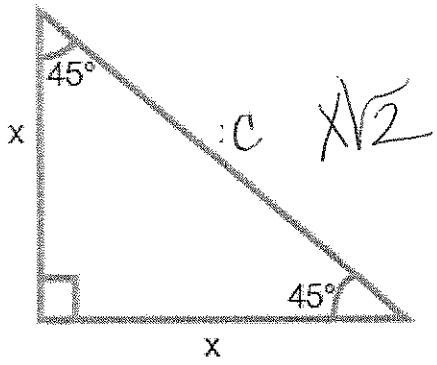
Special Right Triangles

Lessons 5.4 & 5.5

5.4 ~ Properties of 45°-45°-90° Triangles

Special Right Triangles

- The 45°-45°-90° is an isosceles right triangle.
  - This means that the legs are congruent.
  - Use the Pythagorean Theorem to find the length of the third side.

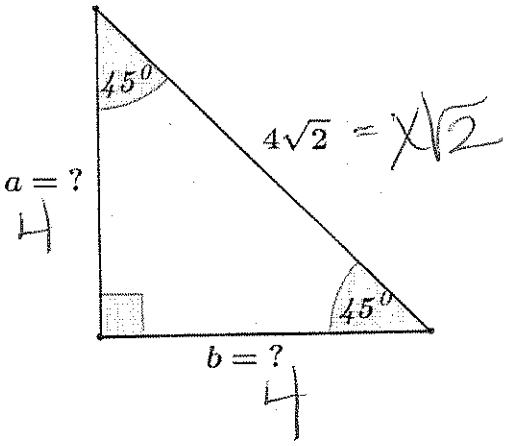


$x^2 + x^2 = c^2$

$\sqrt{2x^2} = \sqrt{c^2}$

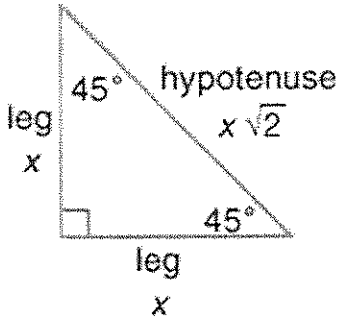
$x\sqrt{2} = c$

  - What do you suppose is the length of the two congruent sides in the triangle below?

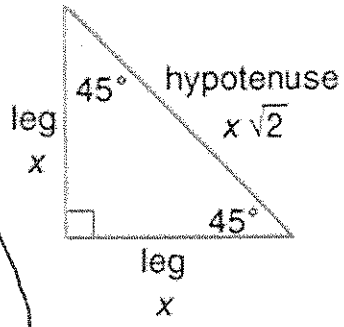


45° - 45° - 90° triangles

- In a triangle whose angles have the measures 45°, 45°, & 90°, the lengths opposite theses angles can be represented by x, x & $x\sqrt{2}$ respectively.



Explain how to calculate the following for a 45°-45°-90° triangle:



1. The length of a leg given the length of the hypotenuse

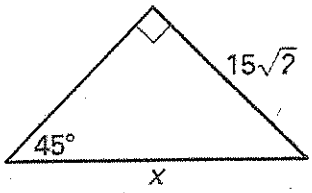
   Divide the hypotenuse by $\sqrt{2}$.

2. The length of the hypotenuse given the leg

   multiply the leg by $\sqrt{2}$.

The triangles below are isosceles and right. Find missing side length indicated. If necessary, express as a radical in simplest form.

3.

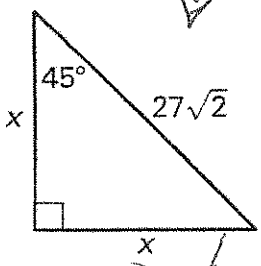


$15\sqrt{2} \cdot \sqrt{2}$

$15 \cdot 2$

30

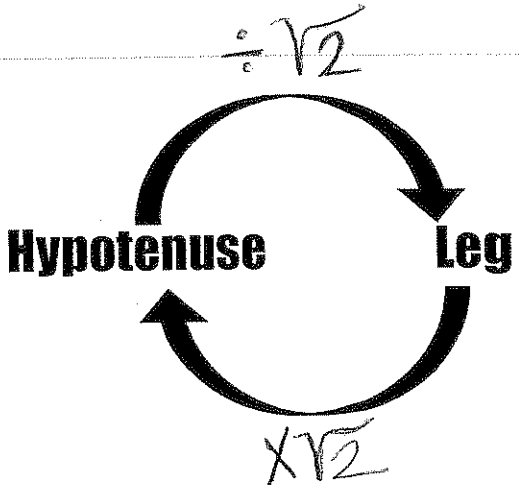
4.



$\frac{27\sqrt{2}}{\sqrt{2}} = 27$

5. Suppose you have an isosceles right triangle whose hypotenuse is 12, what is the length of its leg?

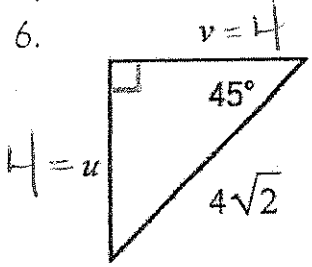
$\frac{12 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2}$



WORK WITH YOUR PARTNER

Find the missing side lengths in each 45°-45°-90° triangle. Express as radicals in simplest form.

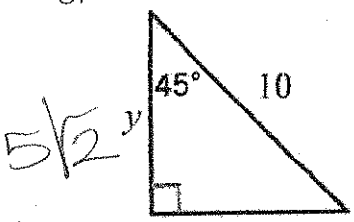
6.



7.

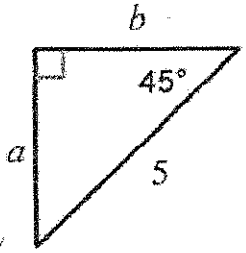
$x = 5\sqrt{2} \cdot \sqrt{2} = 5 \cdot 2 = 10$

8.



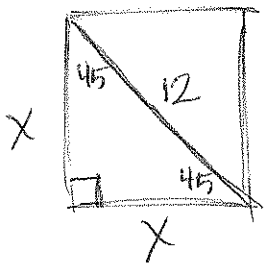
$\frac{10 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$

9.



$\frac{5 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{5\sqrt{2}}{2}$

10. The diagonal of a square is 12 feet. Find the length of each side; express as a radical in simplest form. What is the perimeter of this square? What is the area of this square?



side $= \frac{12 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2}$ feet

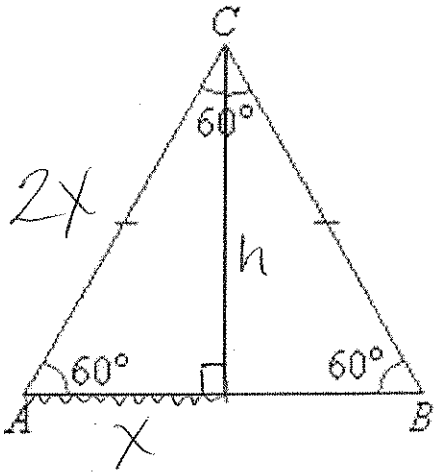
perimeter $= 4 \cdot 6\sqrt{2} = 24\sqrt{2}$ feet

area $= (6\sqrt{2})^2 = 36 \cdot 2 = 72$ feet^2

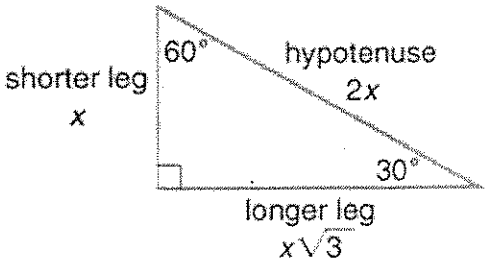
5.5 ~ Properties of 30°-60°-90° Triangles

- Special Right Triangles
  - The 30°-60°-90° is a scalene right triangle.
    - Consider equilateral $\triangle ABC$, with sides measuring $2x$.
    - Draw in a segment connecting C to the midpoint of $\overline{AB}$. (This segment also bisects $\angle C$.)
      - Use the Pythagorean Theorem to find the length of this segment.

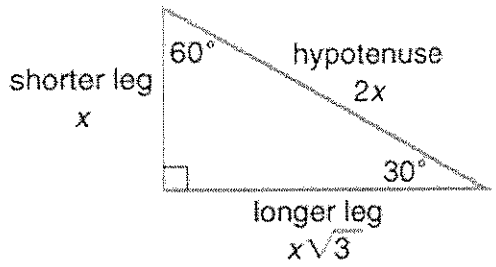
$$x^2 + h^2 = (2x)^2$$
$$x^2 + h^2 = 4x^2$$
$$\sqrt{h^2} = \sqrt{3x^2}$$
$$h = x\sqrt{3}$$



- 30° - 60° - 90° triangles
  - In a triangle whose angles have the measures 30°, 60°, & 90°, the lengths opposite theses angles can be represented by x, $x\sqrt{3}$ & $2x$ respectively.



Explain how to calculate the following for a 30°-60°-90° triangle:



1. The length of the hypotenuse given the length of the shorter leg
   multiply the shorter leg by 2

2. The length of the hypotenuse given the length of the longer leg
   Divide the longer leg by $\sqrt{3}$ then multiply by 2

3. The length of the shorter leg given the length of the longer leg
   Divide the longer leg by $\sqrt{3}$

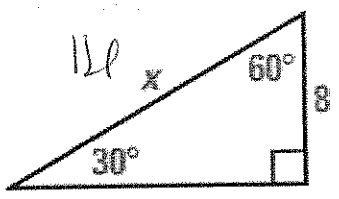
4. The length of the shorter leg given the length of the hypotenuse
   Divide the hypotenuse by 2

5. The length of the longer leg given the length of the shorter leg
   multiply the shorter leg by $\sqrt{3}$

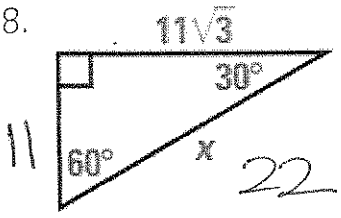
6. The length of the longer leg given the length of the hypotenuse
   Divide the hypotenuse by 2 then multiply by $\sqrt{3}$

Find the length of the hypotenuse. Write your answer in simplest radical form.

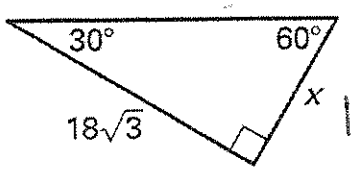
7.



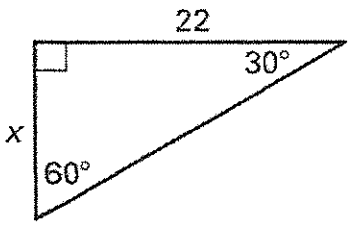
8.



Find the length of the shorter leg. Write your answer in simplest radical form.

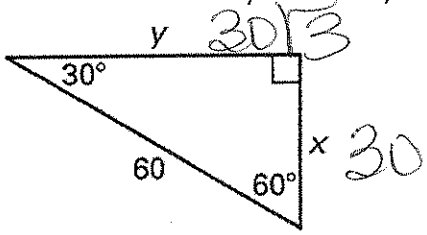
9. 

18

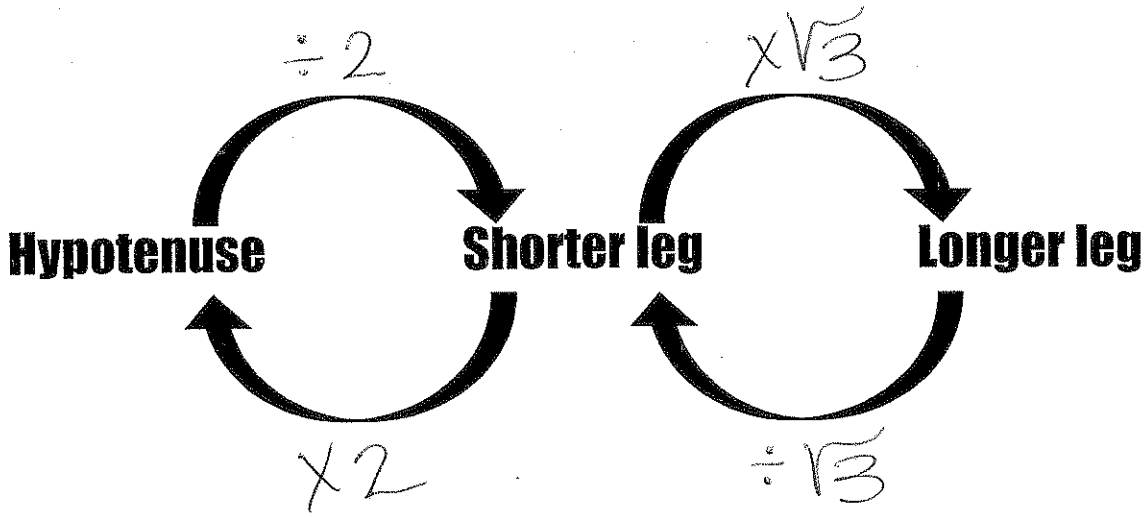
10. 

$\frac{22 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{22\sqrt{3}}{3}$

Find the values of *x* and y. Write your answer as a simplified radical.

11. 

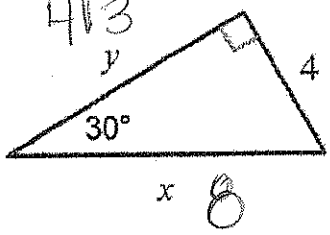
$30\sqrt{3}$
30



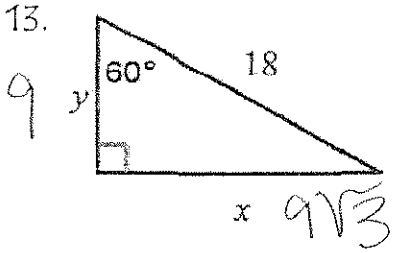
WORK WITH YOUR PARTNER

Find the missing side lengths in each 30°-60°-90° triangle. Express as radicals in simplest form.

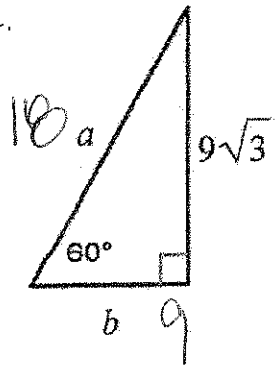
12. $4\sqrt{3}$



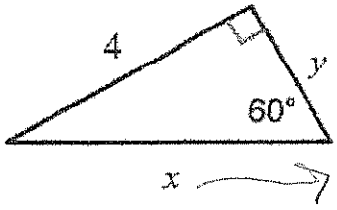
13.



14.



15.

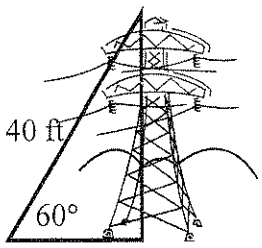


$\frac{4}{3}\sqrt{3}$

$\frac{8}{3}\sqrt{3}$

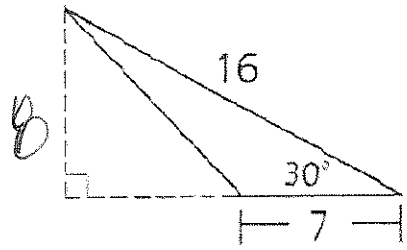
$\frac{4 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{4\sqrt{3}}{3}$

16. A 40 foot cable extends from the top of an electrical tower to the ground. If the cable forms a 60° with the ground, how tall is the tower to the nearest tenth of a foot?



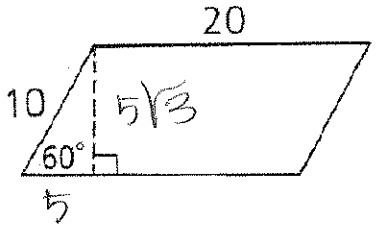
$20\sqrt{3} \approx 34.6$ feet

17. Use special right triangles to find the height of the triangle. What is area of the triangle?



$A = \frac{1}{2}bh = \frac{1}{2} \cdot 7 \cdot 8 = 24$

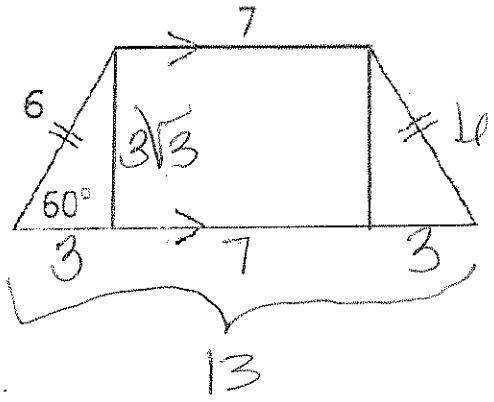
18. Use special right triangles to find the height of the parallelogram. What is area of the parallelogram?



$A = bh = 20 \cdot 5\sqrt{3}$

$100\sqrt{3}$

19. Use special right triangles to find the height of the isosceles trapezoid. What is the length of the second base? What is area of the isosceles trapezoid?



$A = \frac{1}{2}h(b_1 + b_2)$

$A = \frac{1}{2}(3\sqrt{3})(7 + 13)$

$A = \frac{1}{2}(3\sqrt{3})(20)$

$30\sqrt{3}$