle is a line segment with one endpoint on the circle and one endpoint at the center.

- Find the radius of the circle:

dist. from ctr to point

$R = \sqrt{(\underbrace{-7-2}_{-9})^2 + (\underbrace{-1+5}_{4})^2} = \sqrt{81+16} = \sqrt{97}$

- Write the standard form equation of the circle.

$(x-2)^2 + (y+5)^2 = (\sqrt{97})^2$

$(x-2)^2 + (y-5)^2 = 97$

- ***GIVEN: THE ENDPOINTS OF THE DIAMETER***

The diameter of a circle is a line segment with each endpoint on the circle that passes through the center of the circle.

- Consider a circle with a diameter whose endpoints are (18, −13) & (4, −3).
  - Find the center of the circle:

    midpoint

    $\left(\frac{18+4}{2}, \frac{-13+(-3)}{2}\right) = (11, -8)$

  - Find the radius of the circle:

    (11, −8)
    (4, −3)

    dist. from center to one of the endpoints

    $R = \sqrt{(11-4)^2 + (-8+3)^2}$

    $R = \sqrt{49+25} = \sqrt{74}$

  - Write the standard form equation of the circle:

    $(x-11)^2 + (y+8)^2 = 74$

- ***GIVEN: THE CENTER (h, k) & THE CIRCUMFERENCE***
  - Consider a circle whose center is (−13, −8) and whose circumference is 8π.
    - Find the radius of the circle:

      $C = 2\pi R$

      $\frac{8\pi}{2\pi} = \frac{2\pi R}{2\pi}$ $R = 4$

    - Write the standard form equation of the circle:

      $(x+13)^2 + (y+8)^2 = 16$

- ***GIVEN: THE CENTER (h, k) & THE AREA***
  - Consider a circle whose center is (16, 11) and whose area is 4π.
    - Find the radius of the circle:

      $A = \pi R^2$

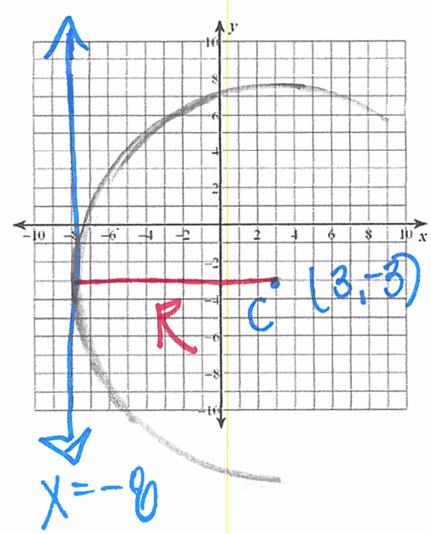
      $4\pi = \pi R^2$
      $4 = R^2$
      $2 = R$

    - Write the standard form equation of the circle:

      $(x-16)^2 + (y-11)^2 = 4$

A tangent of a circle is a line that intersects the circle at exactly one point, called the point of tangency.

- ***GIVEN: THE CENTER (h, k) & A TANGENT***
  - Consider a circle whose center is (3, −3) and that is tangent to $x = -8$
    - Graph the given information on the coordinate plane.



    - Find the radius of the circle:

      $R = 11$

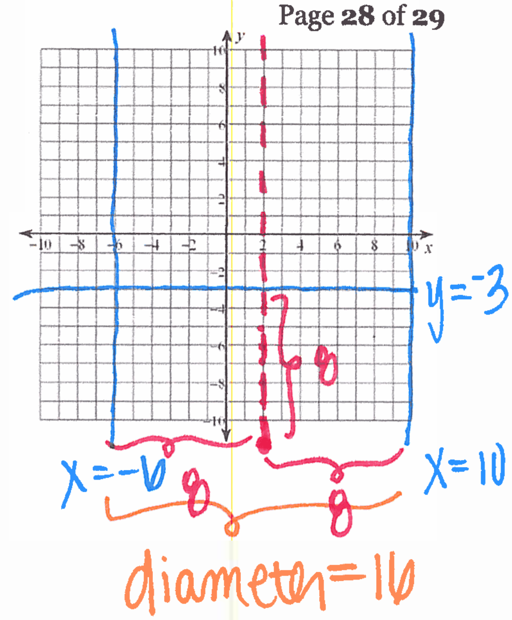
      what is the distance from the center to the tangent?

    - Write the standard form equation of the circle.

      $(x-3)^2 + (y+3)^2 = 121$

- ***GIVEN: LOCATION OF CENTER AND TANGENTS***
  - Consider a circle whose center lies in the fourth quadrant and that is tangent to $y = -3, x = 10$, and $x = -6$
    - Graph the given information on the coordinate plane.
    - Find the radius of the circle:

      $R = 8$



    - Find the center of the circle:

      $(2, -11)$

    - Write the standard form equation of the circle:

      $(x-2)^2 + (y+11)^2 = 64$

10.3 ~ General Form Equation of a Circle

<u>OBJECTIVE:</u>

- Write the standard form equation of a circle given its equation in general form
- Given the equation of a circle in general form, complete the square to find the center and radius

❖ ***GENERAL FORM EQUATION OF A CIRCLE***

- $\boldsymbol{Ax^2 + By^2 + Cx + Dy + E = 0}$, where *A, B, C,* and *D* are constants, *A* = *B*, and $x \neq y$.
  - In order to identify the center and radius of a circle written in general form, it is necessary to rewrite the equation in standard form.

❖ General Form → Standard Form

- Completing the Square

Example:

$$x^2 + y^2 - 4x - 6y + 9 = 0$$

Move the constant term to the right-hand side

Sort/organize the left-hand side, leaving blanks after each linear term

$$\underbrace{x^2 - 4x + \_\_}\ \underbrace{+y^2 - 6y + \_\_} = -9 + \_\_ + \_\_$$

Set up your squares

$$(x - \quad)^2 + (y - \quad)^2$$

(1) Half the middle term; (2) Write it down; (3) Square it; (4) Multiply by a; (5) Add

Repeat for y

$$\underbrace{x^2 - 4x + 4}\ \underbrace{+\,y^2 - 6y + 9} = -9 + 4 + 9$$

Find the sum of the right-hand side of the equation

$$(x - 2)^2 + (y - 3)^2 = 4$$

$R^2 \therefore R = \sqrt{4}$

The center is (2, 3) and the radius is 2.

EXAMPLES: **Write the equation in standard form. Identify the coordinates of the center and the radius.**

1. $x^2 + y^2 - 2x + 4y + 4 = 0$ Center: $(1, -2)$ Radius: 1

$(x^2 - 2x + 1) + (y^2 + 4y + 4) = -4 + 1 + 4$

$(x-1)^2 + (y+2)^2 = 1$

2. $x^2 + y^2 + 10x - 12y + 51 = 0$ Center: $(-5, 6)$ Radius: $\sqrt{10}$

$(x^2 + 10x + 25) + (y^2 - 12y + 36) = -51 + 25 + 36$

$(x+5)^2 + (y-6)^2 = 10$

➤ **Completing the Square (A & $B > 1$)**

Example: $2x^2 + 2y^2 - x + 4y + 2 = 0$

Move the constant term to the right-hand side; FACTOR out the quadratic coefficient

$(2x^2 - x) + (2y^2 + 4y) = -2$

Sort/organize the left-hand side, leaving blanks after each linear term

$2(x^2 - 0.5x + .0625) + 2(y^2 + 2y + 1) = -2 + .125 + 2$

Set up your squares

(1) Half the middle term; (2) write it down; (3) square it; (4) Multiply; (5) Add

$2(x-0.25)^2 + 2(y+1)^2 = 0.125$

Repeat for y

Find the sum of the right-hand side of the equation

$\frac{2(x-0.25)^2}{2} + \frac{2(y+1)^2}{2} = \frac{0.125}{2}$

Divide all three terms by the quadratic coefficient

$(x-0.25)^2 + (y+1)^2 = 0.0625$

Identify the center and radius. Center: $(0.25, -1)$ Radius: 0.25

EXAMPLES: **Write the equation in standard form. Identify the coordinates of the center and the radius.**

3. $4x^2 + 4y^2 - 20x - 32y + 81 = 0$ Center: $(2.5, 4)$ Radius: $\sqrt{2}$

$(4x^2 - 20x) + (4y^2 - 32y) = -81$

$4(x^2 - 5x + 6.25) + 4(y^2 - 8y + 16) = -81 + 25 + 64$

$4(x-2.5)^2 + 4(y-4)^2 = 8$

$(x-2.5)^2 + (y-4)^2 = 2$