EXPONENTIAL, LOGISTIC & LOGARITHMIC FUNCTIONS
Formula Sheet

Properties of Exponents

$a^0 = 1$ $\qquad$ $a^m \bullet a^n = a^{mn}$

$(a^m)^n = a^{mn}$ $\qquad$ $(ab)^m = a^m b^m$

$a^{-m} = \dfrac{1}{a^m}$ $\qquad$ $\dfrac{a^m}{a^n} = a^{m-n}$ $\qquad$ $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

Properties of Logarithms

$\log_b 1 = 0$ $\qquad$ $\log_b b = 1$

$b^{\log_b x} = x$ $\qquad$ $\log_b b^y = y$

If $0 < b \neq 1$, $0 < a \neq 1$, & $x, R, S, > 0$, then:

$$y = \log_b x \Leftrightarrow b^y = x$$

Exponents & Radicals: $\sqrt[n]{x^m} = x^{m/n}$

Exponential Functions

$$f(x) = a \bullet b^x = a \bullet e^{kx}$$

<u>Growth</u>	<u>Decay</u>
b > 1 or k > 0	b < 1 or k < 0

a is always positive in growth/decay functions

Logistic Functions

$$f(x) = \frac{1}{1 + e^{-x}}$$

<u>Logistic Growth Functions</u>

$f(x) = \dfrac{c}{1 + a \bullet b^x}$ $\qquad$ $f(x) = \dfrac{c}{1 + a \bullet e^{-kx}}$

Decay: *if $b > 1$ or $k < 0$*

Basic Properties of Common Logarithms

base 10: $y = \log x$ iff $10^y = x$

Let x & y be real numbers with $x > 0$:

$\log 1 = 0$ $\qquad$ $\log 10 = 1$

$\log 10^y$ $\qquad$ $10^{\log x} = x$

Basic Properties of Natural Logarithms

base e: $y = \ln x$ iff $e^y = x$

Let x & y be real numbers with $x > 0$:

$\ln 1 = 0$ $\qquad$ $\ln e = 1$

$\ln e^y = y$ $\qquad$ $e^{\ln x} = x$

Properties of Logarithmic Functions

Product Rule:

$$\log_b(RS) = \log_b R + \log_b S$$

Power Rule:

$$\log_b R^c = c \log_b R$$

Quotient Rule:

$$\log_b \frac{R}{S} = \log_b R - \log_b S$$

Change-of-Base Formula:

$$\log_b x = \frac{\log x}{\log b} = \frac{\ln x}{\ln b}$$

One-to-One Properties (for solving equations)

For any exponential function: $f(x) = b^x$,
If $b^u = b^v$, then $u = v$.

For any logarithmic function: $f(x) = \log_b x$,
If $\log_b u = \log_b v$, then $u = v$.

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Exponential & Logistic Modeling & Applications Involving Logarithms

Exponential Population Model	Newton's Law of Cooling
$P(t) = P_0(1+r)^t$	$T(t) = T_m + (T_0 - T_m)e^{-kt}$

Measuring Sound – Decibels	Richter Scale – Magnitude	Measuring Acidity – pH
$\beta = 10\log\left(\frac{I}{I_0}\right)$	$R = \log\left(\frac{a}{T}\right) + B$	$pH = -\log[H^+]$

Mathematics of Finance

Interest Compounded:	Annually	*k* times per year	Continuously
P: principal r: fixed annual rate n: number of years t: time (not in years)	$A = P(1+r)^n$	$A = P\left(1+\frac{r}{k}\right)^{kt}$	$A = Pe^{rt}$

Annual Percentage Yield (APY)

<u>APY</u> – The percentage rate that, compounded annually, would yield the same return as the given interest rate w/the given compounding period; used to compare investments

$$APY = \left(1+\frac{r}{k}\right)^k - 1$$

Annuities – Future Value (FV)

<u>Annuity</u> – A sequence of equal periodic (n) payments (R)
Future Value =
all of the periodic payments **+** all interest

$$FV = R\frac{(1+i)^n - 1}{i}$$

Loans & Mortgages – Present Value (PV)

<u>Present Value</u> – The net amount of money put into an annuity

$$PV = R\frac{1-(1+i)^{-n}}{i}$$

<u>Misc.</u>

$$i = \frac{r}{k}$$

$$n = kt$$