

Definition:

- **Quadratic-Type Expression:** is an expression of the form $au^2 + bu + c$ where u is an algebraic expression. For example, the following are all quadratic-type expressions:

$$x^4 + 5x^2 + 6, \quad (x - 3)^2 - 2(x - 3) - 3, \quad x^8 - 7x^4 - 18.$$

Important Properties:

- If you recognize how the expression factors in its original form, then do so. If not, then use a u substitution to create a quadratic that is easier to factor. For example, for $x^4 + 5x^2 + 6$, if we let $u = x^2$ then we have

$$x^4 + 5x^2 + 6 = u^2 + 5u + 6 = (u + 3)(u + 2) = (x^2 + 2)(x^2 + 3).$$

- Only use the substitution if it helps you factor the expression.

Common Mistakes to Avoid:

- If you use the u substitution, you must substitute back to the original variable. So, if the original problem is in terms of x , make sure that your answer is also in terms of x .
- Be on the lookout for the difference of squares, the difference of cubes, and the sum of cubes. Remember that these can be factored further.

PROBLEMS

Factor Completely.

1. $x^4 - 5x^2 - 6$

$$\boxed{\frac{x^4 - 5x^2 - 6}{(x^2 - 6)(x^2 + 1)}}$$

NOTE: You could also use a u substitution on this problem. Letting $u = x^2$, we get

$$\boxed{\frac{x^4 - 5x^2 - 6}{u^2 - 5u - 6} = \frac{(u - 6)(u + 1)}{(x^2 - 6)(x^2 + 1)}}$$

2. $x^4 + 5x^2 - 36$

$$\boxed{\frac{x^4 + 5x^2 - 36}{(x^2 - 4)(x^2 + 9)} = \frac{(x - 2)(x + 2)(x^2 + 9)}{(x - 2)(x + 2)(x^2 + 9)}}$$

OR

Letting $u = x^2$, we get

$$\boxed{\frac{x^4 + 5x^2 - 36}{u^2 + 5u - 36} = \frac{(u - 4)(u + 9)}{(x^2 - 4)(x^2 + 9)} = \frac{(x - 2)(x + 2)(x^2 + 9)}{(x - 2)(x + 2)(x^2 + 9)}}$$

3. $x^4 + 15x^2 - 16$

$$\frac{x^4 + 15x^2 - 16}{(x^2 + 16)(x^2 - 1)}$$

$$\boxed{(x^2 + 16)(x - 1)(x + 1)}$$

OR

Letting $u = x^2$, we get

$$\frac{x^4 + 15x^2 - 16}{u^2 + 15u - 16}$$

$$\frac{(u + 16)(u - 1)}{(x^2 + 16)(x^2 - 1)}$$

$$\boxed{(x^2 + 16)(x - 1)(x + 1)}$$

4. $x^8 - 7x^4 - 18$

$$\frac{x^8 - 7x^4 - 18}{(x^4 - 9)(x^4 + 2)}$$

$$\boxed{(x^2 - 3)(x^2 + 3)(x^4 + 2)}$$

OR

Letting $u = x^4$, we get

$$\frac{x^8 - 7x^4 - 18}{u^2 - 7u - 18}$$

$$\frac{(u - 9)(u + 2)}{(x^4 - 9)(x^4 + 2)}$$

$$\boxed{(x^2 - 3)(x^2 + 3)(x^4 + 2)}$$

5. $x^6 + 4x^3 - 45$

$$\frac{x^6 + 4x^3 - 45}{(x^3 + 9)(x^3 - 5)}$$

OR

Letting $u = x^3$, we get

$$\frac{x^6 + 4x^3 - 45}{u^2 + 4u - 45}$$

$$\frac{(u + 9)(u - 5)}{(x^3 + 9)(x^3 - 5)}$$

6. $6x^6 - 19x^3 - 7$

$$\frac{6x^6 - 19x^3 - 7}{(3x^3 + 1)(2x^3 - 7)}$$

OR

Letting $u = x^3$, we get

$$\frac{6x^6 - 19x^3 - 7}{6u^2 - 19u - 7}$$

$$\frac{(3u + 1)(2u - 7)}{(3x^3 + 1)(2x^3 - 7)}$$

7. $5x^{10} - 19x^5 + 12$

$$\frac{5x^{10} - 19x^5 + 12}{(5x^5 - 4)(x^5 - 3)}$$

OR

Letting $u = x^5$, we get

$$\frac{5x^{10} - 19x^5 + 12}{5u^2 - 19u + 12}$$

$$\frac{(5u - 4)(u - 3)}{(5x^5 - 4)(x^5 - 3)}$$

8. $(x - 3)^2 - 2(x - 3) - 3$

$$\frac{(x - 3)^2 - 2(x - 3) - 3}{[(x - 3) - 3][(x - 3) + 1]}$$

$$\frac{(x - 3 - 3)(x - 3 + 1)}{(x - 6)(x - 2)}$$

OR

Letting $u = (x - 3)$, we get

$$\frac{(x - 3)^2 - 2(x - 3) - 3}{u^2 - 2u - 3}$$

$$\frac{(u - 3)(u + 1)}{[(x - 3) - 3][(x - 3) + 1]}$$

$$\frac{(x - 3 - 3)(x - 3 + 1)}{(x - 6)(x - 2)}$$