10.6 Lesson

Core Vocabulary

segments of a chord, *p. 574*
tangent segment, *p. 575*
secant segment, *p. 575*
external segment, *p. 575*

What You Will Learn

- Use segments of chords, tangents, and secants.

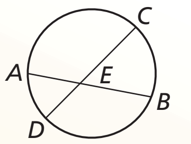
Using Segments of Chords, Tangents, and Secants

When two chords intersect in the interior of a circle, each chord is divided into two segments that are called **segments of the chord**.

Theorem

Theorem 10.18 Segments of Chords Theorem

If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

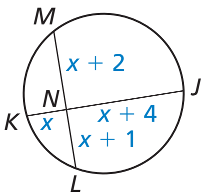


$EA \cdot EB = EC \cdot ED$

Proof Ex. 19, p. 578

EXAMPLE 1 Using Segments of Chords

Find ML and JK.



SOLUTION

$NK \cdot NJ = NL \cdot NM$	Segments of Chords Theorem
$x \cdot (x + 4) = (x + 1) \cdot (x + 2)$	Substitute.
$x^2 + 4x = x^2 + 3x + 2$	Simplify.
$4x = 3x + 2$	Subtract x^2 from each side.
$x = 2$	Subtract $3x$ from each side.

Find ML and JK by substitution.

$$ML = (x + 2) + (x + 1) = 2 + 2 + 2 + 1 = 7$$

$$JK = x + (x + 4) = 2 + 2 + 4 = 8$$

- So, $ML = 7$ and $JK = 8$.

Monitoring Progress

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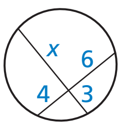
Find the value of x.

1.



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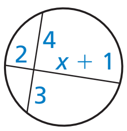
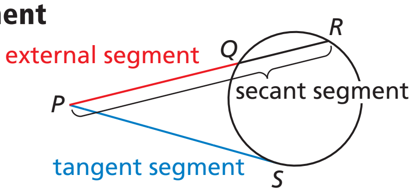


Core Concept

Tangent Segment and Secant Segment

A **tangent segment** is a segment that is tangent to a circle at an endpoint. A **secant segment** is a segment that contains a chord of a circle and has exactly one endpoint outside the circle. The part of a secant segment that is outside the circle is called an **external segment**.

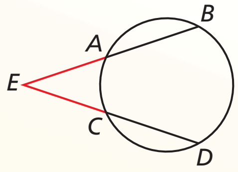


$\overline{PS}$ is a tangent segment.
$\overline{PR}$ is a secant segment.
$\overline{PQ}$ is the external segment of $\overline{PR}$.

Theorem

Theorem 10.19 Segments of Secants Theorem

If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.

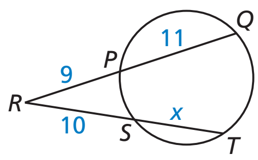


$EA \cdot EB = EC \cdot ED$

Proof Ex. 20, p. 578

EXAMPLE 2 Using Segments of Secants

Find the value of x.



SOLUTION

$RP \cdot RQ = RS \cdot RT$	Segments of Secants Theorem
$9 \cdot (11 + 9) = 10 \cdot (x + 10)$	Substitute.
$180 = 10x + 100$	Simplify.
$80 = 10x$	Subtract 100 from each side.
$8 = x$	Divide each side by 10.

► The value of x is 8.

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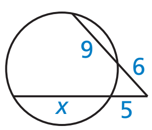
Find the value of x.

3.



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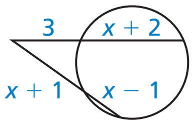
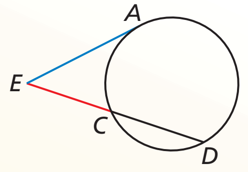


Theorem

Theorem 10.20 Segments of Secants and Tangents Theorem

If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.

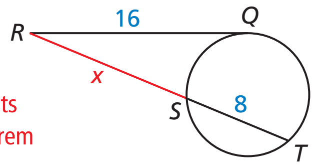


$EA^2 = EC \cdot ED$

Proof Exs. 21 and 22, p. 578

EXAMPLE 3 Using Segments of Secants and Tangents

Find RS.



SOLUTION

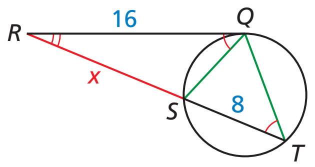
$RQ^2 = RS \cdot RT$	Segments of Secants and Tangents Theorem
$16^2 = x \cdot (x + 8)$	Substitute.
$256 = x^2 + 8x$	Simplify.
$0 = x^2 + 8x - 256$	Write in standard form.
$x = \dfrac{-8 \pm \sqrt{8^2 - 4(1)(-256)}}{2(1)}$	Use Quadratic Formula.
$x = -4 \pm 4\sqrt{17}$	Simplify.

Use the positive solution because lengths cannot be negative.

▶ So, $x = -4 + 4\sqrt{17} \approx 12.49$, and $RS \approx 12.49$.

ANOTHER WAY

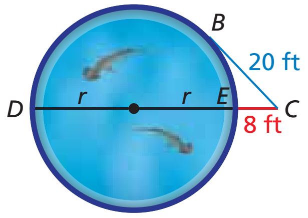
In Example 3, you can draw segments $\overline{QS}$ and $\overline{QT}$.



Because $\angle RQS$ and $\angle RTQ$ intercept the same arc, they are congruent. By the Reflexive Property of Congruence (Theorem 2.2), $\angle QRS \cong \angle TRQ$. So, $\triangle RSQ \sim \triangle RQT$ by the AA Similarity Theorem (Theorem 8.3). You can use this fact to write and solve a proportion to find x.

EXAMPLE 4 Finding the Radius of a Circle

Find the radius of the aquarium tank.



SOLUTION

$CB^2 = CE \cdot CD$	Segments of Secants and Tangents Theorem
$20^2 = 8 \cdot (2r + 8)$	Substitute.
$400 = 16r + 64$	Simplify.
$336 = 16r$	Subtract 64 from each side.
$21 = r$	Divide each side by 16.

▶ So, the radius of the tank is 21 feet.

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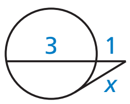
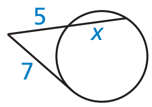
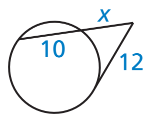
Find the value of x.

5. 

6. 

7. 

8. **WHAT IF?** In Example 4, $CB = 35$ feet and $CE = 14$ feet. Find the radius of the tank.