12.7 – Completing the Square

❖ Perfect Square Trinomials

$25x^2 \qquad + 60x \qquad + 36$

$\Downarrow \qquad \Uparrow \qquad \Downarrow$

$5x \qquad \Uparrow \qquad 6$

$\searrow \quad 2 \cdot 5x \cdot 6 \quad \swarrow$

First you take the square roots of terms one and three;
Double their product, and you check and see;
If it matches term two, then you can declare,
"The three-termed trinomial is a perfect square."

Which of the following are perfect square trinomials?

$16x^2 - 24x + 9$	$9x^2 + 6x + 1$	$25x^2 - 60x + 36$
$4x^2 - 4x - 4$	$x^2 - 8x + 81$	$36x^2 - 84x + 49$

❖ Patterns in Perfect Square Trinomials

- ➢ What do you notice about the constants at the end of each perfect square trinomial? How are they related to the original binomial that was squared?

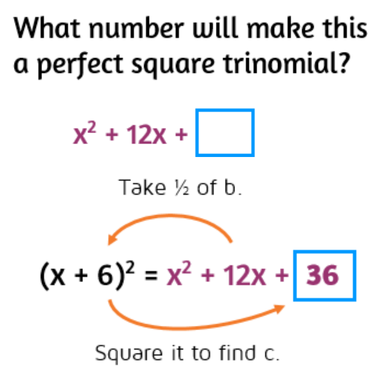
Binomial Squared	→			Perfect Square Trinomial
$(x + 5)^2$	=	$(x + 5)(x + 5)$	=	$x^2 + 10x + 25$
$(x - 8)^2$	=	$(x - 8)(x - 8)$	=	$x^2 - 16x + 64$

- ➢ How is the middle term of the perfect square trinomial related to the original binomial that's being squared?

❖ Completing the Square for $x^2 + bx + c$

- ➢ Take half of b
  - ▪ Write this down inside the parentheses.
- ➢ Square it to find c

$$x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 \;\&\; c = \left(\frac{b}{2}\right)^2$$



EXAMPLES:

Find the value that completes the square and then rewrite as a perfect square in factored form.

1. $x^2 + 18x +$ ____ 2. $x^2 - 30x +$ ____ 3. $x^2 + 5x +$ ____

Notes: Lesson 12.7 – Completing the Square

The equation $y = (x-h)^2 + k$ represents a parabola with vertex (h, k). You can use the method of completing the square to find the vertex of quadratic functions in standard form $y = x^2 + bx + c$.

- Standard Form → Vertex Form
  - Completing the Square $(a = 1)$

Example:

- Sort/organize the right-hand side, leaving a positive blank after the linear term and a negative blank after the constant term
- Set up your square
- Take half the coefficient of the linear term; write it down. Square it; add it after the linear term and subtract it from the constant term: "fill in the blanks"
- Simplify

$$y = x^2 - 4x + 8$$

$$y = \underbrace{x^2 - 4x + \_\_}\, + 8 - \_\_$$

$$y = (x - \quad)^2$$

$$y = \underbrace{x^2 - 4x + 4}\, + 8 - 4$$

$$y = (x-2)^2 + 4$$

Vertex: $(2, 4)$

EXAMPLES:

Complete the square and write the vertex form of the quadratic function. Identify the coordinates of its vertex and state whether the vertex represents a maximum or a minimum value.

4. $y = x^2 + 16x + 71$

5. $y = x^2 - 12x + 46$

More examples of changing forms of quadratic functions:

6. The factored form of a quadratic function is given. Multiply and write the function in standard form.

$$y = -2(x-1)(x+3)$$

7. The standard form of a quadratic function is given. Write the function in factored form and then identify the zeros of the function.

$$y = 3x^2 + 15x + 18$$

Notes: Lesson 12.7 – Completing the Square

- Standard Form → Vertex Form
  - Completing the Square ($a \neq 1$)

Example:

$$y = -3x^2 + 12x - 13$$

- Factor out the coefficient of the quadratic term from the first two terms.

$$y = -3(x^2 - 4x) - 13$$

- Sort/organize the right-hand side, leaving a positive blank after the linear term and a negative blank after the constant term

$$y = -3\underbrace{(x^2 - 4x + \_\_\_)}_{} - 13 - \_\_\_$$

- Set up your square

$$y = -3(x - \quad)^2$$

- Take half the coefficient of the linear term; write it down. Square it; add it after the linear term
- Multiply it by the coefficient factor and subtract the product from the constant term

$$y = -3\underbrace{(x^2 - 4x + 4)}_{} - 13 - (-12)$$

- Simplify

$$y = -3(x - 2)^2 - 1$$

EXAMPLES:

Complete the square and write the vertex form of the quadratic function. Identify the coordinates of its vertex and state whether the vertex represents a maximum or a minimum value.

8. $y = -x^2 - 14x - 59$

9. $y = 2x^2 - 36x + 170$

10. $y = 5x^2 + 70x + 77$

11. $y = -5x^2 + 40x + 1$