FACTORING:

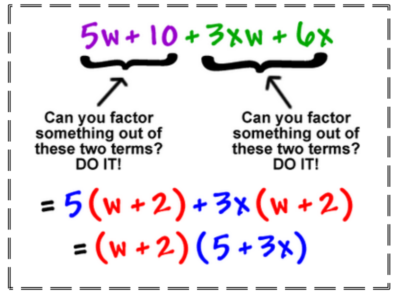
ALWAYS LOOK FOR COMMON FACTORS FIRST!

- **Factoring Binomials**
  - Difference of Two Squares: $a^2 - b^2 = (a+b)(a-b)$
    - $a = \sqrt{a^2}$ & $b = \sqrt{b^2}$
  - Sum of Two Cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
  - Difference of Two Cubes $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$a = \sqrt[3]{a^3}$ & $b = \sqrt[3]{b^3}$

$\mathbf{1^3 = 1, 2^3 = 8, 3^3 = 27, 4^3 = 64, 5^3 = 125}$

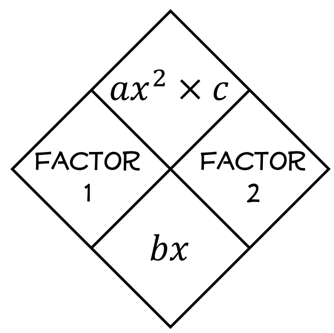
- **Factoring Polynomials w/4 Terms**
  - Factor by grouping
    - Group the first two terms & the last two terms
    - Factor out the common factor of each group
    - Write as a product of two binomials



- **Factoring Trinomials: $x^2 + bx + c$**
  - Find the factors of c that add up to be the middle term b
  - Write as a product of two binomials: $(x \pm \#)(x \pm \#)$

- **Factoring Trinomials: $ax^2 + bx + c$**
  - **THE BOX METHOD**
    1. Multiply the first and last terms: $ax^2 \times c$
    2. Find the factors that multiply to be the product (in step 1) and that add to be the middle term: bx

       Organize this information with an X-box →



    3. Draw a 2×2 square
    4. Put the first term of the trinomial ax^2 in the upper-left corner and the constant term, c, in the lower-right corner.
    5. Put the factors (from step 2) in the two remaining squares.
    6. Find the GCF of each row & each column
    7. Write the result as a product of two binomials.

ax^2	FACTOR 1	GCF of row 1
FACTOR 2	c	GCF of row 2
GCF of column 1	GCF of column 2	

- **DEFOIL/A-C Method**
  - Multiply the first and last terms
  - Find the factors (of the product in step 1) that add up to be the middle termx
  - Replace the middle term with these factors
  - Factor by grouping

AN EXAMPLE: $3x^2 - 4x - 7$

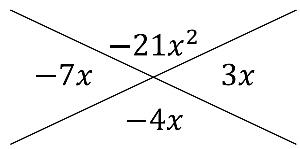
BOX METHOD

$3x^2$	$-7x$	x
$3x$	-7	1
$3x$	-7	

$(3x-7)(x+1)$

1st 2 steps are the same

1. MULTIPLY THE FIRST & LAST TERMS
2. FIND THE FACTORS (OF THE PRODUCT IN STEP 1) THAT ADD UP THE TO BE THE MIDDLE TERM



DEFOIL/A-C METHOD

3. REPLACE THE MIDDLE TERM WITH THESE FACTORS
4. FACTOR BY GROUPING

$$3x^2 - 7x + 3x - 7$$
$$(3x^2 - 7x)(+3x - 7)$$
$$x(3x-7) + 1(3x-7)$$
$$(3x-7)(x+1)$$

- Quadratic Form: $x^4 + bx^2 + c$ OR $ax^4 + bx^2 + c$
  - If you recognize how the expression factors in its original form, then do so.
  - If $a \neq 1$, use the box method or the defoil/a-c method.
    - Example:

$$x^4 - 7x^2 - 18$$

-18	
-9 & 2	-7

Find the factors of c that add up to be the middle term b

$$(x^2-9)(x^2+2)$$

Write as a product of two binomials

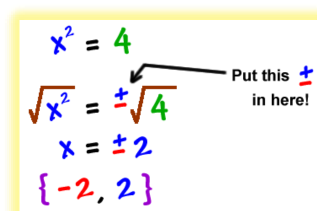
$$(x+3)(x-3)(x^2+2)$$

Be on the lookout for a difference of two squares; the expression can be factored further.

SOLVING QUADRATIC EQUATIONS

THESE METHODS CAN BE USED TO FIND THE ZEROS/ROOTS/ X-INTERCEPTS OF QUADRATIC AND POLYNOMIAL FUNCTIONS!

- Factoring & the Zero Product Property
  - Set the equation – written in standard form – equal to 0
    - $ax^2 + bx + c = 0$
  - Factor
  - Use the Zero-Product Property:
    - Let a and b be real numbers. If $ab = 0$, then $a = 0$ or $b = 0$.
    - Set each factor equal to 0 and solve.



- Square Root Property
  - Given $ax^2 + c = 0$
    - Isolate the quadratic expression on one side of the equation
    - Take the square root of both sides.
      - Don't forget the $\pm$!
    - Solve for x and simplify.

$$(x-5)^2 = 9$$
$$x - 5 = \pm\sqrt{9}$$
$$x - 5 = \pm 3$$
$$x = 5 \pm 3$$
$$x = 8 \text{ or } x = 2$$

- Quadratic Formula
  - Given: $ax^2 + bx + c = 0$
    - The solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$$ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $a \neq 0$ and $b^2 - 4ac \geq 0$