10.5 Lesson

Core Vocabulary

circumscribed angle, *p. 568*

Previous
tangent
chord
secant

What You Will Learn

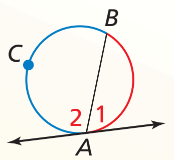
- Find angle and arc measures.
- Use circumscribed angles.

Finding Angle and Arc Measures

Theorem

Theorem 10.14 Tangent and Intersected Chord Theorem

If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one-half the measure of its intercepted arc.



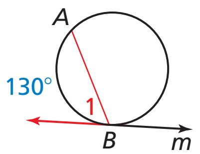
Proof Ex. 33, p. 572

$m\angle 1 = \frac{1}{2}m\overset{\frown}{AB}$ $\quad m\angle 2 = \frac{1}{2}m\overset{\frown}{BCA}$

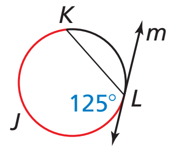
EXAMPLE 1 Finding Angle and Arc Measures

Line *m* is tangent to the circle. Find the measure of the red angle or arc.

a.



b.



SOLUTION

a. $m\angle 1 = \frac{1}{2}(130^\circ) = 65^\circ$

b. $m\overset{\frown}{KJL} = 2(125^\circ) = 250^\circ$

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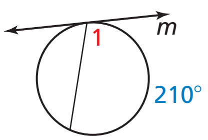
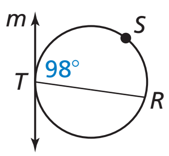
Line *m* is tangent to the circle. Find the indicated measure.

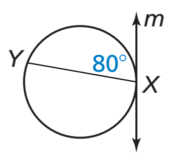
1. $m\angle 1$



2. $m\overset{\frown}{RST}$



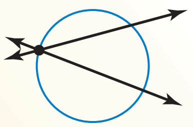
3. $m\overset{\frown}{XY}$



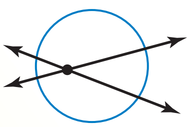
Core Concept

Intersecting Lines and Circles

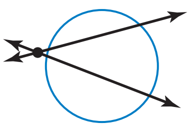
If two nonparallel lines intersect a circle, there are three places where the lines can intersect.



on the circle



inside the circle

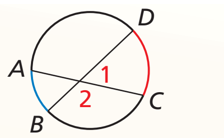


outside the circle

Theorems

Theorem 10.15 Angles Inside the Circle Theorem

If two chords intersect *inside* a circle, then the measure of each angle is one-half the *sum* of the measures of the arcs intercepted by the angle and its vertical angle.



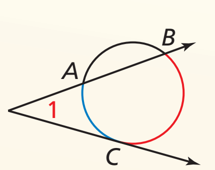
$m\angle 1 = \frac{1}{2}(m\widehat{DC} + m\widehat{AB})$,

$m\angle 2 = \frac{1}{2}(m\widehat{AD} + m\widehat{BC})$

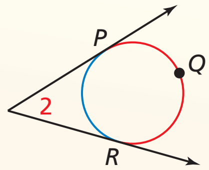
Proof Ex. 35, p. 572

Theorem 10.16 Angles Outside the Circle Theorem

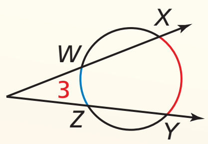
If a tangent and a secant, two tangents, or two secants intersect *outside* a circle, then the measure of the angle formed is one-half the *difference* of the measures of the intercepted arcs.



$m\angle 1 = \frac{1}{2}(m\widehat{BC} - m\widehat{AC})$



$m\angle 2 = \frac{1}{2}(m\widehat{PQR} - m\widehat{PR})$



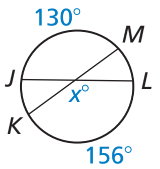
$m\angle 3 = \frac{1}{2}(m\widehat{XY} - m\widehat{WZ})$

Proof Ex. 37, p. 572

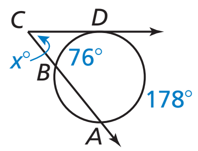
EXAMPLE 2 Finding an Angle Measure

Find the value of x.

a.



b.



SOLUTION

a. The chords $\overline{JL}$ and $\overline{KM}$ intersect inside the circle. Use the Angles Inside the Circle Theorem.

$$x^\circ = \frac{1}{2}(m\widehat{JM} + m\widehat{LK})$$

$$x^\circ = \frac{1}{2}(130^\circ + 156^\circ)$$

$$x = 143$$

▶ So, the value of x is 143.

b. The tangent $\overleftrightarrow{CD}$ and the secant $\overleftrightarrow{CB}$ intersect outside the circle. Use the Angles Outside the Circle Theorem.

$$m\angle BCD = \frac{1}{2}(m\widehat{AD} - m\widehat{BD})$$

$$x^\circ = \frac{1}{2}(178^\circ - 76^\circ)$$

$$x = 51$$

▶ So, the value of x is 51.

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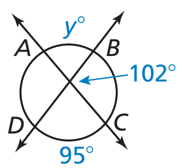
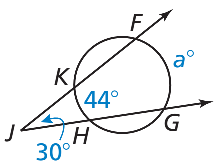
Find the value of the variable.

4.



5.

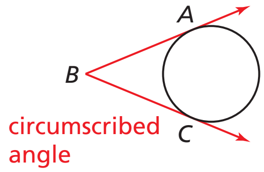


Using Circumscribed Angles

Core Concept

Circumscribed Angle

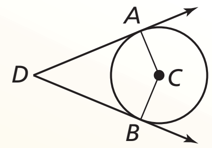
A **circumscribed angle** is an angle whose sides are tangent to a circle.



Theorem

Theorem 10.17 Circumscribed Angle Theorem

The measure of a circumscribed angle is equal to 180° minus the measure of the central angle that intercepts the same arc.



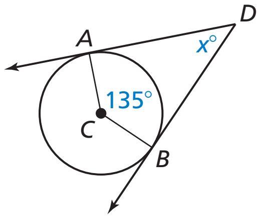
$m\angle ADB = 180^\circ - m\angle ACB$

Proof Ex. 38, p. 572

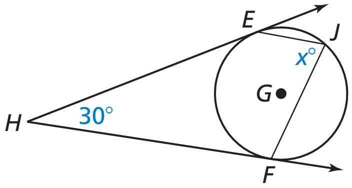
EXAMPLE 3 **Finding Angle Measures**

Find the value of x.

a.



b.



SOLUTION

a. By definition, $m\overset{\frown}{AB} = m\angle ACB = 135^\circ$. Use the Circumscribed Angle Theorem to find $m\angle ADB$.

$m\angle ADB = 180^\circ - m\angle ACB$	Circumscribed Angle Theorem
$x^\circ = 180^\circ - 135^\circ$	Substitute.
$x = 45$	Subtract.

▶ So, the value of x is 45.

b. Use the Measure of an Inscribed Angle Theorem (Theorem 10.10) and the Circumscribed Angle Theorem to find $m\angle EJF$.

$m\angle EJF = \frac{1}{2}m\overset{\frown}{EF}$	Measure of an Inscribed Angle Theorem
$m\angle EJF = \frac{1}{2}m\angle EGF$	Definition of minor arc
$m\angle EJF = \frac{1}{2}(180^\circ - m\angle EHF)$	Circumscribed Angle Theorem
$m\angle EJF = \frac{1}{2}(180^\circ - 30^\circ)$	Substitute.
$x = \frac{1}{2}(180 - 30)$	Substitute.
$x = 75$	Simplify.

▶ So, the value of x is 75.