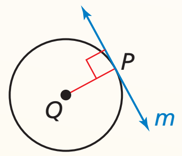
Using Properties of Tangents

Theorems

Theorem 10.1 Tangent Line to Circle Theorem

In a plane, a line is tangent to a circle if and only if the line is perpendicular to a radius of the circle at its endpoint on the circle.

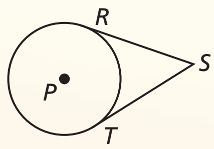


Line m is tangent to $\odot Q$ if and only if $m \perp \overline{QP}$.

Proof Ex. 47, p. 540

Theorem 10.2 External Tangent Congruence Theorem

Tangent segments from a common external point are congruent.

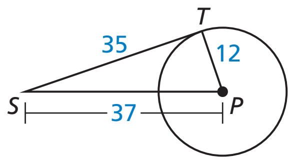


If $\overline{SR}$ and $\overline{ST}$ are tangent segments, then $\overline{SR} \cong \overline{ST}$.

Proof Ex. 46, p. 540

EXAMPLE 3 Verifying a Tangent to a Circle

Is $\overline{ST}$ tangent to $\odot P$?



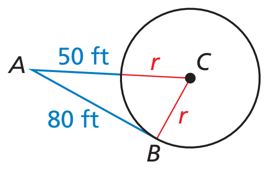
SOLUTION

Use the Converse of the Pythagorean Theorem (Theorem 9.2). Because $12^2 + 35^2 = 37^2$, $\triangle PTS$ is a right triangle and $\overline{ST} \perp \overline{PT}$. So, $\overline{ST}$ is perpendicular to a radius of $\odot P$ at its endpoint on $\odot P$.

▶ By the Tangent Line to Circle Theorem, $\overline{ST}$ is tangent to $\odot P$.

EXAMPLE 4 Finding the Radius of a Circle

In the diagram, point B is a point of tangency. Find the radius r of $\odot C$.



SOLUTION

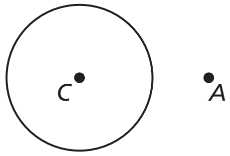
You know from the Tangent Line to Circle Theorem that $\overline{AB} \perp \overline{BC}$, so $\triangle ABC$ is a right triangle. You can use the Pythagorean Theorem (Theorem 9.1).

$AC^2 = BC^2 + AB^2$	Pythagorean Theorem
$(r + 50)^2 = r^2 + 80^2$	Substitute.
$r^2 + 100r + 2500 = r^2 + 6400$	Multiply.
$100r = 3900$	Subtract r^2 and 2500 from each side.
$r = 39$	Divide each side by 100.

▶ The radius is 39 feet.

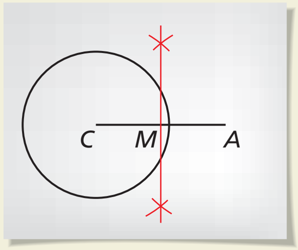
CONSTRUCTION Constructing a Tangent to a Circle

Given ⊙C and point A, construct a line tangent to ⊙C that passes through A. Use a compass and straightedge.



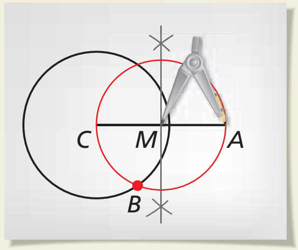
SOLUTION

Step 1



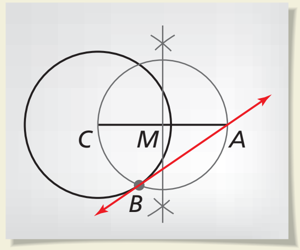
Find a midpoint
Draw $\overline{AC}$. Construct the bisector of the segment and label the midpoint M.

Step 2



Draw a circle
Construct ⊙M with radius MA. Label one of the points where ⊙M intersects ⊙C as point B.

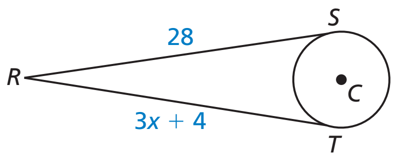
Step 3



Construct a tangent line
Draw $\overleftrightarrow{AB}$. It is a tangent to ⊙C that passes through A.

EXAMPLE 5 Using Properties of Tangents

$\overline{RS}$ is tangent to ⊙C at S, and $\overline{RT}$ is tangent to ⊙C at T. Find the value of x.



SOLUTION

$RS = RT$	External Tangent Congruence Theorem
$28 = 3x + 4$	Substitute.
$8 = x$	Solve for x.

▶ The value of x is 8.

Monitoring Progress

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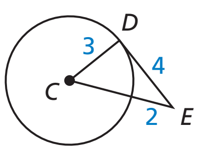
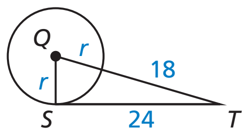
6. Is $\overline{DE}$ tangent to ⊙C?



7. $\overline{ST}$ is tangent to ⊙Q. Find the radius of ⊙Q.



8. Points M and N are points of tangency. Find the value(s) of x.