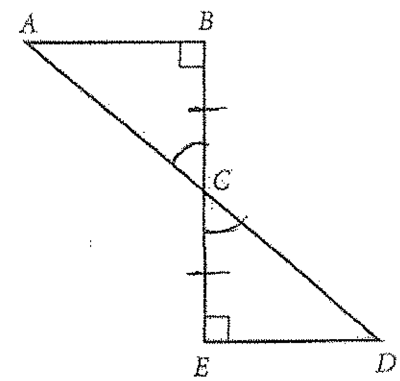
7.7.D2 ~ USING CONGRUENT TRIANGLES

ACCESSING PRIOR KNOWLEDGE

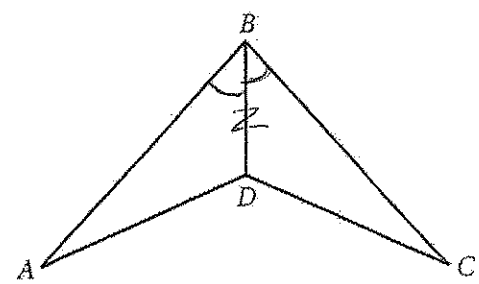
a. If C is the midpoint of $\overline{BE}$, then what two segments are congruent? $\overline{BC} \cong \overline{EC}$



b. If $\overline{BE}$ & $\overline{AD}$ intersect at C, what two angles must be congruent and why?
$\angle BCA \cong \angle ECD$
vert. ∠s are ≅

c. Name two other congruent angles and explain why they are congruent. $\angle B \cong \angle E$
Rt. ∠s are ≅

d. If $\overrightarrow{BD}$ bisects $\angle ABC$, then what two angles are congruent
$\angle ABD \cong \angle CBD$



e. Why is $\overline{BD} \cong \overline{BD}$?
Reflexive

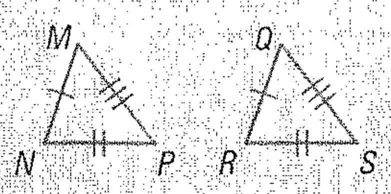
PROVING TRIANGLES CONGRUENT COULD BE A VERY TEDIOUS TASK IF WE HAD TO VERIFY THE CONGRUENCE OF EVERY ONE OF THE SIX PAIRS OF CORRESPONDING PARTS.

TRIANGLES HAVE SOME SPECIAL PROPERTIES THAT WILL ENABLE US TO PROVE TWO TRIANGLES ARE CONGRUENT BY COMPARING <u>ONLY THREE</u> SPECIALLY CHOSEN PAIRS OF CORRESPONDING PARTS.

Side-Side-Side Congruence Postulate (SSS)

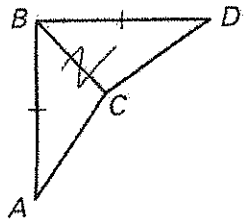
Words If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

Symbols If Side $\overline{MN} \cong \overline{QR}$, and
Side $\overline{NP} \cong \overline{RS}$, and
Side $\overline{PM} \cong \overline{SQ}$,
then $\triangle MNP \cong \triangle QRS$.



Examples: Using the SSS Congruence Postulate

Does the diagram give enough information to use the SSS Congruence Postulate? Explain your reasoning.

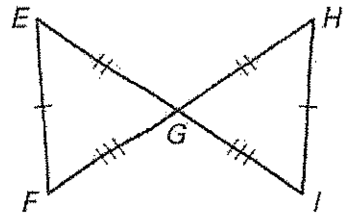
1\. 

List those angles &/or sides you know to be congruent:

$\overline{AB} \cong \overline{DB}$

$\overline{BC} \cong \overline{BC}$

Not $\cong$, need to know whether $\overline{AC} \cong \overline{DC}$

2\. 

List those angles &/or sides you know to be congruent:

$\overline{EF} \cong \overline{HI}$

$\overline{EG} \cong \overline{HG}$

$\overline{FG} \cong \overline{IG}$

$\triangle FEG \cong \triangle IHG$ by SSS

Side-Angle-Side Congruence Postulate (SAS)

Words If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

Symbols If Side $\overline{PQ} \cong \overline{WX}$, and

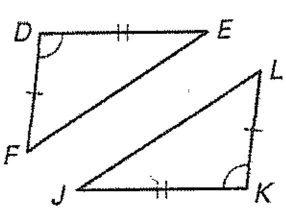
Angle $\angle Q \cong \angle X$, and

Side $\overline{QR} \cong \overline{XY}$,

then $\triangle PQR \cong \triangle WXY$.

Examples: Using the SAS Congruence Postulate

Does the diagram give enough information to use the SAS Congruence Postulate? Explain your reasoning.

3\. 

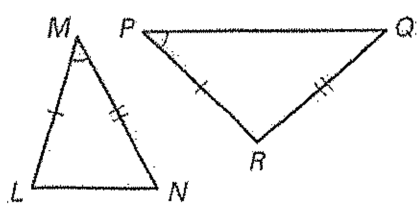
List those angles &/or sides you know to be congruent:

$\overline{FD} \cong \overline{LK}$

$\angle D \cong \angle K$

$\overline{DE} \cong \overline{KJ}$

$\triangle FDE \cong \triangle LKJ$ by SAS

4\. 

List those angles &/or sides you know to be congruent:

$\overline{LM} \cong \overline{RP}$

$\angle M \cong \angle P$ not the included $\angle$

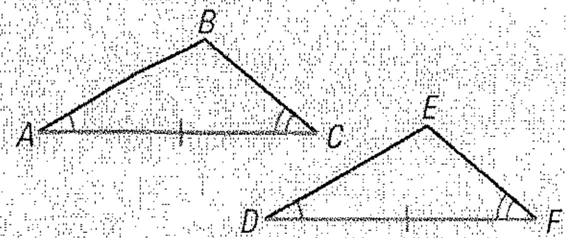
$\overline{MN} \cong \overline{QR}$

NOT $\cong$

Angle-Side-Angle Congruence Postulate (ASA)

Words If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

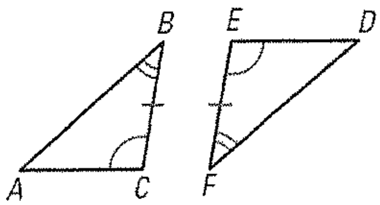
Symbols If Angle $\angle A \cong \angle D$, and
Side $\overline{AC} \cong \overline{DF}$, and
Angle $\angle C \cong \angle F$,
then $\triangle ABC \cong \triangle DEF$.



Examples: Using the ASA Congruence Postulate

Does the diagram give enough information to use the ASA Congruence Postulate? Explain your reasoning.

5.

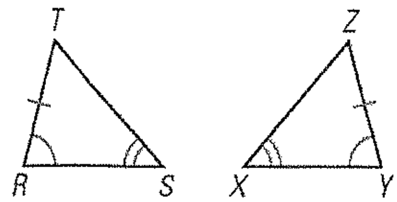


List those angles &/or sides you know to be congruent:

$\angle C \cong \angle E$
$\overline{CB} \cong \overline{EF}$
$\angle B \cong \angle F$

$\triangle CBA \cong \triangle EFD$ by ASA

6.



List those angles &/or sides you know to be congruent:

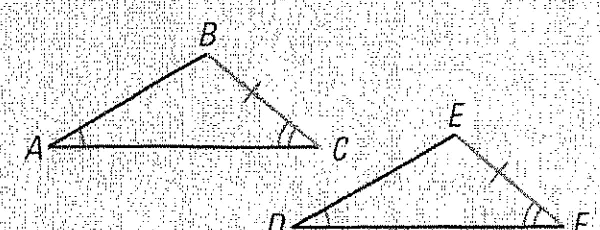
$\angle R \cong \angle Y$
$\overline{RT} \cong \overline{YZ}$
$\angle S \cong \angle X$

NOT $\cong$ by ASA b/c $\overline{RT}$ & $\overline{YZ}$ are NOT the included sides

Angle-Angle-Side Congruence Theorem (AAS)

Words If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.

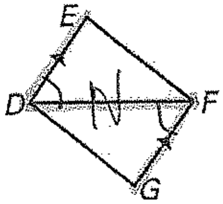
Symbols If Angle $\angle A \cong \angle D$, and
Angle $\angle C \cong \angle F$, and
Side $\overline{BC} \cong \overline{EF}$,
then $\triangle ABC \cong \triangle DEF$.



Examples: Using the AAS Congruence Theorem

Based on the diagram, can you use the AAS Congruence Theorem to show that the triangles are congruent? If not, what additional congruence is needed?

7.



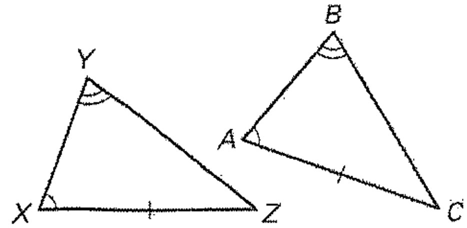
List those angles &/or sides you know to be congruent:

$\angle EDF \cong \angle GFD$ (alt. int $\angle$s are $\cong$)

$\overline{DF} \cong \overline{FD}$

*Not enough info – need to know if $\angle E \cong \angle G$

8.



List those angles &/or sides you know to be congruent:

$\angle X \cong \angle A$

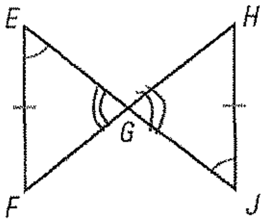
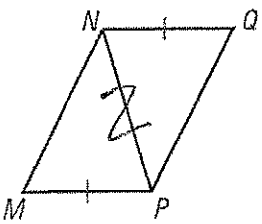
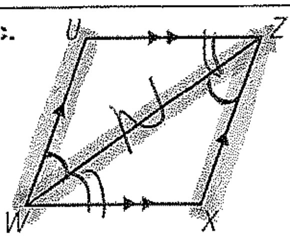
$\angle Y \cong \angle B$

$\overline{XZ} \cong \overline{AC}$

$\triangle XYZ \cong \triangle ABC$ by AAS

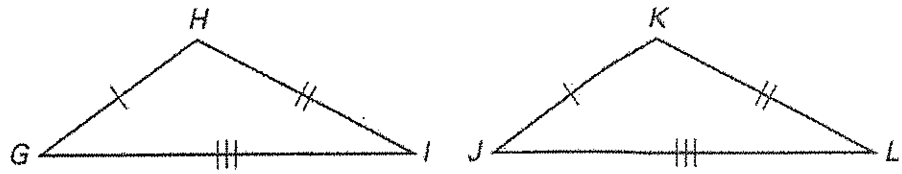
Examples: Deciding Whether Triangles are Congruent

Does the diagram given enough information to show that the triangles are congruent? If so, state the method – SSS, SAS, ASA or AAS – you would use.

Diagram	Congruences	Method
9. 	$\overline{EF} \cong \overline{JH}$ $\angle E \cong \angle J$ $\angle EGF \cong \angle JGH$ (vert. $\angle$s are $\cong$)	AAS
10. 	$\overline{QN} \cong \overline{MP}$ $\overline{NP} \cong \overline{PN}$	Not enough info
11. 	$\angle UWZ \cong \angle XZW$ $\angle UZW \cong \angle XWZ$ (alt. int. $\angle$s are $\cong$) $\overline{WZ} \cong \overline{ZW}$	ASA

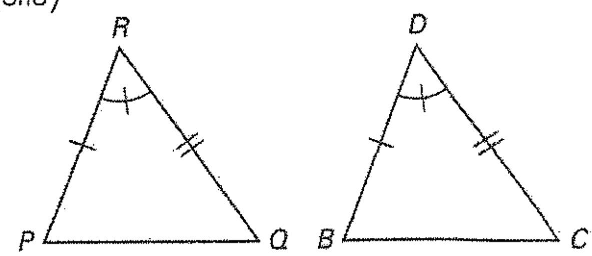
7.8 ~ CONGRUENT TRIANGLE PROOFS

- Methods of Proving Triangles Congruent
  - The Side-Side-Side Congruence Theorem
    - If three sides of one triangle are congruent to the corresponding sides of another triangle, then the triangles are congruent. (SSS)



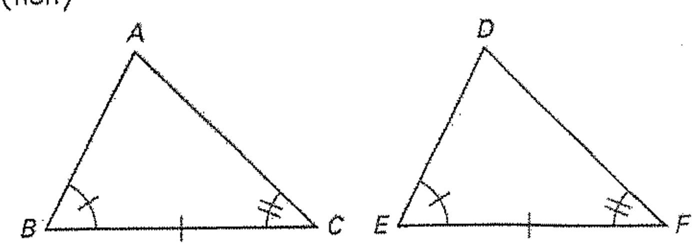
Side	Side	Side
$\overline{GH} \cong \overline{JK}$	$\overline{IG} \cong \overline{LJ}$	$\overline{HI} \cong \overline{KL}$

  - The Side-Angle-Side Congruence Theorem
    - If two sides and the included angle of one triangle are congruent to the corresponding two sides and the included angle of a second triangle, then the triangles are congruent. (SAS)



Side	Angle	Side
$\overline{RP} \cong \overline{DB}$	$\angle QRP \cong \angle CDB$	$\overline{RQ} \cong \overline{DC}$

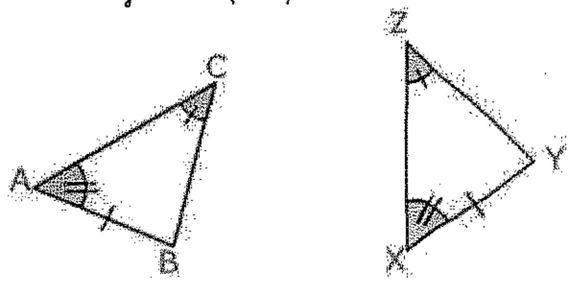
  - The Angle-Side-Angle Congruence Theorem
    - If two angles and the included side of one triangle are congruent to the corresponding two angles and included side of another triangle, then the triangles are congruent. (ASA)



Angle	Side	Angle
$\angle CBA \cong \angle FED$	$\overline{BC} \cong \overline{EF}$	$\angle ACB \cong \angle DFE$

➢ The Angle-Angle-Side Congruence Theorem

- If two angles and a non-included side of one triangle are congruent to the corresponding two angles and the corresponding non-included side of a second triangle, then the triangles are congruent. (AAS)



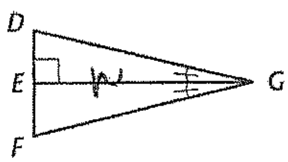
Angle	Angle	Side
$\angle CAB \cong \angle ZXY$	$\angle ACB \cong \angle XZY$	$\overline{AB} \cong \overline{XY}$

BE ON THE LOOKOUT FOR: (1) VERTICAL ANGLES & (2) SHARED SIDES – REFLEXIVE PROPERTY

Examples: Congruent? SSS, SAS, ASA or AAS?

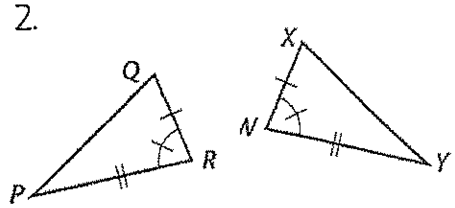
Determine whether you could prove that the triangles are congruent. If so, write a congruence statement & identify the postulate you could use.

1.



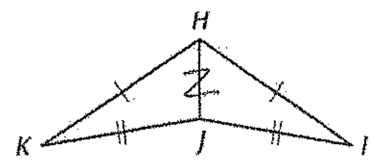
$\triangle DEG \cong \triangle FEG$
by ASA

2.



$\triangle QRP \cong \triangle XNY$
by SAS

3.



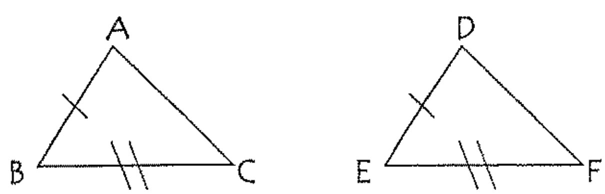
$\triangle KHJ \cong \triangle IHJ$
by SSS

Examples:

In 4 – 7, you are given the congruent angles and sides shown by the tick marks. Name the additional congruent sides or angles needed to prove that the triangles are congruent by each specified method.

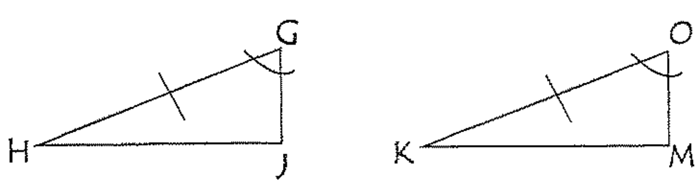
4. a. SSS $\overline{AC} \cong \overline{DF}$

b. SAS $\angle B \cong \angle E$

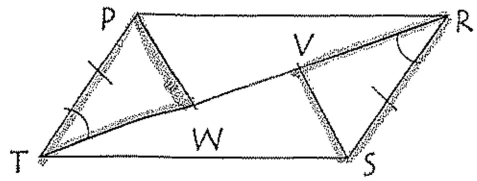


5. a. ASA $\angle H \cong \angle K$

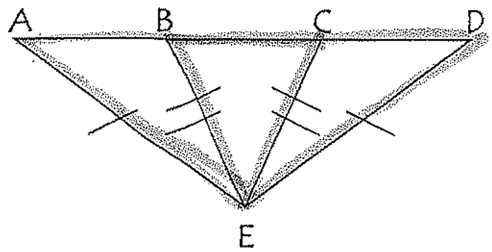
b. AAS $\angle J \cong \angle M$



6. Prove: $\triangle PWT \cong \triangle SVR$
a. SAS $\overline{TW} \cong \overline{RV}$
b. ASA $\angle TPW \cong \angle RSV$
c. AAS $\angle PWT \cong \angle SVR$

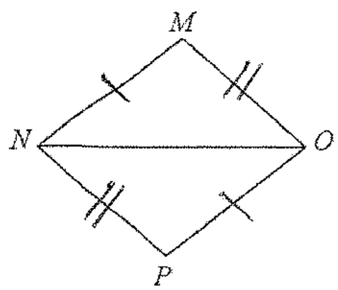


7. Prove: $\triangle AEC \cong \triangle DEB$
a. SSS $\overline{AC} \cong \overline{DB}$
b. SAS $\angle AEC \cong \angle DEB$



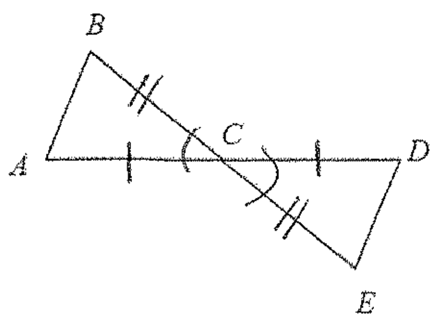
PROOFS:

8. Given: $\overline{MN} \cong \overline{PO}$
$\overline{MO} \cong \overline{PN}$
Prove: $\triangle MNO \cong \triangle PON$



	Statements	Reasons
S	1. $\overline{MN} \cong \overline{PO}$	1. Given
S	2. $\overline{MO} \cong \overline{PN}$	2. Given
S	3. $\overline{NO} \cong \overline{ON}$	3. Reflexive
	4. $\triangle MNO \cong \triangle PON$	4. SSS

9. Given: $\overline{AC} \cong \overline{DC}$
$\overline{BC} \cong \overline{CE}$
Prove: $\triangle ABC \cong \triangle DEC$



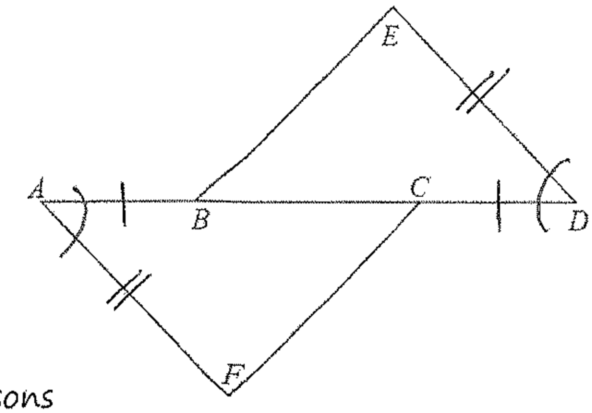
	Statements	Reasons
S	1. $\overline{AC} \cong \overline{DC}$	1. Given
S	2. $\overline{BC} \cong \overline{CE}$	2. Given
A	3. $\angle ACB \cong \angle DCE$	3. Vert. ∠s are ≅
	4. $\triangle ABC \cong \triangle DEC$	4. SAS

10. Given: $\overline{AB} \cong \overline{CD}$

$\overline{AF} \cong \overline{DE}$

$\angle A \cong \angle D$

Prove: $\triangle FAC \cong \triangle EDB$



	Statements	Reasons
	1. $\overline{AB} \cong \overline{CD}$	1. Given
S	2. $\overline{AF} \cong \overline{DE}$	2. Given
A	3. $\angle A \cong \angle D$	3. Given
	4. $\overline{BC} \cong \overline{CB}$	4. Reflexive
S	5. $\overline{AC} \cong \overline{DB}$	5. Seg. Add. Prop.
	6. $\triangle FAC \cong \triangle EDB$	6. SAS

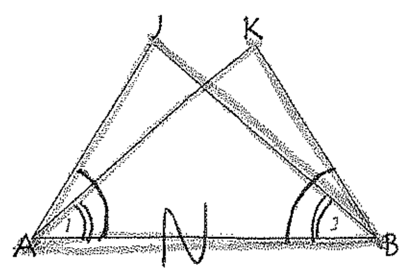
Proofs 11 & 12: Save for 7.8D3

11. Overlapping Triangles

Given: $\angle JAB \cong \angle KBA$

$\angle 1 \cong \angle 2$

Prove: $\triangle JAB \cong \triangle KBA$

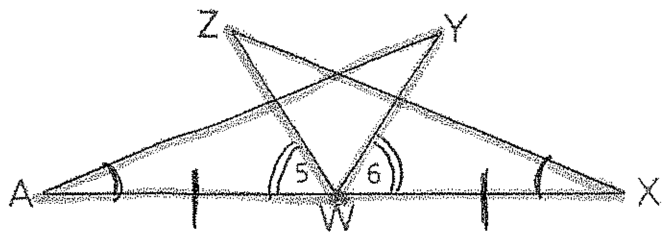


	Statements	Reasons
A	1. $\angle JAB \cong \angle KBA$	1. Given
A	2. $\angle 1 \cong \angle 2$	2. Given
S	3. $\overline{AB} \cong \overline{BA}$	3. Reflexive
	4. $\triangle JAB \cong \triangle KBA$	4. ASA

Helpful Hints w/Overlapping Triangles

- Draw the triangles separately.
- Outline the two triangles in different colors.
- ALSO…there will be a reflexive step—that shared side or angle.

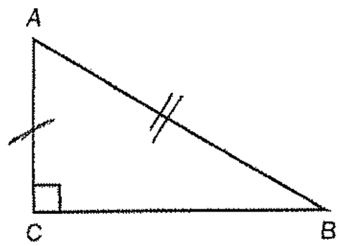
12. Given: $\overline{YW}$ bisects $\overline{AX}$
$\angle A \cong \angle X$
$\angle 5 \cong \angle 6$
Prove: $\triangle AWY \cong \triangle XWZ$

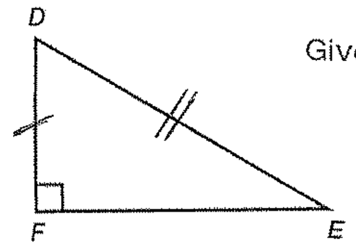


	Statements	Reasons
	1. $\overline{YW}$ bisects $\overline{AX}$	1. Given
S	2. $\overline{AW} \cong \overline{XW}$	2. Def. of bisects
A	3. $\angle A \cong \angle X$	3. } Given
	4. $\angle 5 \cong \angle 6$	4.
	5. $\angle ZWY \cong \angle ZWY$	5. Reflexive
A	6. $\angle AWY \cong \angle XWZ$	6. Angle Addition Prop.
	7. $\triangle AWY \cong \triangle XWZ$	7. ASA

8.1 ~ HL CONGRUENCE THEOREM

- The Hypotenuse-Leg Congruence Theorem
  - If the hypotenuse and leg of one right triangle are congruent to the hypotenuse and leg of another right triangle, then the triangles are congruent. (HL)





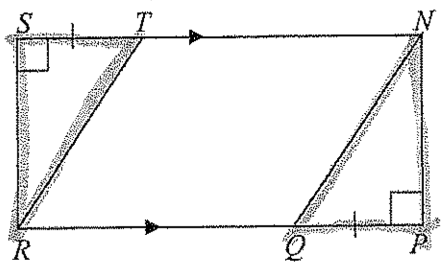
Given: $\angle C$ and $\angle F$ are right angles

$\overline{AC} \cong \overline{DF}$ (legs)

$\overline{AB} \cong \overline{DE}$ (hyp.)

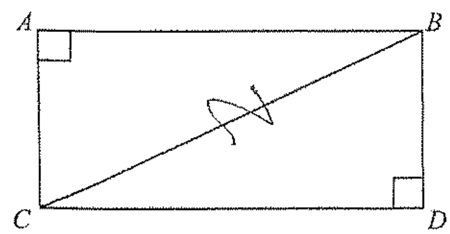
Examples: What additional information would you need to prove the triangles congruent by the HL Congruence Theorem?

1. $\triangle STR \cong \triangle PQN$



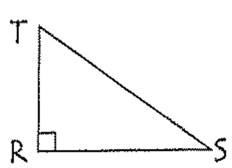
$\overline{TR} \cong \overline{QN}$

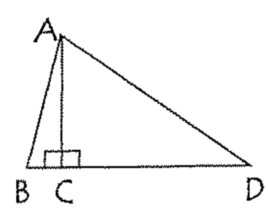
2. $\triangle ABC \cong \triangle DCB$

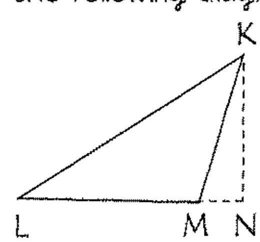


$\overline{AC} \cong \overline{DB}$ OR $\overline{AB} \cong \overline{DC}$

- Altitudes of Triangles
  - If a segment is an altitude of a triangle, then it forms right angles with the side to which it is drawn.
    - Every triangle has three altitudes.
    - An altitude of a triangle forms right angles with one of the sides.
      - Identify the altitude shown & the right angles formed in the following diagrams:







$\triangle TRS$ ~ altitude: $\overline{TR}$
right angle(s): $\angle TRS$

$\triangle ABD$ ~ altitude: $\overline{AC}$
right angle(s): $\angle ACB$ & $\angle ACD$

$\triangle KLM$ ~ altitude: $\overline{KN}$
right angle(s): $\angle N$

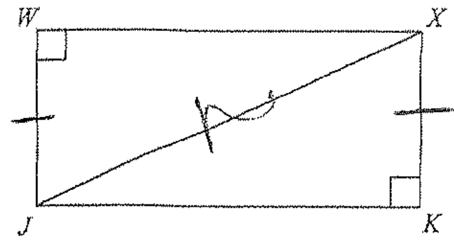
PROOFS:

3. Given: $\overline{WJ} \cong \overline{KX}$

$\angle JWX$ is a right angle

$\angle XKJ$ is a right angle

Prove: $\triangle WJX \cong \triangle KJX$

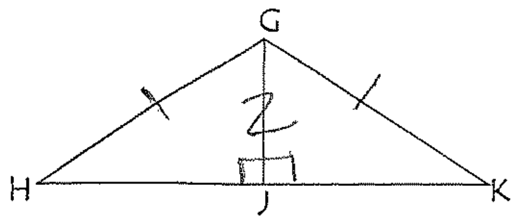


Statements	Reasons
L 1. $\overline{WJ} \cong \overline{KX}$	1. Given
2. $\angle JWX$ is a Rt $\angle$	2.
3. $\angle XKJ$	3.
4. $\angle JWX \cong \angle XKJ$	4. Rt $\angle$s are $\cong$
H 5. $JX \cong XJ$	5. Reflexive
6. $\triangle WJX \cong \triangle KJX$	6. HL

4. Given: $\overline{GH} \cong \overline{GK}$

$\overline{GJ}$ is an altitude

Prove: $\triangle GHJ \cong \triangle GKJ$



Statements	Reasons
H 1. $\overline{GH} \cong \overline{GK}$	1. Given
2. GJ is an altitude	2.
3. $\angle GJH$ & $\angle GJK$ are Rt $\angle$s	3. Def. of altitude
4. $\angle GJH \cong \angle GJK$	4. Rt $\angle$s are $\cong$
L 5. $\overline{GJ} \cong \overline{GJ}$	5. Reflexive
6. $\triangle GHJ \cong \triangle GKJ$	6. HL

8.2 ~ CPCTC & CIRCLES

If two triangles are congruent, then each part of one triangle is congruent to the corresponding part of the other triangle. "**Corresponding parts of congruent triangles are congruent**," is abbreviated as **CPCTC**, is often used as reasons in proofs. CPCTC states that corresponding angles or sides in two congruent triangles are congruent. This reason can only be used after you have proven that the triangles are congruent.

To use CPCTC in a proof, follow these steps:

Step 1: Identify two triangles in which segments or angles are corresponding parts.

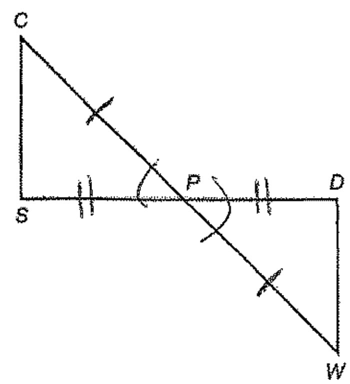
Step 2: Prove the triangles congruent.

Step 3: State the two parts are congruent using CPCTC as the reason.

A PROOF:

1. Given: $\overline{CW}$ & $\overline{SD}$ bisect each other

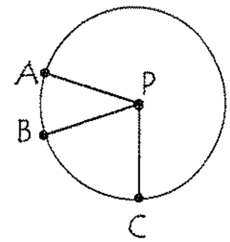
   Prove: $\overline{CS} \cong \overline{WD}$



Statements	Reasons
1. $\overline{CW}$ & $\overline{SD}$ bisect each other	1. Given
S 2. $\overline{CP} \cong \overline{WP}$	2. Def. of bisects
S 3. $\overline{SP} \cong \overline{DP}$	3. Def. of bisects
A 4. $\angle CPS \cong \angle WPD$	4. Vert. ∠s are ≅
5. $\triangle CPS \cong \triangle WPD$	5. SAS
6. $\overline{CS} \cong \overline{WD}$	6. CPCTC

Circles

- A circle is named by its center; this circle is called circle P (or ⊙P)



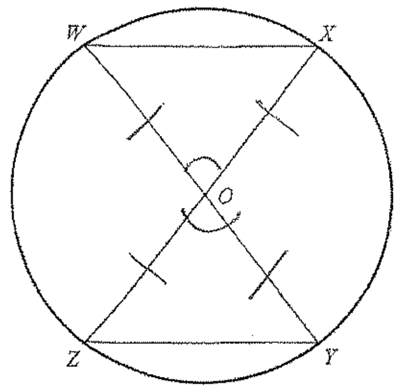
➢ Radii

- Points A, B, and C lie on circle P (⊙P)
  - $\overline{PA}$ is called a radius
  - $\overline{PA}, \overline{PB}, \& \overline{PC}$ are called radii
- Theorem: All radii of a circle are congruent.

A PROOF:

2. Given: ⊙O

   Prove: $\overline{XW} \cong \overline{ZY}$



Statements	Reasons
1. ⊙O	1. Given
S 2. $\overline{OW} \cong \overline{OX} \cong \overline{OY} \cong \overline{OZ}$	2. All radii are ≅
A 3. $\angle XOW \cong \angle ZOY$	3. Vert. ∠s are ≅
4. $\triangle XOW \cong \triangle ZOY$	4. SAS
5. $\overline{XW} \cong \overline{ZY}$	5. CPCTC

Auxiliary Lines

➢ Need there to be line connecting two points? No problem!

- Auxiliary lines connect two points already in the diagram.

Whenever we use an auxiliary line in a proof, we must be able to show that such a line can be drawn & then justify it with the following postulate:

Two points determine a line.

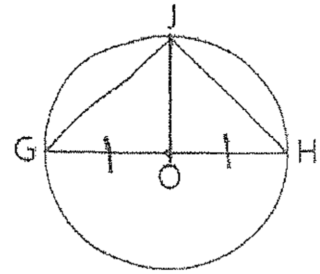
Statements	Reasons
⋮	⋮
Draw $\overline{AL}$	Two points determine a line.

A PROOF:

3. Given: $\odot O$

$\overline{GJ} \cong \overline{HJ}$

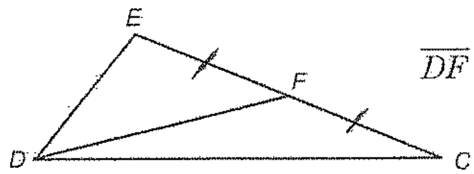
Prove: $\angle G \cong \angle H$



	Statements	Reasons
	1. $\odot O$	1. Given
S	2. $\overline{GO} \cong \overline{HO}$	2. All radii are $\cong$
S	3. $\overline{GJ} \cong \overline{HJ}$	3. Given
	4. Draw $\overline{JO}$	4. 2 pts. det. a line
S	5. $\overline{JO} \cong \overline{JO}$	5. Reflexive
	6. $\triangle GJO \cong \triangle HJO$	6. SSS
	7. $\angle G \cong \angle H$	7. CPCTC

8.3 ~ TRIANGLES IN PROOFS

- Medians of Triangles
  - If a segment is a <u>median</u> of a triangle, then it divides the opposite side into two congruent segments.

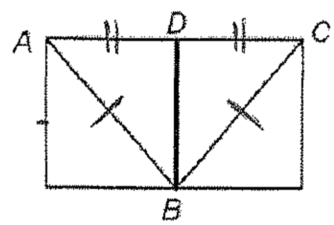


$\overline{DF}$ is a median of $\triangle DEC$.

Conclusion: $\overline{EF} \cong \overline{CF}$

A PROOF:

1. Given: $\overline{AB} \cong \overline{CB}$
   $\overline{BD}$ is a median of $\triangle ABC$
   Prove: $\triangle ABD \cong \triangle CBD$



Statements	Reasons
S 1. $\overline{AB} \cong \overline{CB}$	1. Given
2. $\overline{BD}$ is a median of $\triangle ABC$	2. Given
S 3. $\overline{AD} \cong \overline{CD}$	3. Def. of median
S 4. $\overline{DB} \cong \overline{DB}$	4. Reflexive
5. $\triangle ABD \cong \triangle CBD$	5. SSS

- Triangles in Proofs
  - Isosceles Triangles
    - If at least two sides of a triangle are congruent, then the triangle is an isosceles triangle.
  - Equilateral Triangles
    - If all sides of a triangle are congruent, then the triangle is an equilateral triangle.
  - Right Triangles
    - If a triangle has a right angle, then it is a right triangle.

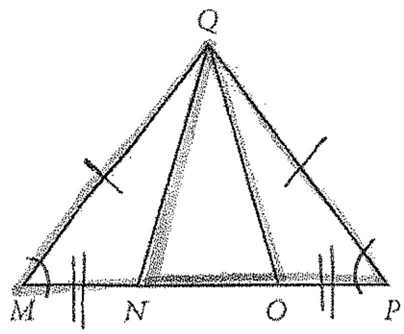
Isosceles Triangle Theorems

- **Isosceles Triangle Base Angle Theorem**
  - If two sides of a triangle are congruent, then the angles opposite these sides are congruent.
- **Isosceles Triangle Base Angle Converse Theorem**
  - If two angles of a triangle are congruent, then the sides opposite these angles are congruent.

PROOFS:

2. Given: $\overline{QM} \cong \overline{QP}$
   $\overline{MN} \cong \overline{PO}$

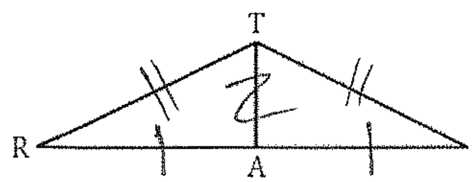
   Prove: $\angle QNP \cong \angle QOM$



Statements	Reasons
S 1. $\overline{QM} \cong \overline{QP}$	1., 2. Given
2. $\overline{MN} \cong \overline{PO}$	
A 3. $\angle M \cong \angle P$	3. Iso. △ Base ∠s Thm
4. $\overline{NO} \cong \overline{ON}$	4. Reflexive
S 5. $\overline{MO} \cong \overline{PN}$	5. Seg. Add. Prop.
6. $\triangle QNP \cong \triangle QOM$	6. SAS
7. $\angle QNP \cong \angle QOM$	7. CPCTC

3. Given: $\overline{TA}$ is a median of $\triangle RIT$
   $\triangle RIT$ is isosceles with base $\overline{RI}$

   Prove: $\triangle TRA \cong \triangle TIA$



Statements	Reasons
1. $\overline{TA}$ is a med. of △RIT	1. Given
S 2. $\overline{RA} \cong \overline{IA}$	2. Def. of median
3. △RIT is isosceles	3. Given
S 4. $\overline{RT} \cong \overline{IT}$	4. Def. of isosceles △
S 5. $\overline{TA} \cong \overline{TA}$	5. Reflexive
6. $\triangle TRA \cong \triangle TIA$	6. SSS