10.4 Lesson

Core Vocabulary

inscribed angle, *p. 558*
intercepted arc, *p. 558*
subtend, *p. 558*
inscribed polygon, *p. 560*
circumscribed circle, *p. 560*

What You Will Learn

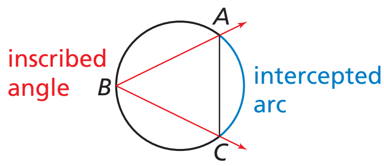
- Use inscribed angles.
- Use inscribed polygons.

Using Inscribed Angles

Core Concept

Inscribed Angle and Intercepted Arc

An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. An arc that lies between two lines, rays, or segments is called an **intercepted arc**. If the endpoints of a chord or arc lie on the sides of an inscribed angle, then the chord or arc is said to **subtend** the angle.

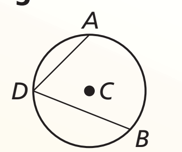


$\angle B$ intercepts $\overset{\frown}{AC}$.
$\overset{\frown}{AC}$ subtends $\angle B$.
$\overline{AC}$ subtends $\angle B$.

Theorem

Theorem 10.10 Measure of an Inscribed Angle Theorem

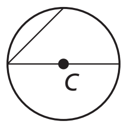
The measure of an inscribed angle is one-half the measure of its intercepted arc.



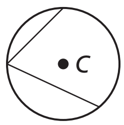
$m\angle ADB = \frac{1}{2}m\overset{\frown}{AB}$

Proof Ex. 37, p. 564

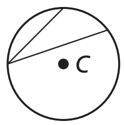
The proof of the Measure of an Inscribed Angle Theorem involves three cases.



Case 1 Center C is on a side of the inscribed angle.



Case 2 Center C is inside the inscribed angle.



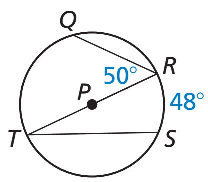
Case 3 Center C is outside the inscribed angle.

EXAMPLE 1 **Using Inscribed Angles**

Find the indicated measure.

a. $m\angle T$

b. $m\overset{\frown}{QR}$



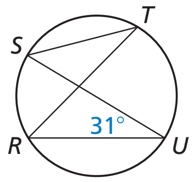
SOLUTION

a. $m\angle T = \frac{1}{2}m\overset{\frown}{RS} = \frac{1}{2}(48°) = 24°$

b. $m\overset{\frown}{TQ} = 2m\angle R = 2 \cdot 50° = 100°$
Because $\overset{\frown}{TQR}$ is a semicircle, $m\overset{\frown}{QR} = 180° - m\overset{\frown}{TQ} = 180° - 100° = 80°$.

EXAMPLE 2 Finding the Measure of an Intercepted Arc

Find $m\overset{\frown}{RS}$ and $m\angle STR$. What do you notice about $\angle STR$ and $\angle RUS$?



SOLUTION

From the Measure of an Inscribed Angle Theorem, you know that $m\overset{\frown}{RS} = 2m\angle RUS = 2(31^\circ) = 62^\circ$.

Also, $m\angle STR = \frac{1}{2}m\overset{\frown}{RS} = \frac{1}{2}(62^\circ) = 31^\circ$.

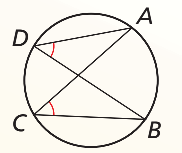
▶ So, $\angle STR \cong \angle RUS$.

Example 2 suggests the Inscribed Angles of a Circle Theorem.

Theorem

Theorem 10.11 Inscribed Angles of a Circle Theorem

If two inscribed angles of a circle intercept the same arc, then the angles are congruent.

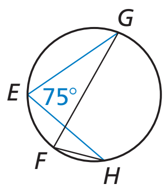


$\angle ADB \cong \angle ACB$

Proof Ex. 38, p. 564

EXAMPLE 3 Finding the Measure of an Angle

Given $m\angle E = 75^\circ$, find $m\angle F$.



SOLUTION

Both $\angle E$ and $\angle F$ intercept $\overset{\frown}{GH}$. So, $\angle E \cong \angle F$ by the Inscribed Angles of a Circle Theorem.

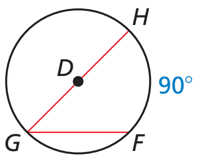
▶ So, $m\angle F = m\angle E = 75^\circ$.

Monitoring Progress

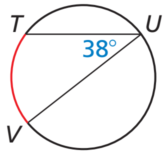
Help in English and Spanish at *BigIdeasMath.com*

Find the measure of the red arc or angle.

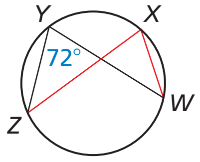
1.



2.



3.

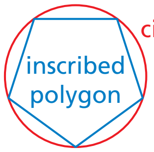


Using Inscribed Polygons

Core Concept

Inscribed Polygon

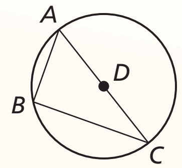
A polygon is an **inscribed polygon** when all its vertices lie on a circle. The circle that contains the vertices is a **circumscribed circle**.



Theorems

Theorem 10.12 Inscribed Right Triangle Theorem

If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.

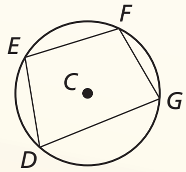


$m\angle ABC = 90^\circ$ if and only if $\overline{AC}$ is a diameter of the circle.

Proof Ex. 39, p. 564

Theorem 10.13 Inscribed Quadrilateral Theorem

A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.



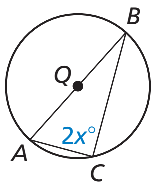
D, E, F, and G lie on $\odot C$ if and only if $m\angle D + m\angle F = m\angle E + m\angle G = 180^\circ$.

Proof Ex. 40, p. 564

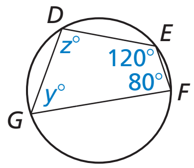
EXAMPLE 4 Using Inscribed Polygons

Find the value of each variable.

a.



b.



SOLUTION

a. $\overline{AB}$ is a diameter. So, $\angle C$ is a right angle, and $m\angle C = 90^\circ$ by the Inscribed Right Triangle Theorem.

$$2x^\circ = 90^\circ$$

$$x = 45$$

▶ The value of x is 45.

b. $DEFG$ is inscribed in a circle, so opposite angles are supplementary by the Inscribed Quadrilateral Theorem.

$$m\angle D + m\angle F = 180^\circ \qquad m\angle E + m\angle G = 180^\circ$$

$$z + 80 = 180 \qquad 120 + y = 180$$

$$z = 100 \qquad y = 60$$

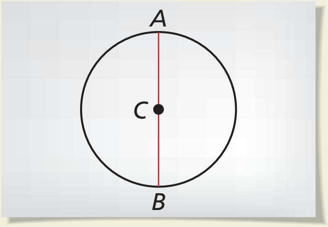
▶ The value of z is 100 and the value of y is 60.

CONSTRUCTION Constructing a Square Inscribed in a Circle

Given $\odot C$, construct a square inscribed in a circle.

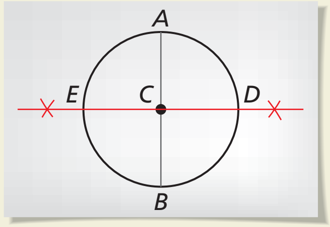
SOLUTION

Step 1



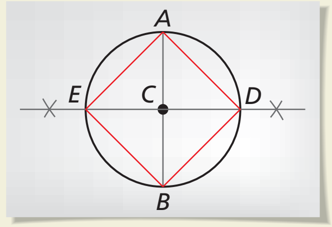
Draw a diameter
Draw any diameter. Label the endpoints A and B.

Step 2



Construct a perpendicular bisector
Construct the perpendicular bisector of the diameter. Label the points where it intersects $\odot C$ as points D and E.

Step 3



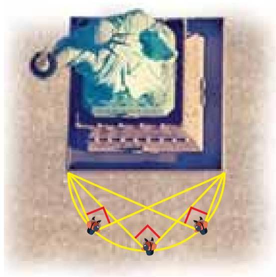
Form a square
Connect points A, D, B, and E to form a square.

EXAMPLE 5 Using a Circumscribed Circle

Your camera has a 90° field of vision, and you want to photograph the front of a statue. You stand at a location in which the front of the statue is all that appears in your camera's field of vision, as shown. You want to change your location. Where else can you stand so that the front of the statue is all that appears in your camera's field of vision?

SOLUTION

From the Inscribed Right Triangle Theorem, you know that if a right triangle is inscribed in a circle, then the hypotenuse of the triangle is a diameter of the circle. So, draw the circle that has the front of the statue as a diameter.



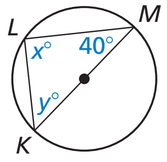
▶ The statue fits perfectly within your camera's 90° field of vision from any point on the semicircle in front of the statue.

Monitoring Progress

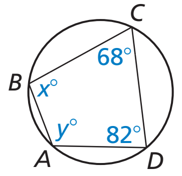
Help in English and Spanish at *BigIdeasMath.com*

Find the value of each variable.

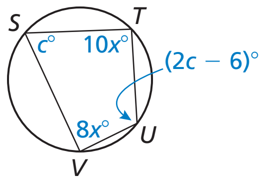
4.



5.



6.



7. In Example 5, explain how to find locations where the front and left side of the statue are all that appears in your camera's field of vision.