How do you simplify radical expressions?

Ex. $\sqrt[3]{448uv^6}$

$$\sqrt[3]{448uv^6} = \sqrt[3]{448} \times \sqrt[3]{u} \times \sqrt[3]{v^6}$$

1. Separate the radical

   Let's look at each part separately.

$\sqrt[3]{448}$

$448 = 4 \times 112$

$= (2 \times 2) \times (2 \times 56)$

$= (2 \times 2) \times (2 \times 7 \times 8)$

$= (2 \times 2) \times (2 \times 7 \times 2 \times 4)$

$= (2 \times 2) \times (2 \times 7 \times 2 \times 2 \times 2)$

$\sqrt[3]{448} = \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7}$

2. First check to see if 448 is a perfect cube: Calculate 448 ^ (1/3)
3. Since it's not, we have to factor 448 into its prime factors
4. Look for a trio of the same factor.
5. There are 2 trios: 2 groups of three 2s

$\sqrt[3]{448} = 2 \times 2\sqrt[3]{7} = 4\sqrt[3]{7}$

6. What comes out? Two 2s, which we multiply together.

$\sqrt[3]{u}$

7. This part is already in simplest form.

$\sqrt[3]{v^6} \rightarrow 6 \div 3 = 2$

$\sqrt[3]{v^6} = v^2$

8. Divide your exponent by your index. (If you don't have a remainder, then you have a perfect cube.)

$4\sqrt[3]{7} \times \sqrt[3]{u} \times v^2$

$4v^2\sqrt[3]{7u}$

9. Combine the three simplified parts. Combine the outsides; combine the insides

How do you multiply radical expressions containing variables?

Ex. $5\sqrt[3]{10n^8} * 4\sqrt[3]{25n^3}$

$$5\sqrt[3]{10n^8} * 4\sqrt[3]{25n^3} = 20\sqrt[3]{250n^{11}}$$

1. Use the product property of radicals: $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$

$$20\sqrt[3]{250n^{11}} = 20 \times \sqrt[3]{250} \times \sqrt[3]{n^{11}}$$

2. Simplify the expression in the radicand. (Follow the steps above.)

$\sqrt[3]{250}$

3. (See steps 2 – 6 above.)

$$250 = 10 \times 25$$

$$250 = 2 \times 5 \times 5 \times 5$$

$$\sqrt[3]{250} = \sqrt[3]{2 \times 5 \times 5 \times 5}$$

$$\sqrt[3]{250} = 5\sqrt[3]{2}$$

$$\sqrt[3]{n^{11}} \rightarrow 11 \div 3 = 3R2$$

$$\sqrt[3]{n^{11}} = n^3\sqrt[3]{n^2}$$

4. Divide the exponent by the index.

   The quotient is the exponent of the variable on the outside of the radical.

   The remainder is the exponent of the variable on the inside of the radical.

$$20 \times 5\sqrt[3]{2} \times n^3\sqrt[3]{n^2} = 100n^3\sqrt[3]{2n^2}$$

5. Combine the two simplified parts w/the term on the outside of the original radical.

**How do you simplify a radical expression – containing variables – by rationalizing the denominator?**

Ex. $\dfrac{\sqrt[3]{5m}}{\sqrt[3]{2m^2}}$

$$\frac{\sqrt[3]{5m}}{\sqrt[3]{2m^2}} = \sqrt[3]{\frac{5m}{2m^2}} = \sqrt[3]{\frac{5}{2m}}$$

1. Use the quotient property to determine whether the fraction in the radicand can be simplified.

   $$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

   An *m* can be cancelled out of both the numerator <u>and</u> the denominator.

2. Check to see if you can take the cube root of the numerator and/or the denominator.

   ~ No can do. ~

$$\frac{\sqrt[3]{5} \times \sqrt[3]{4m^2}}{\sqrt[3]{2m} \times \sqrt[3]{2^2m^2}} = \frac{\sqrt[3]{20m^2}}{2m}$$

3. Separate and rationalize the denominator: Multiply both the numerator and denominator by the cube root that makes the denominator equal to $\sqrt[3]{2^3m^3}$

4. Check whether you can take the cube root of the radical in the numerator.

   ~ No can do. ~

## How do you simplify the quotient of two radical expressions?

Ex. $\frac{5\sqrt[4]{15}}{\sqrt[4]{48}}$

$$\frac{5\sqrt[4]{15}}{\sqrt[4]{48}} = 5\sqrt[4]{\frac{15}{48}} = 5\sqrt[4]{\frac{5}{16}}$$

1. Use the quotient property to determine whether the fraction in the radicand can be simplified.

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

Both the numerator <u>and</u> the denominator are divisible by 3.

$$\frac{5\sqrt[4]{5}}{\sqrt[4]{16}} = \frac{5\sqrt[4]{5}}{2}$$

2. Separate. Check to see if you can take the $4^{th}$ root of the numerator and/or the denominator.

$$16 = 4^2 = (2^2)^2 = 2^4$$

There's no need to rationalize the denominator because we eliminated the radical from the denominator by simplifying fractions and looking for $4^{th}$ roots.

## How do you simplify a radical expression by rationalizing the denominator?

Ex. $\frac{3\sqrt[3]{3}}{\sqrt[3]{5}}$

$$\frac{3\sqrt[3]{3} \times \sqrt[3]{25}}{\sqrt[3]{5} \times \sqrt[3]{5^2}} = \frac{3\sqrt[3]{75}}{5}$$

1. Multiply both the numerator and denominator by the cube root that makes the denominator equal to $\sqrt[3]{5^3}$
2. Check whether you can take the cube root of the radical in the numerator.

~ No can do. ~

How do you simplify the sum (or difference) of two (or more) radical expressions?

Ex. $3\sqrt[4]{486} - 2\sqrt[4]{96}$

$$3\sqrt[4]{486}$$

$486 = 2 \times 243$

$486 = 2 \times 9 \times 27$

$486 = 2 \times 3 \times 3 \times 3 \times 9$

$486 = 2 \times 3 \times 3 \times 3 \times 3 \times 3$

$$3\sqrt[4]{486} = 3\sqrt[4]{2 \times 3 \times 3 \times 3 \times 3 \times 3}$$

$$3\sqrt[4]{486} = 3 \times 3\sqrt[4]{6} = 9\sqrt[4]{6}$$

$$2\sqrt[4]{96}$$

$96 = 3 \times 32$

$96 = 3 \times 4 \times 8$

$96 = 3 \times 2 \times 2 \times 2 \times 4$

$96 = 3 \times 2 \times 2 \times 2 \times 2 \times 2$

$$2\sqrt[4]{96} = 2\sqrt[4]{3 \times 2 \times 2 \times 2 \times 2 \times 2}$$

$$2\sqrt[4]{96} = 4\sqrt[4]{6}$$

$$3\sqrt[4]{486} - 2\sqrt[4]{96} = 9\sqrt[4]{6} - 4\sqrt[4]{6} = 5\sqrt[4]{6}$$

1. Simplify each radical expression.
2. Since 486 is not a perfect 4th, factor into its prime factors.
3. Look for a grouping of 4 of the same factor.
4. What comes out? A 3, which is multiplied w/the 3 on the outside.

   What stays in? The product of the remaining factors: $2 \times 3 = 6$
5. Repeat w/the second radical.
6. Since the indexes and the radicands are the same for both radicals, we can subtract.

## How do you multiply binomial radical expressions?

Ex. $(\sqrt{2}-4)(4\sqrt{2}+3)$

$$(\sqrt{2}-4)(4\sqrt{2}+3)$$

$$\sqrt{2}\times 4\sqrt{2} = 4\sqrt{4} = 4\times 2 = 8$$

$$\sqrt{2}\times 3 = 3\sqrt{2}$$

$$-4\times 4\sqrt{2} = -16\sqrt{2}$$

$$-4\times 3 = -12$$

$$8 + 3\sqrt{2} - 16\sqrt{2} - 12$$

$$-4 - 13\sqrt{2}$$

FOIL & use the product property of radicals:

$$\sqrt[n]{a}\times\sqrt[n]{b} = \sqrt[n]{ab}$$

(Multiply the insides w/the insides & the outsides w/the outsides.)

1. Multiply the first terms.
2. Multiply the outside terms.
3. Multiply the inside terms.
4. Multiply the last terms.
5. Combine like terms.

## How do you rationalize the denominator if it's a binomial radical expression?

Ex. $\dfrac{-2+\sqrt{3}}{4+\sqrt{3}}$

$$\frac{-2+\sqrt{3}}{4+\sqrt{3}}\frac{\times(4-\sqrt{3})}{\times(4-\sqrt{3})}$$

$$\frac{-8+2\sqrt{3}+4\sqrt{3}-\sqrt{9}}{16-4\sqrt{3}+4\sqrt{3}-\sqrt{9}}$$

$$\frac{-8+6\sqrt{3}-3}{16-3} = \frac{-11+6\sqrt{3}}{13}$$

1. Multiply the numerator and denominator by the conjugate of the denominator.
2. (See multiplying process above.)
3. Combine like terms; Make the substitution: $\sqrt{9} = 3$
4. Check to see if there's a common factor that can be taken out of ALL 3 outside terms.