12.7 – COMPLETING THE SQUARE

❖ Perfect Square Trinomials

$$\begin{array}{ccc} 25x^2 & +60x & +36 \\ \Downarrow & \Uparrow & \Downarrow \\ 5x & \Uparrow & 6 \\ & 2\cdot 5x\cdot 6 & \end{array}$$

First you take the square roots of terms one and three;
Double their product, and you check and see;
If it matches term two, then you can declare,
"The three-termed trinomial is a perfect square."

Which of the following are perfect square trinomials?

$16x^2 - 24x + 9$ ☆ $(4x - 3)^2$

$4x^2 - 4x - 4$ ~~2x 2~~ NOPE

$9x^2 + 6x + 1$ ☆ $(3x+1)^2$

$x^2 - 8x + 81$ ~~x-9~~ NOPE

$25x^2 - 60x + 36$ ☆ $(5x-6)^2$

$36x^2 - 84x + 49$ ☆ $(6x-7)^2$

❖ Patterns in Perfect Square Trinomials

➢ What do you notice about the constants at the end of each perfect square trinomial? How are they related to the original binomial that was squared?

The constants are positive perfect squares of the constant of the binomial.

Binomial Squared	→			Perfect Square Trinomial
$(x+5)^2$	=	$(x+5)(x+5)$	=	$x^2 + 10x + 25$
$(x-8)^2$	=	$(x-8)(x-8)$	=	$x^2 - 16x + 64$

➢ How is the middle term of the perfect square trinomial related to the original binomial that's being squared?

the middle term is twice the product of the 2 terms of the binomial.
$10x = 2\cdot x\cdot 5$ & $-16x = 2\cdot x\cdot -8$

❖ Completing the Square for $x^2 + bx + c$

➢ Take half of b

- Write this down inside the parentheses.

➢ Square it to find c

$$x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 \ \& \ c = \left(\frac{b}{2}\right)^2$$

What number will make this a perfect square trinomial?

$x^2 + 12x + \boxed{}$

Take ½ of b.

$(x+6)^2 = x^2 + 12x + \boxed{36}$

Square it to find c.

EXAMPLES:

Find the value that completes the square and then rewrite as a perfect square in factored form.

1. $x^2 + 18x +$ 81 $(x+9)^2$
2. $x^2 - 30x +$ 225 $(x-15)^2$
3. $x^2 + 5x +$ 25/4 $(x+5/2)^2$

Notes: Lesson 12.7 – Completing the Square

The equation $y = (x-h)^2 + k$ represents a parabola with vertex (h, k). You can use the method of completing the square to find the vertex of quadratic functions in standard form $y = x^2 + bx + c$.

❖ Standard Form → Vertex Form

➢ Completing the Square ($a = 1$)

Example:

- Sort/organize the right-hand side, leaving a positive blank after the linear term and a negative blank after the constant term
- Set up your square
- Take half the coefficient of the linear term; write it down. Square it; add it after the linear term and subtract it from the constant term: "fill in the blanks"
- Simplify

$$y = x^2 - 4x + 8$$

$$y = \underbrace{x^2 - 4x + \_\_}\ + 8 - \_\_$$

$$y = (x - \quad)^2$$

$$y = \underbrace{x^2 - 4x + 4}\ + 8 - 4$$

$$y = (x-2)^2 + 4$$

Vertex: $(2, 4)$

EXAMPLES:

Complete the square and write the vertex form of the quadratic function. Identify the coordinates of its vertex and state whether the vertex represents a maximum or a minimum value.

4. $y = x^2 + 16x + 71$

$y = x^2 + 16x + \underline{64} + 71 - \underline{64}$

$y = (x+8)^2 + 7$

vertex $(-8, 7)$ minimum

5. $y = x^2 - 12x + 46$

$y = x^2 - 12x + \underline{36} + 46 - \underline{36}$

$y = (x-6)^2 + 10$

vertex $(6, 10)$ minimum

More examples of changing forms of quadratic functions:

6. The factored form of a quadratic function is given. Multiply and write the function in standard form.

$$y = -2(x-1)(x+3)$$

$y = -2(x^2 + 2x - 3)$

$y = -2x^2 - 4x + 6$

7. The standard form of a quadratic function is given. Write the function in factored form and then identify the zeros of the function.

$$y = 3x^2 + 15x + 18$$

$y = 3(x^2 + 5x + 6)$

$y = 3(x+2)(x+3)$

zeros: -2 & -3

Notes: Lesson 12.7 – Completing the Square

- ❖ Standard Form → Vertex Form
  - ➢ Completing the Square ($a \neq 1$)

Example:

$$y = -3x^2 + 12x - 13$$

- Factor out the coefficient of the quadratic term from the first two terms. $y = -3(x^2 - 4x) - 13$
- Sort/organize the right-hand side, leaving a positive blank after the linear term and a negative blank after the constant term $y = \underbrace{-3(x^2 - 4x + \_\_)}\, - 13 - \_\_$
- Set up your square $y = -3(x - \quad)^2$
- Take half the coefficient of the linear term; write it down. Square it; add it after the linear term $y = -3\underbrace{(x^2 - 4x + 4)} - 13 - (-12)$
- Multiply it by the coefficient factor and subtract the product from the constant term $y = -3(x - 2)^2 - 1$
- Simplify

EXAMPLES:

Complete the square and write the vertex form of the quadratic function. Identify the coordinates of its vertex and state whether the vertex represents a maximum or a minimum value.

8. $y = -x^2 - 14x - 59$

$y = -1(x^2 + 14x + \underline{49}) - 59 - \underline{-49}$

$y = -1(x+7)^2 - 10$

vertex $(-7, -10)$

maximum

9. $y = 2x^2 - 36x + 170$

$y = 2(x^2 - 18x + \underline{81}) + 170 - \underline{162}$

$y = 2(x-9)^2 + 8$

vertex $(9, 8)$

minimum

10. $y = 5x^2 + 70x + 77$

$y = 5(x^2 + 14x + \underline{49}) + 77 - \underline{245}$

$y = 5(x+7)^2 - 168$

vertex $(-7, -168)$

minimum

11. $y = -5x^2 + 40x + 1$

$y = -5(x^2 - 8x + \underline{16}) + 1 - \underline{-80}$

$y = -5(x-4)^2 + 81$

vertex $(4, 81)$

maximum