Chapter 2: Name: ______________________________

Introduction to Proof

2.6 BEGINNING PROOFS

OBJECTIVES:

- PROVE A CONJECTURE THROUGH THE USE OF A TWO-COLUMN PROOF
- STRUCTURE STATEMENTS AND REASONS TO FORM A LOGICAL ARGUMENT
- INTERPRET GEOMETRIC DIAGRAMS

Why Study Proofs?

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You will need them every day, I hope, without knowing it. Geometry is beautifully logical, and it teaches you how to think and prove that things are so, step by step by step. Proofs are excellent lessons in reasoning. Without logic and reasoning, you are dependent on jumping to conclusions or - worse - having empty opinions.

Assumptions from Diagrams

➢ You should assume:

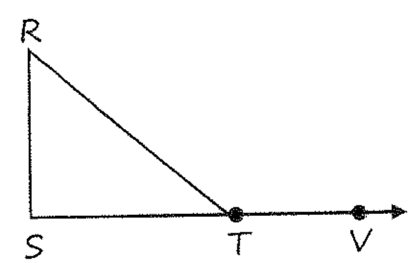
- Straight lines & angles ← linear pairs
- Collinearity of points
- Betweenness of points
- Relative positions of points

* vertical ∠s

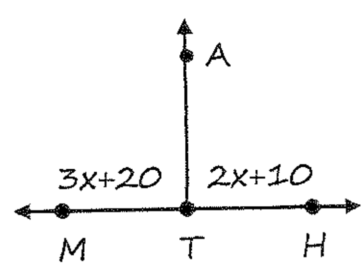
You should NEVER assume:

- Right angles
- Congruent segments
- Congruent angles
- Relatives sizes of segments & angles

Examples ~



1. Should we assume that S, T, and V are collinear in the diagram? yes
2. Should we assume that $m\angle S = 90$? NO!
3. What can we assume from this diagram?
   ∠MTH is a straight ∠
4. Use that assumption to set up and solve an equation to find x. $3x+20+2x+10=180$
   $5x+30=180$
   $5x=150$
   $x=30$
5. Find $m\angle MTA$
   $3 \cdot 30+20=110°$

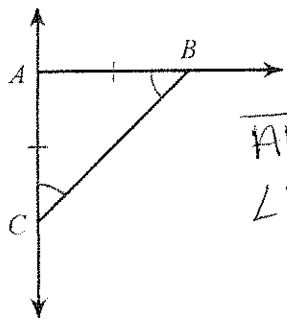


OFTEN, WE USE IDENTICAL <u>TICK MARKS</u> TO INDICATE CONGRUENT SEGMENTS AND <u>ARC MARKS</u> TO INDICATE CONGRUENT ANGLES.

Examples ~

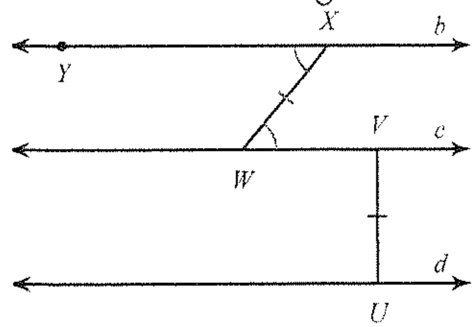
6. Identify the congruent segments and/or angles in each diagram. $\overline{WX} \cong \overline{UV}$ $\angle YXW \cong \angle VWX$

a)



$\overline{AB} \cong \overline{AC}$
$\angle ABC \cong \angle ACB$

b)



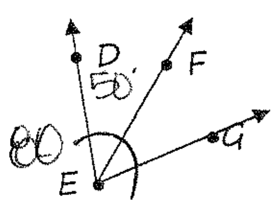
c) What kind of triangle is $\triangle ABC$? How do you know?

isosceles - it has @ least 2 $\cong$ sides

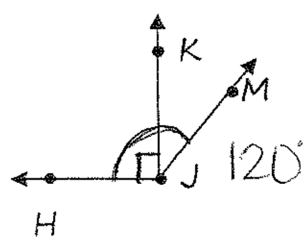
d) Is $b \parallel c$? Explain why or why not.

yes - they are $\cong$ alternate interior ∠s

7. In the diagram below, $\angle DEG = 80°$, $\angle DEF = 50°$, $\angle HJM = 120°$, and $\angle HJK = 90°$. Draw a conclusion about $\angle FEG$ & $\angle KJM$.



$m\angle FEG = 30°$



$m\angle KJM = 30°$

conclusion: $\angle FEG \cong \angle KJM$

Writing Two-Column Proofs

- <u>Proof</u> – A convincing argument that shows why a statement is true
  - The proof begins with the given information and ends with the statement you are trying to prove.
  - Two-Column Proof:

Statements	Reasons
• Specific – applies only to <u>this</u> proof	• General – can apply to <u>any</u> proof

Procedure for Drawing Conclusions

1. Memorize theorems, definitions, & postulates.
2. Look for key words & symbols in the given information.
3. Think of all the theorems, definitions, & postulates that involve those keys.
4. Decide which theorem, definition, or postulate allows you to draw a conclusion.
5. Draw a conclusion, & give a reason to justify the conclusion. Be certain that you have not used the *reverse* of the correct reason.
   - The "If…" part of the reason matches the given information, and the "then…" part matches the conclusion being justified.

Schultz says: We write our reasons—if they are not theorems, postulates, or properties—as "if…then" statements.

Try this thought process:

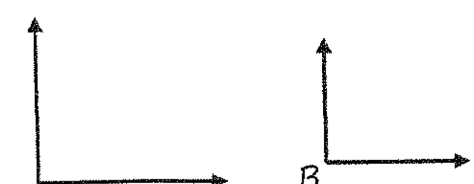
If **what I just said**, *then* **what I'm trying to prove**.

Theorem—A mathematical statement that can be proved

Theorem: If two angles are right angles, then they are congruent.

Given: $\angle A$ is a right angle
$\angle B$ is a right angle

Prove: $\angle A \cong \angle B$

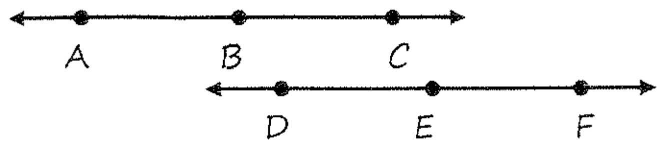


Statements	Reasons
1. $\angle A$ is a right angle	1. Given
2. $m\angle A = 90$	2. Def. of right $\angle$
3. $\angle B$ is a right angle	3. Given
4. $m\angle B = 90$	4. Def. of right $\angle$
5. $\angle A \cong \angle B$	5. Def. of $\cong$ $\angle$s

Theorem: If two angles are straight angles, then they are congruent.

Given: Diagram as shown.

Prove: $\angle ABC \cong \angle DEF$



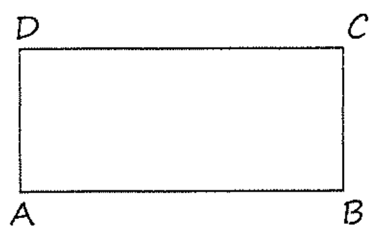
Statements	Reasons
1. Diagram	1. Given
2. ∠ABC is a strt ∠	2. Assumed from diagram.
3. $m\angle ABC = 180$	3. Def. of straight ∠s
4. ∠DEF is a strt ∠	4. Assumed from diagram.
5. $m\angle DEF = 180$	5. Def. of straight ∠s
6. $\angle ABC \cong \angle DEF$	6. Def. of ≅ ∠s

NOW THAT WE HAVE PROVEN THEOREMS 1 & 2, WE CAN USE THEM IN PROOFS.

Example #8

Given: $\angle A$ is a right angle
$\angle C$ is a right angle

Prove: $\angle A \cong \angle C$

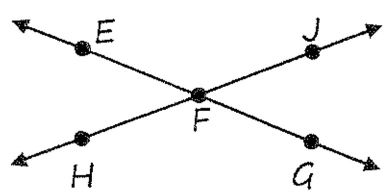


Statements	Reasons
1. ∠A is a rt ∠	1. } Given
2. ∠C is a rt ∠	2.
3. ∠A ≅ ∠C	3. Right ∠s are ≅

Example #9

Given: Diagram as shown

Prove: $\angle EFG \cong \angle HFJ$



Statements	Reasons
1. Diagram	1. Given
2. ∠EFG is a strt ∠	2. } Assumed from diagram
3. ∠HFJ is a strt ∠	3.
4. ∠EFG ≅ ∠HFJ	4. Straight ∠s are ≅.

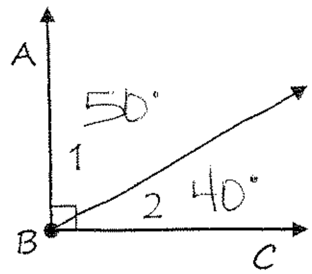
Example #10

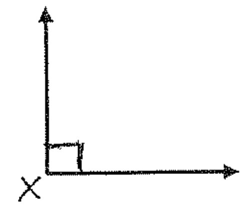
Given: $m\angle 1 = 50$

$m\angle 2 = 40$

$\angle X$ is a right angle

Prove: $\angle ABC \cong \angle X$



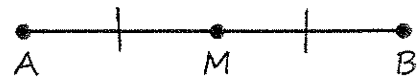


Statements	Reasons
1. $m\angle 1 = 50$	1. Given
2. $m\angle 2 = 40$	2. Given
3. $m\angle 1 + m\angle 2 = 90$	3. Angle Addition
4. $m\angle 1 + m\angle 2 = m\angle ABC$	4. Angle Addition Postulate
5. $m\angle ABC = 90$	5. Substitution
6. $\angle ABC$ is a rt $\angle$	6. Def. of right $\angle$
7. $\angle X$ is a rt $\angle$	7. Given
8. $\angle ABC \cong \angle X$	8. Right $\angle$s are $\cong$

2.7 MIDPOINTS, BISECTORS, & PERPENDICULARITY

Midpoints & Bisectors of Segments

- A point (or segment, ray, or line) that divides a segment into two congruent segments <u>bisects</u> the segment.
  - The bisection point is called the <u>midpoint</u> of the segment.
  - Only segments have midpoints.
    - Given: M is the midpoint of $\overline{AB}$
    - Conclusion: $\overline{AM} \cong \overline{MB}$

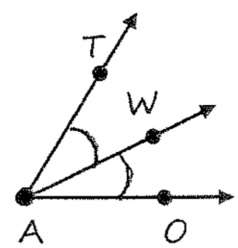


Trisection Points & Trisecting a Segment

- Two points (or segments, rays, or lines) that divide a segment into three congruent segments <u>trisect</u> the segment.
  - The two points at which the segment is divided are called the <u>trisection points</u> of the segment.
  - Only segments have trisection points.
    - Given: R and S are trisection points of $\overline{AC}$
    - Conclusion: $\overline{AR} \cong \overline{RS} \cong \overline{SC}$

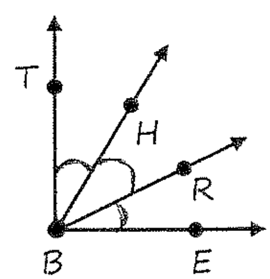
Angle Bisectors

- A ray that divides an angle into two congruent angles <u>bisects</u> the angle.
  - The dividing ray is called the <u>bisector</u> of the angle.
    - Given: $\overrightarrow{AW}$ bisects $\angle TAO$
    - Conclusion: $\angle TAW \cong \angle WAO$



Angle Trisectors

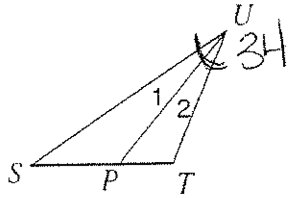
- Two rays that divide an angle into three congruent angles <u>trisect</u> the angle.
  - The two dividing rays are called the <u>trisectors</u> of the angle.
    - Given: $\overrightarrow{BH}$ and $\overrightarrow{BR}$ trisect $\angle TBE$
    - Conclusion: $\angle TBH \cong \angle HBR \cong \angle RBE$



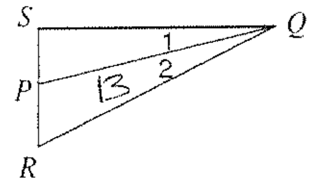
Examples ~

1. Each figure shows a triangle with one of its angle bisectors. → 26°

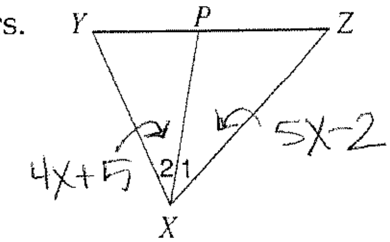
   a) $m\angle SUT = 34°$. Find $m\angle 1$. = 17°



   b) Find $m\angle SQR$ if $m\angle 2 = 13°$.



2. The figure shows a triangle with one of its angle bisectors. Find x if $m\angle 2 = 4x + 5$ and $m\angle 1 = 5x - 2$.



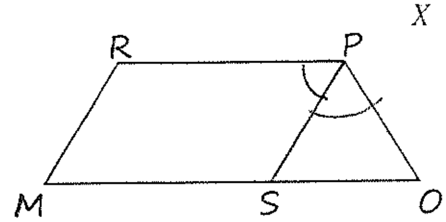
$\angle 1 \cong \angle 2$

$5x - 2 = 4x + 5$

$x = 7$

3. Given: $\overrightarrow{PS}$ bisects $\angle RPO$

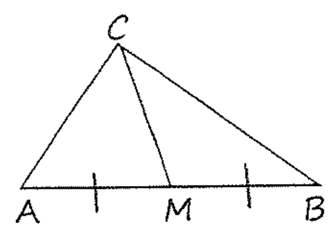
   Prove: $\angle RPS \cong \angle OPS$



Statements	Reasons
1. $\overrightarrow{PS}$ bisects $\angle RPO$	1. Given
2. $\angle RPS \cong \angle OPS$	2. Def. of bisects

4. Given: $\overline{CM}$ bisects $\overline{AB}$

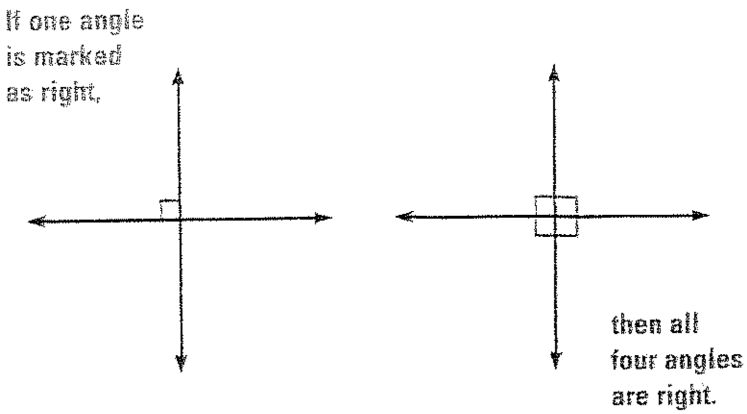
   Prove: $\overline{AM} \cong \overline{MB}$



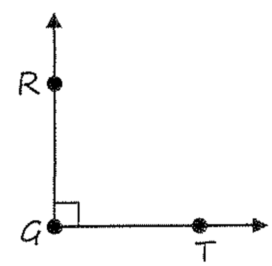
Statements	Reasons
1. $\overline{CM}$ bisects $\overline{AB}$	1. Given
2. $\overline{AM} \cong \overline{MB}$	2. Def. of bisects

Perpendicular Lines, Rays, & Segments

- Perpendicularity, right angles, & 90° measurements all go together.
- Lines, rays, or segments that intersect at right angles are <u>perpendicular</u> (⊥).
  - A pair of perpendicular lines forms four right angles.

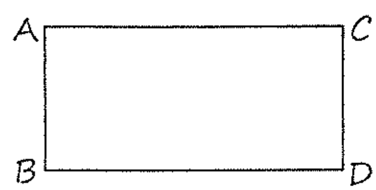


- Do not assume perpendicularity from a diagram!
  - In the figure at the right, the mark inside the angle (¬) indicates that ∠G is a right angle.
    - Given: $\overrightarrow{GR} \perp \overrightarrow{GT}$
    - Conclusion: ∠RGT is a rt ∠



Examples ~

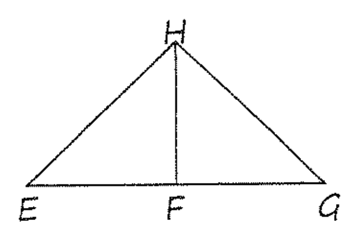
5. Given: $\overline{AB} \perp \overline{BD}$
   $\overline{DC} \perp \overline{AC}$
   Prove: $\angle B \cong \angle C$



Statements	Reasons
1. $\overline{AB} \perp \overline{BD}$	1. Given
2. ∠B is a rt ∠	2. Def. of ⊥
3. $\overline{DC} \perp \overline{AC}$	3. Given
4. ∠C is a rt ∠	4. Def. of ⊥
5. ∠B ≅ ∠C	5. Right ∠s are ≅

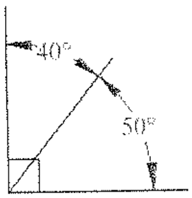
6. Given: $\overline{EH} \perp \overline{HG}$
   Name all the angles you can **prove** to be right angles.

   ∠EHG

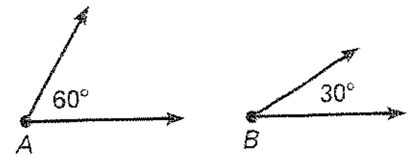


2.8 COMPLEMENTARY & SUPPLEMENTARY ANGLES

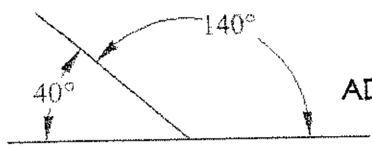
- <u>Complementary angles</u> – two angles whose sum is 90° (a right angle)
  - Each of the two angles is called the <u>complement</u> of the other.



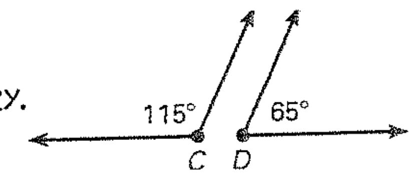
NO NEED FOR ANGLES TO BE ADJACENT TO BE COMPLEMENTARY.



- <u>Supplementary angles</u> – two angles whose sum is 180° (a straight angle)
  - Each of the two angles is called the <u>supplement</u> of the other.



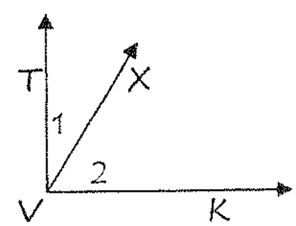
NO NEED FOR ANGLES TO BE ADJACENT TO BE SUPPLEMENTARY.



- **Linear Pair Postulate** ~ If two angles form a linear pair, then they are supplementary.
- If two angles are congruent and supplementary, then each is a right angle.

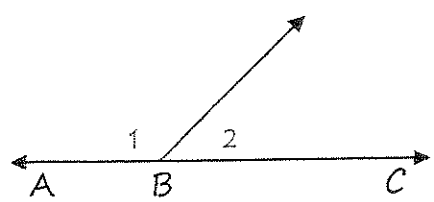
Examples ~

1. Given: $\angle TVK$ is a right $\angle$.
   Prove: $\angle 1$ is complementary to $\angle 2$.



Statements	Reasons
1. ∠TVK is a rt ∠	1. Given
2. ∠1 + ∠2 = ∠TVK	2. Angle Addition Postulate
3. ∠1 is comp. to ∠2	3. Def. of comp. ∠s

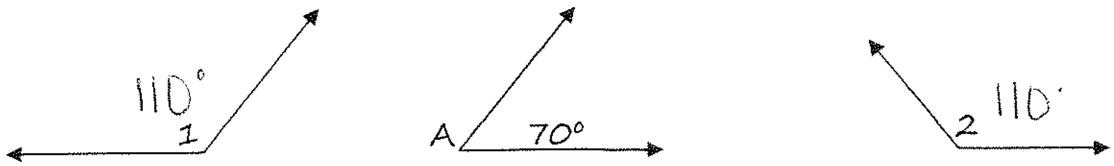
2. Given: Diagram as shown
   Prove: $\angle 1$ is supplementary to $\angle 2$.



Statements	Reasons
1. Diagram	1. Given
2. ∠1 ; ∠2 form a linear pair	2. Assumed from diagram
3. ∠1 is supp to ∠2	3. Linear Pair Postulate

Congruent Complements & Supplements

- In the diagram below, $\angle 1$ is supplementary to $\angle A$, and $\angle 2$ is also supplementary to $\angle A$.

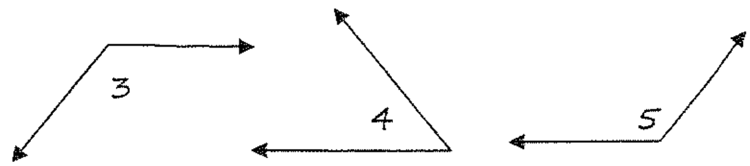


- How large is $\angle 1$? How large is $\angle 2$? How does $\angle 1$ compare with $\angle 2$?

110°   110° $\angle 1 \cong \angle 2$

Theorem : If angles are supplementary to the same angle, then they are congruent.

3. Given: $\angle 3$ is supp. to $\angle 4$
   $\angle 5$ is supp. to $\angle 4$
   Prove: $\angle 3 \cong \angle 5$



Statements	Reasons
1. $\angle 3$ is supp. to $\angle 4$	1. Given
2. $m\angle 3 + m\angle 4 = 180$	2. Def. of Supp. ∠s
3. $m\angle 3 = 180 - m\angle 4$	3. Subtraction Prop of Equality
4. $\angle 5$ is supp. to $\angle 4$	4. Given
5. $m\angle 5 + m\angle 4 = 180$	5. Def. of Supp. ∠s
6. $m\angle 5 = 180 - m\angle 4$	6. Subtraction Prop. of Equality
7. $\angle 3 \cong \angle 5$	7. Substitution

Congruent Supplements Theorems:

- If angles are supplementary to the same angle, then they are congruent.
- If angles are supplementary to congruent angles, then they are congruent.

Congruent Complements Theorems:

- If angles are complementary to the same angle, then they are congruent.
- If angles are complementary to congruent angles, then they are congruent.

4. Given: $\angle 1$ is supp. to $\angle 2$

$\angle 3$ is supp. to $\angle 4$

$\angle 1 \cong \angle 4$

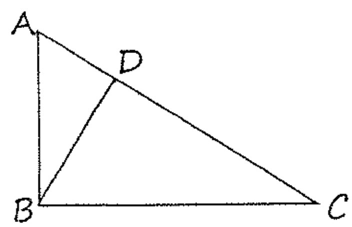
Prove: $\angle 2 \cong \angle 3$

Statements	Reasons
1. $\angle 1$ is supp. to $\angle 2$	1. } Given
2. $\angle 3$ is supp to $\angle 4$	2. } Given
3. $\angle 1 \cong \angle 4$	3. } Given
4. $\angle 2 \cong \angle 3$	4. Congruent Supplements theorem

5. Given: $\angle A$ is comp. to $\angle C$

$\angle DBC$ is comp. to $\angle C$

Prove: $\angle A \cong \angle DBC$

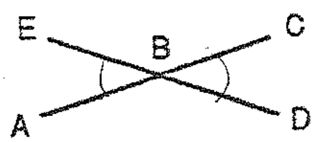


Statements	Reasons
1. $\angle A$ is comp. to $\angle C$	1. } Given
2. $\angle DBC$ is comp. to $\angle C$	2. } Given
3. $\angle A \cong \angle DBC$	3. Congruent Complements theorem

6. Given: Diagram as shown

Prove: $\angle ABE \cong \angle DBC$

Do not use vertical angles.



Statements	Reasons
1. Diagram	1. Given
2. $\angle ABE$ & $\angle EBC$ form a linear pair	2. Assumed from diagram
3. $\angle ABE$ is supp $\angle EBC$	3 Linear Pair Postulate
4. $\angle DBC$ & $\angle EBC$ form a linear pair	4. Assumed from diagram
5. $\angle DBC$ is supp $\angle EBC$	5. Linear Pair Postulate
6. $\angle ABE \cong \angle DBC$	6. Congruent Supplements theorem

2.9 PROPERTIES OF SEGMENTS & ANGLES

- The Addition Properties

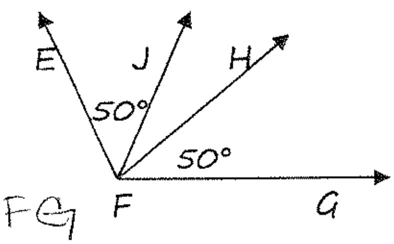
Segment Addition Property ~ If a segment is added to two congruent segments, the sums are congruent.

- If $\overline{AB} \cong \overline{CD}$, does $\overline{AC} \cong \overline{BD}$? Explain.

yes, b/c $\overline{BC}$ (a segment) is added to two congruent segments: $\overline{AB}$ & $\overline{CD}$

Angle Addition Property ~ If an angle is added to two congruent angles, the sums are congruent.

- Does a similar relationship hold for angles?
  - Does $m\angle EFH = m\angle JFG$? Explain.



yes, b/c $\angle JFH$ (an $\angle$) is added to a pair of congruent $\angle$s: $\angle EFH$ & $\angle JFG$

More Addition Properties

- If congruent segments are added to congruent segments, the sums are congruent.
- If congruent angles are added to congruent angles, the sums are congruent.

- Using the Addition Properties Proofs:
  - An addition property is used when the segments or angles in the conclusion are *greater than* those in the given information.

- **Reflexive Property**: Any segment or angle is congruent to itself.
  - Whenever a segment or an angle is shared by two figures, we can say that the segment or angle is congruent to itself.

The Subtraction Properties & Proofs

- A subtraction property is used when the segments or angles in the conclusion are *smaller than* those in the given information.

Segment and Angle Subtraction Properties

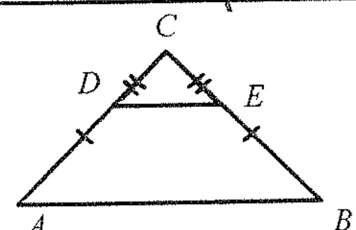
- If a segment (or angle) is subtracted from congruent segments (or angles), the differences are congruent.
- If congruent segments (or angles) are subtracted from congruent segments (or angles), the differences are congruent.

1. Given: Diagram as shown.
   $\overline{AB} \cong \overline{CD}$

   Explain how $\overline{AC} \cong \overline{BD}$.

   A segment, $\overline{BC}$, was added onto a pair of $\cong$ segments: $\overline{AB}$; $\overline{CD}$

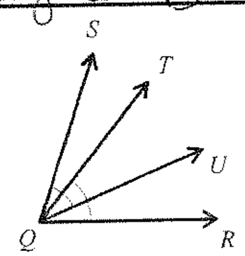
2. Given: Diagram as shown (w/tick marks).



   Explain how $\overline{CA} \cong \overline{CB}$.

   $\cong$ segments were added to $\cong$ segments
   AD + DC = CA ; BE + EC = CB

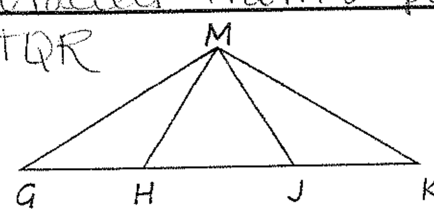
3. Given: Diagram as shown.
   $\angle SQU \cong \angle TQR$



   Explain how $\angle SQT \cong \angle UQR$.

   An angle, $\angle TQU$, was subtracted from a pair of $\cong$ $\angle$s: $\angle SQU$; $\angle TQR$

4. Given: $\overline{GJ} \cong \overline{HK}$
   Prove: $\overline{GH} \cong \overline{JK}$

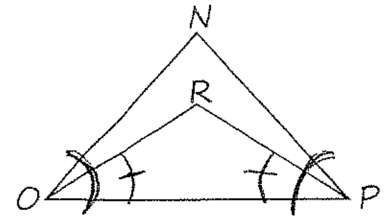


Statements	Reasons
1. $\overline{GJ} \cong \overline{HK}$	1. Given
2. $\overline{HJ} \cong \overline{HJ}$	2. Reflexive
3. $\overline{GH} \cong \overline{JK}$	3. Segment Subtraction Property

5. Given: $\angle NOP \cong \angle NPO$

$\angle ROP \cong \angle RPO$

Prove: $\angle NOR \cong \angle NPR$



Statements	Reasons
1. $\angle NOP \cong \angle NPO$	1. } Given
2. $\angle ROP \cong \angle RPO$	2. } Given
3. $\angle NOR \cong \angle NPR$	3. Angle Subtraction Property

Transitive Properties of Congruence

- Suppose that $\angle A \cong \angle B$ and $\angle A \cong \angle C$. Is $\angle B \cong \angle C$? YES
  - If angles (or segments) are congruent to the same angle (or segment), they are congruent to each other.
  - If angles (or segments) are congruent to congruent angles (or segments), they are congruent to each other.

Substitution Property

- (Solving for a variable x & *substituting* the value found for that variable.)

6. Given: $m\angle 1 + m\angle 2 = 90$,

$\angle 1 \cong \angle 3$

Prove: $m\angle 3 + m\angle 2 = 90°$

Statements	Reasons
1. $m\angle 1 + m\angle 2 = 90$	1. } Given
2. $\angle 1 \cong \angle 3$	2. } Given
3. $m\angle 1 = m\angle 3$	3. Def. of $\cong$ $\angle$s
4. $m\angle 3 + m\angle 2 = 90$	4. Substitution

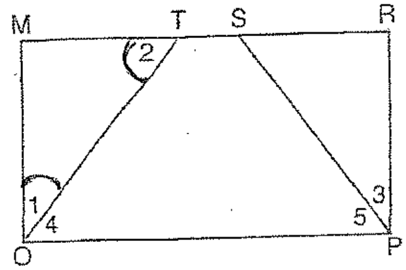
7. Given: $\angle 1 \cong \angle 2$

$\angle 1$ is comp. to $\angle 4$

$\overline{RP} \perp \overline{OP}$

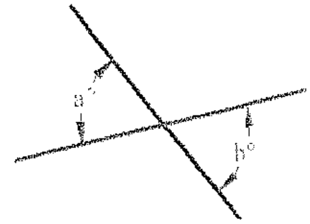
$\angle 4 \cong \angle 5$

Prove: $\angle 2 \cong \angle 3$



Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. 2. } Given
2. $\angle 1$ is comp to $\angle 4$	
3. $\angle 2$ is comp to $\angle 4$	3. Substitution
4. $\overline{RP} \perp \overline{OP}$	4. Given
5. $\angle RPO$ is a rt $\angle$	5. Def. of $\perp$
6. $\angle 3 + \angle 5 = \angle RPO$	6. Angle Addition Postulate
7. $\angle 3$ is comp to $\angle 5$	7. Def. of comp. $\angle$s
8. $\angle 4 \cong \angle 5$	8. Given
9. $\angle 3$ is comp to $\angle 4$	9. Substitution
10. $\angle 2 \cong \angle 3$	10. Congruent Complements theorem

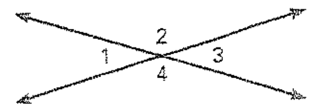
2.10 ANGLES FORMED BY INTERSECTING LINES



- Opposite rays – two collinear rays that have a common endpoint & extend in different directions

Vertical Angles

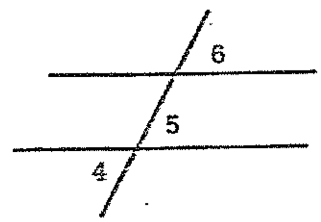
- Whenever two lines intersect, two pairs of vertical angles are formed.
- Two angles are vertical angles if the rays forming the sides of one & the rays forming the sides of the other are opposite rays.



∠1 and ∠3 are vertical angles.
∠2 and ∠4 are vertical angles.

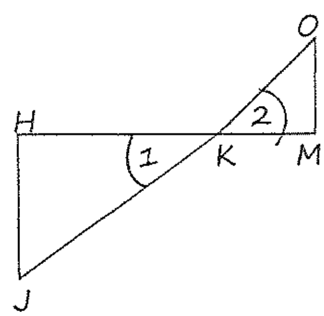
Vertical Angles Theorem ~ Vertical angles are congruent.

1. Given: $\angle 4 \cong \angle 6$
   Prove: $\angle 5 \cong \angle 6$



Statements	Reasons
1. $\angle 4 \cong \angle 6$	1. Given
2. $\angle 4 \cong \angle 5$	2. Vert. ∠s are ≅
3. $\angle 5 \cong \angle 6$	3. transitive Prop.

2. Given: $\angle O$ is comp. to $\angle 2$
   $\angle J$ is comp. to $\angle 1$
   Prove: $\angle O \cong \angle J$



Statements	Reasons
1. ∠O is comp to ∠2	1. } Given
2. ∠J is comp to ∠1	2. }
3. $\angle 1 \cong \angle 2$	3. Vert. ∠s are ≅
4. $\angle O \cong \angle J$	4. congruent complements thm

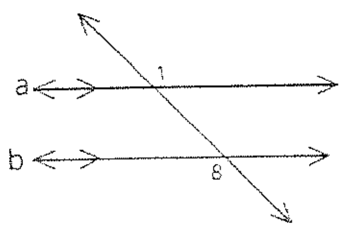
Theorems Involving Parallel Lines

- The Parallel Postulate
  - Through a point not on a line there is exactly one parallel to the given line.

- Theorems on Parallel Lines, Transversals &/or Angles
  - If two parallel lines are cut by a transversal, then any pair of the angles formed are either congruent or supplementary.

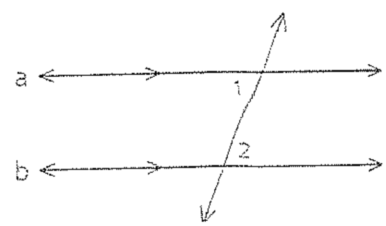
- Angles formed from Parallel Lines
  - If two parallel lines are cut by a transversal…

Alternate Exterior Angles



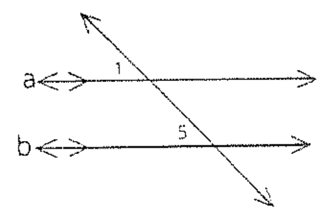
Congruent: $\angle 1 \cong \angle 8$

Alternate Interior Angles



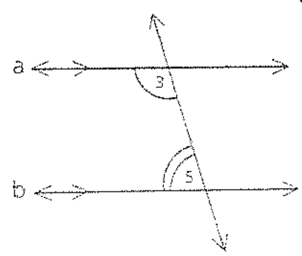
Congruent: $\angle 1 \cong \angle 2$

Corresponding Angles



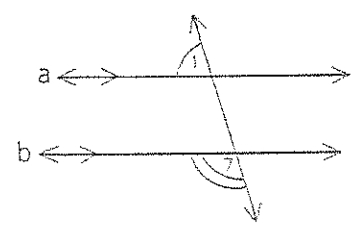
Congruent: $\angle 1 \cong \angle 5$

Same-Side Interior Angles



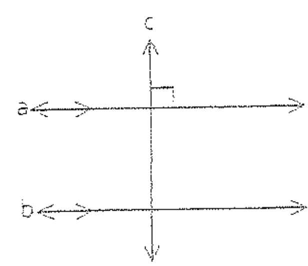
Supplementary: $m\angle 3 + m\angle 5 = 180°$

Same-Side Exterior Angles

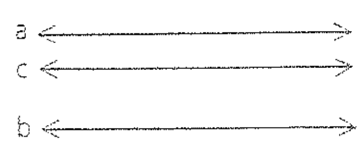


Supplementary: $m\angle 1 + m\angle 7 = 180°$

- In a plane, if a line is perpendicular to one of two parallel lines, it is perpendicular to the other.
  - If $a \parallel b$ and $c \perp a$, then $c \perp b$.



- If two lines are parallel to a third line, they are parallel to each other.
  - If $a \parallel b$ and $b \parallel c$, then $a \parallel c$.



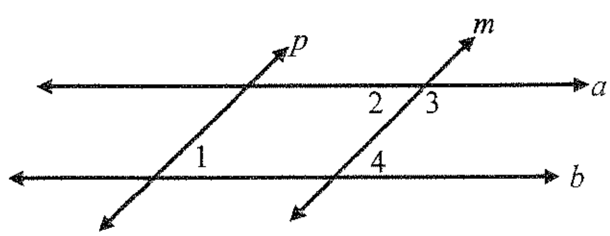
Theorems & Postulates Related to Parallel Lines

- **Corresponding Angles Postulate** ~ If a transversal intersects two parallel lines, then corresponding angles are congruent.
- **Converse of the Corresponding Angles Postulate** ~ If two lines and a transversal form corresponding angles that are congruent, then the two lines are parallel.
- **Alternate Interior Angles Theorem** ~ If a transversal intersects two parallel lines, then alternate interior angles are congruent.
- **Same-Side Interior Angles Theorem** ~ If a transversal intersects two parallel lines, then -same-side interior angles are supplementary.
- **Converse of the Alternate Interior Angles Theorem** ~ If two lines and a transversal form alternate interior angles that are congruent, then the two lines are parallel.
- **Converse of the Same-side Interior Angles Theorem** ~ If two lines and a transversal form same-side interior angles that are supplementary, then the two lines are parallel.

3. Given: $a \parallel b$

   $\angle 1$ is supplementary to $\angle 3$

   Prove: $m \parallel p$

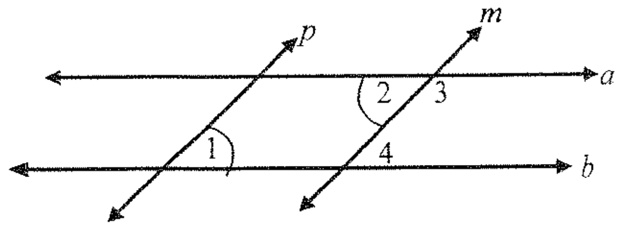


Statements	Reasons
1. $a \parallel b$	1. Given
2. $\angle 1$ is supp to $\angle 3$	2. " "
3. $\angle 3$ is supp to $\angle 4$	3. Same-side interior ∠s theorem
4. $\angle 1 \cong \angle 4$	4. Congruent Supplements thm
5. $m \parallel p$	5. Converse of the Corresponding ∠s Postulate

4. Given: $a \parallel b$

$\angle 1 \cong \angle 2$

Prove: $m \parallel p$



Statements	Reasons
1. $a \parallel b$	1. Given
2. $\angle 1 \cong \angle 2$	2.
3. $\angle 2 \cong \angle 4$	3. Alternate interior ∠s theorem
4. $\angle 1 \cong \angle 4$	4. Transitive Property
5. $m \parallel p$	5. Converse of the Corresponding ∠s Postulate