Quadratics Review

Multiplying Polynomials

- **The Distributive Property**
  - $a(b+c) = ab + ac$
    - An example: $3x^2(x+5)$

$$3x^2(x+5) = 3x^2(x) + 3x^2(5)$$
$$= 3x^2x^1 + 3 \cdot 5x^2$$
$$= 3x^3 + 15x^2$$

- **The FOIL Pattern:**

First, Outer, Inner, Last

$$(2x+3)(4x+1) = 8x^2 + 2x + 12x + 3 = 8x^2 + 14x + 3$$

FACTORING: THE PROCESS

- **Always Look for Common Factors**
  - Use Divisibility Rules to spot the GCF!
    - These rules let you test if one number is divisible by another – with little, to no, calculation!

DIVISIBLE BY...	IF...
2	THE LAST DIGIT IS EVEN: 0, 2, 4, 6, 8
3	THE SUM OF THE DIGITS IS DIVISIBLE BY 3
4	THE NUMBER FORMED BY THE LAST TWO DIGITS IS DIVISIBLE BY 4
5	THE LAST DIGIT IS 0 OR 5
6	THE NUMBER IS EVEN AND DIVISIBLE BY 3
8	THE NUMBER FORMED BY THE LAST THREE DIGITS IS DIVISIBLE BY 8
9	THE SUM OF THE DIGITS IS DIVISIBLE BY 9
10	THE LAST DIGIT IS 0

❖ Look for Special Cases Involving Binomials

$1^2 = 1$	$2^2 = 4$	$3^2 = 9$	$4^2 = 16$
$5^2 = 25$	$6^2 = 36$	$7^2 = 49$	$8^2 = 64$
$9^2 = 81$	$10^2 = 100$	$11^2 = 121$	$12^2 = 144$

- Difference of Two Squares: $a^2 - b^2$
  - Recognizing a Difference of Two Squares
    - There must be two terms – both perfect squares
    - There must be a minus sign between the two terms
- Factoring a Difference of Two Squares: $a^2 - b^2 = (a + b)(a - b)$
  - *An example:*

$4x^2 - 36$	
$4(x^2 - 9)$	ALWAYS LOOK FOR A COMMON FACTOR.
$4(x^2 - 3^2)$	WRITE AS $a^2 - b^2$
$4(x + 3)(x - 3)$	FACTORING USING PATTERN

$$x^2 - 9 = x^2 - 3^2 = (x - 3)(x + 3)$$

❖ Factoring Flow Chart

ALWAYS LOOK FOR A COMMON FACTOR FIRST!

↓

THEN LOOK AT THE NUMBER OF TERMS

↙ ↓ ↘

BINOMIAL → ↓ → IS IT A DIFFERENCE OF TWO SQUARES?

- ↙ NO
- ↘ YES ↓ $a^2 - b^2 = (a + b)(a - b)$

FOUR TERMS → ↓ → FACTOR BY GROUPING

TRINOMIAL → ↓ → DEFOIL

❖ DEFOIL – Factoring Trinomials: $ax^2 + bx + c$

- Multiply the first and last terms
- Find the factors (of the product in step 1) that add up to be the middle term
- Replace the middle term with these factors
- Factor by grouping

- *An example:* $\boldsymbol{x^2 + bx + c}$

$$n^2 + 5n - 24$$

$$n^2 \cdot (-24) = -24n^2$$

MULTIPLY THE FIRST AND LAST TERMS

$-24n^2$	
$8n \,\&\, -3n$	$5n$

FIND THE FACTORS (OF THE PRODUCT IN STEP 1) THAT ADD UP THE TO BE THE MIDDLE TERM

$$n^2 + 8n - 3n - 24$$

REPLACE THE MIDDLE TERM WITH THESE FACTORS

$$(n^2 + 8n)(-3n - 24)$$

$$n(n + 8) - 3(n + 8)$$

FACTOR BY GROUPING

$$(n + 8)(n - 3)$$

- *An example:* $\boldsymbol{ax^2 + bx + c}$

$$3x^2 - 4x - 7$$

$$3x^2 \cdot (-7) = -21x^2$$

MULTIPLY THE FIRST AND LAST TERMS

$-21x^2$	
$-7x \,\&\, 3x$	$-4x$

FIND THE FACTORS (OF THE PRODUCT IN STEP 1) THAT ADD UP THE TO BE THE MIDDLE TERM

$$3x^2 - 7x + 3x - 7$$

REPLACE THE MIDDLE TERM WITH THESE FACTORS

$$(3x^2 - 7x)(+3x - 7)$$

$$x(3x - 7) + 1(3x - 7)$$

FACTOR BY GROUPING

$$(3x - 7)(x + 1)$$

Solving Quadratic Equations by Factoring

- Solving Quadratic Equations by Factoring
  - Set the equation – written in standard form – equal to 0
    - $ax^2 + bx + c = 0$
  - Factor
  - Use the Zero-Product Property:
    - Let a and b be real numbers. If $ab = 0$, then $a = 0$ or $b = 0$.
    - Set each factor equal to 0 and solve

Examples:

ALREADY IN FACTORED FORM:

Solve $(x - 4)(x + 2) = 0$.

$(x - 4)(x + 2) = 0$		**Write original equation.**
$x - 4 = 0$ *or* $x + 2 = 0$		**Zero-product property**
$x = 4$ *or* $x = -2$		**Solve for x.**

▶ The solutions of the equation are 4 and −2.

TWO TERMS:

Solve $6n^2 = 15n$.

$6n^2 - 15n = 0$	**Subtract $15n$ from each side.**
$3n(2n - 5) = 0$	**Factor left side.**
$3n = 0$ *or* $2n - 5 = 0$	**Zero-product property**
$n = 0$ *or* $n = \frac{5}{2}$	**Solve for n.**

▶ The solutions of the equation are 0 and $\frac{5}{2}$.

THREE TERMS:

Solve the equation $x^2 + 3x = 18$.

$x^2 + 3x = 18$	**Write original equation.**
$x^2 + 3x - 18 = 0$	**Subtract 18 from each side.**
$(x + 6)(x - 3) = 0$	**Factor left side.**
$x + 6 = 0$ *or* $x - 3 = 0$	**Zero-product property**
$x = -6$ *or* $x = 3$	**Solve for x.**

▶ The solutions of the equation are −6 and 3.