Review of Exponential and Logarithmic Functions

An exponential function is a function in the form of $f(x) = b^x$ for a fixed base b, where $b > 0$ and $b \neq 1$.

b is called the base of the exponential function.
The domain of the function is all real numbers and the range is all positive real numbers.
Note: In the exponential function the variable x is an exponent.

If the base is $b = e \approx 2.71828183$, then $f(x) = e^x$. This is called the natural exponential function.

Evaluating an Exponential Function

Example: $f(x) = 3^{x+2}$ Find $f(-4)$ without using a calculator.

Identification/Analysis	This is an exponential function with base 3.	
Solution	$f(-4) = 3^{-4+2}$ $= 3^{-2}$ $= \frac{1}{3^2} = \frac{1}{9}$	Substitute -4 for x in the function, Simplify. Recall the definition of a negative exponent is $a^{-n} = \frac{1}{a^2}$.

Example: Evaluate $f(x) = 4^x$ at $x = 3$, using a calculator. Round your answer to the nearest thousandth.

Identification/Analysis	This is an exponential function with base 4. Find the appropriate calculator key.	
Solution	<u>Caret key type</u>: 4 [^] 3 [ENTER] <u>Power key type</u>: 4 [y^x] 3 [=] $f(3) = 4^3 = 64$	Calculators use one of two keys. (Note: Square brackets indicate a calculator key.) [^] Caret or power operator key [y^x] or [x^y] base raised to a power key. Check your calculator manual if you do not obtain 64.

Example: Evaluate $f(x) = -e^{3x-4}$ at $x = 2$, using a calculator. Round your answer to the nearest thousandth.

Identification/Analysis	This is an exponential function with base e, called the natural exponential function.	
Solution	$f(2) = -e^{3(2)-4}$ $= -e^{2}$	Substitute 2 for x. Simplify.
	For most calculators types: $-[e]\ 2$ [ENTER] $f(2) = -e^{2} \approx -7.389$	Find the e^x key on your calculator. Enter the – sign e followed by 2. Check your calculator manual if needed to determine the evaluation sequence.

Logarithms

A logarithm is an exponent.

It is the exponent to which the base must be raised to produce a given number.

For example, since $2^3 = 8$, then 3 is called the logarithm with base 2. It is written as: $\log_2 8 = 3$

3 is the exponent to which 2 must be raised to produce 8.

"$2^3 = 8$" is called the exponential form.

"$\log_2 8 = 3$" is called the logarithmic form.

General definition of the **logarithm function with base *b***, $b > 0$ and $b \neq 1$, $x > 0$ is given by:

$\log_b x = y$ is equivalent to $b^y = x$	For example: $\log_3 9 = 2$ is equivalent to $3^2 = 9$

The function is written as $f(x) = \log_b x$ and is read as "log base b of x" or "logarithm of x with base b."

The two most often used bases are 10 and e. These are called the common logarithm and natural logarithm.

Common logarithm A common logarithm is a logarithm with a base 10.
When the base is not indicated, such as $f(x) = \log x$, a base 10 is implied.

$y = \log x$ is equivalent to $10^y = x$

Natural logarithm A natural logarithm is a logarithm with base e.
It is written as $f(x) = \ln x$.

$y = \ln x$ is equivalent to $e^y = x$

Example: Write $\log_5 125 = 3$ in exponential form.

Identification/Analysis	Use the definition of a logarithm to write the logarithm in exponential form. The base on the logarithm is 5 so the base for exponential form is also 5.	
Solution	$\log_5 125 = 3$ $5^3 = 125$	The logarithm is base 5, so the base of the exponential is also 5. The logarithm is equal to 3, so the exponent will be 3

Example: Write $\log 0.01 = -2$ in exponential form.

Identification/Analysis	Use the definition of a logarithm to write the logarithm in exponential form. The base on the logarithm is not indicated so base 10 is implied.	
Solution	$\log 0.01 = -2$ $10^{-2} = 0.01$	The logarithm is base 10, so the base of the exponential is 10. The logarithm is equal to -2, so the exponent will be -2

Example: Write $7^3 = 343$ in logarithmic form.

Identification/Analysis	Use the definition of a logarithm to write the exponential as a logarithm. The base of the exponential is 7, so the base of the logarithm is also 7.	
Solution	$7^3 = 343$ $\log_7 343 = 3$	The exponential is base 7, so the base of the logarithm is also 7. The exponent is 3, so the logarithm will be equal to 3.

Example: Write $\sqrt{10} = x$ in logarithmic form.

Identification/Analysis	Write the radical as an exponent. Use the definition of a logarithm to write in logarithmic form.	
Solution	$\sqrt{10} = x$ $(10)^{1/2} = x$ $\log x = \frac{1}{2}$	Rewrite the radical as an exponent. The base of the exponential is 10 so the base of the logarithm is also 10. The base of the common logarithm is also 10. The exponent is $1/2$, so the logarithm will be equal to $1/2$.

Example: Evaluate $\log_3 81$

Identification/Analysis	Use the definition of a logarithm to write the expression in exponential form. The base on the logarithm is 3, so the base for exponential form is also 3.	
Solution	$\log_3 81 = y$ $3^y = 81$ $3^y = 81 = 3*3*3*3 = 3^4$ $\log_3 81 = 4$	Set the logarithm equal to y. Rewrite the logarithm in exponential form. The base is 3 and the exponent is y. Determine an appropriate exponent. (Factor 81 or write 81 as a power of 3.) The logarithm is the exponent so the logarithm equals 4

Example: Evaluate $\log 1000$

Identification/Analysis	Use the definition of a logarithm to write the expression in exponential form. The base on the logarithm is not indicated so the base is 10.	
Solution	$\log 1000 = y$ $10^y = 1000$ $10^y = 10^3$ $\log 1000 = 3$	Set the logarithm equal to y. Rewrite the logarithm in exponential form. Factor1000 or write 1000 as a power of 10 to determine the exponent. The logarithm is the exponent so the logarithm equals 3.

Example: Evaluate $\log(0.01)$

Identification/Analysis	Use the definition of a logarithm to write the expression in exponential form. The base on the logarithm is not indicated so the base is 10.	
Solution	$\log 0.01 = y$ $10^y = 0.01$ $10^y = \frac{1}{100}$ $10^y = \frac{1}{10^2} = 10^{-2}$ $\log 0.01 = -2$	Set the logarithm equal to y. Rewrite the logarithm in exponential form. Write 0.01 as a fraction. Write 100 as a power of 10 to help determine the exponent. The logarithm is the exponent so the logarithm equals -2.

Example: Evaluate $\ln e^5$

Identification/Analysis	This is a natural logarithm with base e. Use the definition of a logarithm to write the expression in exponential form with base e.	
Solution	$\ln e^5 = y$ $e^y = e^5$ $\ln e^5 = 5$	Set the logarithm equal to y. Rewrite the logarithm in exponential form. The logarithm is the exponent so the logarithm equals 3.

Properties of Logarithm Functions

The first column lists a property of logarithms. The second column provides a rational for the property based on the definition of a logarithm. While the third column provides an example of the property.

Property For $b > 0$, $b \neq 1$	Rational	Example
$\log_b 1 = 0$	$b^0 = 1$	$\log 1 = 0$ Since $10^0 = 1$
$\log_b b = 1$	$b^1 = b$	$\log_3 3 = 1$ Since $3^1 = 3$
$\log_b b^x = x$	$b^x = b^x$	$\log_4 4^{3x} = 3x$ Since $4^{3x} = 4^{3x}$
$b^{\log_b x} = x$	$b^y = x$ is equivalent to $y = \log_b x$ Substitute $y = \log_b x$ into the first statement to validate the identity.	$e^{\ln x} = x$ $10^{\log(5x)} = 5x$
If $\log_b x = \log_b y$ then $x = y$ For $x > 0$ and $y > 0$.	The logarithms have the same base and are equal so the arguments are equal.	If $\log(x+7) = \log(2x)$ then $(x+7) = 2x$ Solve for x. $7 = x$

Example: Evaluate $\log_3 81$

Identification/Analysis	This is a logarithm with base 3, so the base of the exponential will also be base 3.	
Solution	$\log_3 81 = \log_3 3^4$	Write 81 as a power of three since the base on the logarithm is 3.
	$\log_3 3^4 = 4$	Evaluate the logarithm using $\log_b b^x = x$.

Example: Evaluate $\ln\sqrt{e}$

Identification/Analysis	This is the natural logarithm with base e, so the base of the exponential will also be e.	
Solution	$\ln\sqrt{e} = \ln e^{1/2}$ $\ln e^{1/2} = \frac{1}{2}$	Write the radical in exponential form. Evaluate the logarithm since the base of the logarithm and the exponential are both e.

Example: Evaluate log 0.0001

Identification/Analysis	The base on the logarithm is not indicated so the base is 10.	
Solution	$\log 0.0001 = \log \frac{1}{10000}$ $= \log 10^{-4}$ $\log 0.0001 = -4$	Write 0.0001 as a fraction. Write the fraction as a power of 10 since the base is 10 Evaluate the logarithm since the bases of both are 10.

Example: Evaluate: $10^{\log(5x+2)}$

Identification/Analysis	This is an exponential with base 10. In the exponent is a common logarithm with base 10.	
Solution	$10^{\log(5x+2)} = (5x+2)$	Since the base of the exponential and the base of the logarithm are the same, apply $b^{\log_b x} = x$.

Additional Properties of Logarithms

Listed below are properties of logarithms used to expand and condense logarithmic expressions.
The properties apply to any base b but most applications use common $(\log x)$ or natural logarithms $(\ln x)$.

Name of the Property	General Rule Assume $b>0$, $b \neq 1$, $m>0$ and $n>0$	Example Assume $x>0$ and $y>0$	Stated in Words
Product Rule	$\log_b(mn) = \log_b m + \log_b n$	$\log(3x) = \log 3 + \log x$	*The logarithm of a product equals the logarithm of the first factor plus the logarithm of the second factor.*
Quotient Rule	$\log_b \frac{m}{n} = \log_b m - \log_b n$	$\ln \frac{2}{y} = \ln 2 - \ln y$	*The logarithm of a quotient equals the logarithm of the numerator minus the logarithm of the denominator.*
Power Rule	$\log_b m^n = n \log_b m$	$\log x^3 = 3 \log x$	*The logarithm of a power equals the exponent times the logarithm.*
Change of Base Formula	$\log_b m = \frac{\log m}{\log b}$ or $\log_b m = \frac{\ln m}{\ln b}$ Note: The quotient can be written with any base as long as the base in numerator and denominator are the same.	$\log_3 5 = \frac{\log 5}{\log 3}$	*The logarithm base b of m equals the common logarithm of m divided by the common logarithm of the original base b*

For the examples below, only common and natural logarithms are used even though the properties apply to any base.
Assume are variables used are positive numbers.

Example: Expand the logarithm $\ln(2x)$

Identification/Analysis	Expand means to write the logarithm as a sum or difference of simpler logarithms. This is a natural logarithm of a product.	
Solution	$\ln(2x) = \ln 2 + \ln x$	Apply the product rule. The log of a product is the sum of the logs.

Example: Expand. $\log\left(\frac{a^2}{b}\right)$

Identification/Analysis	Expand means to write the logarithm as a sum or difference of simpler logarithms. This common logarithm contains a quotient and a power.	
Solution	$\log\left(\frac{a^2}{b}\right) = \log a^2 - \log b$ $= 2\ \log a - \log b$	Apply the quotient rule. The log of a quotient is the difference of the logs. Apply the power rule. The log of an exponential is the exponent times the log.

Example: Expand. $\ln\left(\frac{x}{3y}\right)$

Identification/Analysis	Expand means to write the logarithm as a sum or difference of simpler logarithms. This natural logarithm contains a quotient and a product.	
Solution	$\ln\left(\frac{x}{3y}\right) = \ln x - \ln(3y)$ $= 2\ \ln x - (\ln 3 + \ln y)$ $= 2\ln x - \ln 3 - \ln y$	Apply the quotient rule. The log of a quotient is the difference of the logs. Apply the product rule. The log of a product is the sum of logs. Be sure to put the sum in parentheses. Distribute the negative. (This is not a required step.)

Example: Expand the logarithm and simplify. $\log\left(\frac{100y}{\sqrt{x}}\right)$

Identification/Analysis	Expand means to write the logarithm as a sum or difference of simpler logarithms. This is a common logarithm contains a quotient, product and a power.	
Solution	$\log\left(\frac{100y}{\sqrt{x}}\right) = \log(100y) - \log\sqrt{x}$ $= \log 100 + \log y - \log\sqrt{x}$ $= \log 10^2 + \log y - \log x^{1/2}$ $= 2 + \log y - \frac{1}{2}\log x$	Apply the quotient rule. Apply the product rule. Write 100 as a power of 10 since the base on the logarithm is 10. Write the radical as an exponent. Apply $\log_b b^x = x$ since the bases are both 10. Apply the power rule.

The following examples use the properties of logarithms in the opposite manner. In other words you start with an expression containing sums and difference of logarithms and condense to a single logarithm

Example: Condense. Write as a single logarithm. $\log x + \log y - \log z$

Identification/Analysis	Condense means to write as a single logarithm. There is a sum and difference of common logarithms. In general when there are several terms, apply the rules one at a time moving from left to right.	
Solution	$\log x + \log y - \log z = \log(xy) - \log z$ $= \log\left(\frac{xy}{z}\right)$	Since there is a sum and difference two rules apply. Apply the product rule in the opposite manner. The sum of logs equals the log of a product. Apply the quotient rule. The difference of logs equals the log of a quotient. The original is written as a single logarithm.

Example: Condense. Write as a single logarithm. $3\ln x - \frac{1}{2}\ln 16$

Identification/Analysis	Condense means to write as a single logarithm. There is a difference of logarithms and a power times a logarithm.	
Solution	$3\ln x - \frac{1}{2}\ln 16 = \ln x^3 - \ln 16^{1/2}$	Apply the power rule.
	$= \ln\left(\frac{x^3}{16^{1/2}}\right)$	Apply the quotient rule.
	$= \ln\left(\frac{x^3}{4}\right)$	Simplify. $16^{1/2} = \sqrt{16}$ The original is written as a single logarithm.

Example: Condense. $\frac{1}{2}(\log x - 3\log y + \log z)$

Identification/Analysis	Condense means to write as a single logarithm. There is a sum and difference of logarithms. There are also parenthesis and a power times a logarithm.	
Solution	$\frac{1}{2}(\log x - 3\log y + \log z)$	Since there is parenthesis, first condense within the parenthesis.
	$= \frac{1}{2}(\log x - \log y^3 + \log z)$	Apply the power rule.
	$= \frac{1}{2}\left(\log \frac{x}{y^3} + \log z\right)$	In general apply the rules one at a time from left to right. Apply the quotient rule.
	$= \frac{1}{2}\left(\log \frac{xz}{y^3}\right)$	Apply the product rule.
	$= \log\left(\frac{xz}{y^3}\right)^{1/2}$ or $\log\sqrt{\frac{xz}{y^3}}$	Apply the power rule. Leave as an exponent or write as a radical. The original is written as a single logarithm.

Example: Condense $4\log x - \log y - 2\log z$.

<table>
<tr><td>Identification/Analysis</td><td colspan="2">Condense means to write as a single logarithm.
There are two differences of logarithms and a logarithm times a power.</td></tr>
<tr><td>Solution</td><td>$4\log x - \log y - 2\log z$
$= \log x^4 - \log y - \log z^2$
$= \log \frac{x^4}{y} - \log z^2$
$= \log \frac{x^4}{yz^2}$</td><td>Apply the power rule.
There are three terms apply the rules from left to right.
Apply the quotient rule.
Apply the quotient rule.
The original is written as a single logarithm.</td></tr>
</table>

Example: Use a calculator to evaluate $\log_7 3$. Round to the nearest thousandth.

<table>
<tr><td>Identification/Analysis</td><td colspan="2">The logarithm is base 7. To evaluate with the calculator it must be written with a base of 10 or e.
Use the change of base formula.</td></tr>
<tr><td>Solution</td><td>$\log_7 3 = \frac{\log 3}{\log 7}$ or $\log_7 3 = \frac{\ln 3}{\ln 7}$
$\log_7 3 = \frac{\log 3}{\log 7} \approx 0.565$</td><td>Apply the change of base formula.
Use either the common log or natural log.
Type [log] 3 [÷] [log] 7 [ENTER] and round appropriately.</td></tr>
</table>

Domain of the logarithmic function

The function $f(x) = \log_b x, \quad b > 0,\ b \neq 1$ has a domain $x > 0$ Also written $\{x \mid x \in \mathbb{R},\ x > 0\}$ or using interval notation $(0, \infty)$
In words *the argument of the logarithm must be positive*. The base of the logarithm does not affect the domain.

Example: Find the domain of $f(x) = \log(x - 7)$

Identification/Analysis	The argument of the logarithm has to be positive. The base of the logarithm does not affect the domain.	
Solution	$(x-7) > 0$ $x > 7$ $\{x \mid x \in \mathbb{R},\ x > 7\}$. or $(7, \infty)$	The argument of a logarithm must be positive. Solve for x. State the domain using set builder or interval notation.

Example: Find the domain of $f(x) = \ln(2 - 3x)$

Identification/Analysis	The argument of the logarithm has to be positive. The base of the logarithm does not affect the domain.	
Solution	$(2-3x) > 0$ $-3x > -2$ $x < \frac{2}{3}$ $\left\{x \mid x \in \mathbb{R},\ x < \frac{2}{3}\right\}$ or $\left(-\infty, \frac{2}{3}\right)$	The argument of a logarithm must be positive. Solve for x. Remember when you divide both sides by a negative number the direction of the inequality is reversed. State the domain using set builder or interval notation.

Solving Exponential and Logarithmic Equations

An exponential equation is an equation where the variable is an exponent. In general, consolidate the exponential terms, then isolate the exponential term containing the variable. Take the logarithm of both sides of the equation. Simplify the logarithm or use the power rule to write the variable as a factor instead of an exponent.

Example: Solve $e^x + 4 = 27$

Identification/Analysis	This is an equation where the variable is an exponent. The base of the exponential is e so use the natural logarithm.	
Solution	$e^x + 4 = 27$ $e^x = 23$ $\ln e^x = \ln 23$ $x = \ln 23$	Isolate the term containing the variable. Take the natural logarithm of both sides. Simplify the natural logarithm of e to a power Solve for x.
Check/verification	$e^{\ln 23} + 4 = 27$ $23 + 4 = 27$	Substitute $x = \ln 23$ in the original equation. Simplify. True, so $x = \ln 23$ is a solution.

Example: Solve $10^{x+3} = 15$ Solve exactly then use your calculator to estimate the solution to the nearest thousandth.

Identification/Analysis	This is an equation where the variable is an exponent. The base of the exponential is 10 so use common logarithms.	
Solution	$10^{x+3} = 15$ $\log 10^{x+3} = \log 15$ $x + 3 = \log 15$ $x = \log 15 - 3 \approx -1.824$	The term with the variable is isolated. Take the common logarithm of both sides. Simplify the common logarithm of 10 to a power. Solve exactly for x. Approximate with your calculator.
Check/verification	$10^{(\log 15 - 3)+3} = 15$ $10^{\log 15} = 15$ $15 = 15$	Substitute $x = \log 15 - 3$ into the original equation. Simplify the right side. Combine $-3 + 3$. The base of the exponential and the logarithm are the same. True, so $x = \log 15 - 3$ is a solution.

Example: Solve $3^{2x+1} = 27$

Identification/Analysis	This is an equation where the variable is an exponent. There are two distinct bases, 3 and 27. Both bases can be written using base 3.	
Solution	$3^{2x+1} = 27$ $3^{2x+1} = 3^3$ $2x+1=3$ $2x=2$ $x=1$	Both exponentials can be written using base 3. Write each exponential with base 3. Since the bases are equal the exponents are equal. Solve for x.
Check/verification	$3^{2(1)+1} = 27$ $3^3 = 27$	Substitute $x=1$ into the original equation. Simplify. True so, $x=1$ is a solution.

Example: Solve $3^x = 2^{x+1}$ Find the exact

Identification/Analysis	This is an equation where the variable is an exponent. There are two distinct bases, 3 and 2. The bases do not have a common factor so we cannot write them using the same base	
Solution	$3^x = 2^{x+1}$ $\log 3^x = \log 2^{(x+1)}$ $x\log 3 = (x+1)\log 2$ $x\log 3 = x\log 2 + \log 2$ $x\log 3 - x\log 2 = \log 2$ $x(\log 3 - \log 2) = \log 2$ $x = \dfrac{\log 2}{\log 3 - \log 2}$ $x = \dfrac{\log 2}{\log 3 - \log 2} \approx 1.710$	The bases are distinct and cannot be written using the same base. Take the log of both sides. Use either common or natural log. Apply the power rule. Solve for x. Distribute $\log 2$. Collect the variable x on one side of the equation. Factor out the variable x. Divide both sides by $(\log 3 - \log 2)$, the coefficient of x. Approximate with a calculator. Round to thousandths.
Check/verification	$3^{1.7095} = 2^{1.7095+1}$ so $x \approx 1.710$ $6.54092 \approx 6.54095$	Substitute the approximated value for x in the original equation. Evaluate with a calculator. The two sides are $\approx$.

Logarithmic Equations

A logarithmic equation is an equation that has a logarithmic expression containing a variable.

Example: Solve $\log(x-5)+6=8$

Identification/Analysis	This is a logarithmic equation with a common logarithm. There is one logarithmic term.	
Solution	$\log(x-5)+6=8$ $\log(x-5)=2$ $10^2=(x-5)$ $10^2+5=x$ $x=105$	There is a logarithmic term which contains a variable. Isolate the logarithmic term. Use the definition of a logarithm to write in exponential form. Solve for x. Simplify.
Check/verification	$\log(105-5)+6=8$ $\log 100+6=8$ $2+6=8$ $8=8$	Substitute $x=105$ in the original equation. Simplify each side independently. Use $100=10^2$ to simplify the common logarithm. True so $x=105$ is a solution.

Example: Solve $3+\ln(x-2)=5$ Solve exactly then use your calculator to estimate the solution to the nearest thousandth.

Identification/Analysis	This is a logarithmic equation with a natural logarithm. There is one logarithmic term.	
Solution	$3+\ln(x-2)=5$ $\ln(x-2)=2$ $e^2=(x-2)$ $x=e^2+2\approx 9.389$	There is a logarithmic term which contains a variable. Isolate the logarithmic term. Use the definition of a logarithm to write in exponential form. Solve for x. Approximate with a calculator.
Check/verification	$3+\ln(9.389-2)=5$ $4.999992\approx 5$	Substitute $x=9.389$ in the original equation. Simplify the left side using a calculator. The two sides are approximately equal.

Example: Solve $\log x - \log 4 = 2$

Identification/Analysis	This is a logarithmic equation with a common logarithm. There are two logarithmic terms and a term without a logarithm. Use properties of logarithms to condense to a single logarithmic term.	
Solution	$\log x - \log 4 = 2$ $\log \frac{x}{4} = 2$ $10^2 = \frac{x}{4}$ $x = 4 * 10^2 = 400$	There are two logarithmic terms. Apply the quotient rule. Use the definition of a logarithm to write in exponential form. Solve for x and simplify.
Check/Verification	$\log 400 - \log 4 = 2$ $2 = 2$	Substitute $x = 400$ in the original equation. Evaluate the left side using a calculator. True so $x = 400$ is a solution.

Example: Solve $\ln(3x+1) = \ln 7$

Identification/Analysis	This is a logarithmic equation with a natural logarithm. There are two logarithmic terms and no terms without a logarithm.	
Solution	$\ln(3x+1) = \ln 7$ $(3x+1) = 7$ $3x = 6$ $x = 2$	There is a single logarithm on each side of the equation. The logarithms are equal so the arguments are equal. Solve for x.
Check/Verification	$\ln(3*2+1) = \ln 7$ $\ln 7 = \ln 7$	Substitute $x = 2$ in the original equation. Simplify. True so $x = 2$.

Example: Solve $\log(x+21)+\log x=2$ Solve exactly then use a calculator to approximate the solution to the nearest thousandth.

Identification/Analysis	This is a logarithmic equation with a common logarithm. There are two logarithmic terms. Use properties of logarithms to condense to a single logarithmic term.	
Solution	$\log(x+21)+\log x=2$ $\log[(x+21)(x)]=2$ $10^2=[(x+21)(x)]$ $100=x^2+21x$ $x^2-21x-100=0$ $(x+25)(x-4)=0$ $x=-25$ or $x=4$	There are two logarithmic terms. Apply the product rule. Use the definition of a logarithm to write in exponential form. Solve for x. Second degree equation Factor or use the quadratic equation.
Check/Verification	$\log(-25+21)+\log-25=2$ $\log(2+21)+\log 4=2$ $2=2$	Substitute $x=-25$ in the original equation. Notice this requires taking the logarithm of a negative number. The domain of the logarithm requires the argument to be positive so $x=-25$ is not a solution to the equation. Substitute $x=4$ in the original equation. Evaluate the left side using a calculator. True so $x=4$ is a solution.

Example: Solve $\log(x+3) = \log(x-1) + \log 3$

Identification/Analysis	This is a logarithmic equation with a common logarithm. There are two logarithmic terms and no terms without a logarithm	
Solution	$\log(x+3) = \log(x-1) + \log 3$ $\log(x+3) = \log[(x-1)*3]$ $(x+3) = (x-1)*3$ $x+3 = 3x-3$ $6 = 2x$ $x = 3$	There are three logarithmic terms. Condense each side. Apply the product rule. The logarithms are equal so the arguments are equal. Solve for x.
Check/Verification	$\log(3+3) = \log(3-1) + \log 3$ $\log(6) = \log(2) + \log 3$ $\log 6 = \log(2*3)$ $0.778 = 0.301 + 0.477$	Substitute $x = 2$ in the original equation. Simplify. Apply the product rule to the right side. Or approximate each side using a calculator. In either case the statement is true so $x = 3$ is a solution.